# 2.10: Make Inferences–Normal Distribution

### Introduction

In this section of Lesson 2, you will learn about the properties of normal distributions and how to calculate z-scores and normal probabilities.

### Normal Distribution

When you learned about histograms, it was established that a histogram that was bell shaped and symmetric represented a normal distribution. A curve superimposed over the histogram (as shown in the following image) is called a density curve.

Because this curve is bell-shaped and symmetric, it is called a normal density curve. The area under the density curve will always be equal to one. In addition, the density curve always lies on or above the horizontal axis. Because of these two properties, the area of a region under the curve can be treated as a probability.

Let’s examine another example of a normal distribution. Recall that the mean occurs at the center of the normal distribution, so the population mean mu (µ) for the following example is 5.

If you look about halfway down one of the sides of our distribution, you find a point called an inflection point. The distance between the value that lines up with that point and the mean will be the population standard deviation sigma. Sigma in this example is the distance between 6.2 and 5, which is 1.2. The mean and standard deviation for a normal distribution will usually be different for each problem that you encounter.

Each data value from a normal distribution can be converted to a z-score.

Z-Score:  A measurement that indicates how many standard deviations a data value is from the mean.

If the z-score is positive, the corresponding data value is greater than the mean. If the z-score is negative, the corresponding data value is less than the mean. So the mean of the normal distribution will always have a z-score of zero because it is zero standard deviations from the mean. The data value that is one standard deviation higher than the mean will always have a z-score of 1. The data value that is one standard deviation lower than the mean will always have a z-score of -1.

The distribution created when the data values are converted to z-scores is called a standard normal distribution. The mean for the standard normal distribution will always be zero, and the standard deviation for a standard normal distribution will always be one.

### Calculating Z-Scores

The formula for calculating a z-score is z = (x-µ)/σ.

For our example, µ = 5 and σ = 1.2. Now let's use the z-score formula to find the z-score associated with the data value 6.2.

The resulting z-score of 1 makes sense because it was previously determined that 6.2 is one standard deviation higher than the mean for this example.

So you can see in our standard normal distribution that the mean is zero because the population mean of 5 corresponds to a z-score of 0. Also the z-score of 1 is in the same location in the standard normal distribution as the data value 6.2 is in the normal distribution.

### Finding the Area

The normal and standard normal distributions can also be used to find the area of a region under the density curve. Recall that an area under the density curve is equivalent to the probability of a randomly chosen data value being in that part of the distribution.

For example, let’s say you want to find the probability that a z-score is greater than one for this problem.  Follow these steps to find the probability:

1. Open the [normal probability applet](http://byuimath.com/apps/normprob.html) (bookmark this site if you have not yet done so).
2. Shade the right tail in the applet (because we are finding a "greater than" probability).
3. Enter the z-score of 1 in the box on the edge of the shaded region and press enter.
4. The area of the shaded region will be shown in the area box at the top. The area for this problem is 0.1587. Thus, the probability of a randomly chosen z-score being greater than 1 is 0.1587.  
     
   Note that this is the same thing as saying the probability of a random chosen data value being greater than 6.2 is 0.1587.

### Practice Problem

For this practice problem, you will use the same distribution with the following values:  
  
µ = 5, σ = 1.2, x = 2.9

First, try to find the z score that corresponds to a data value of 2.9 on your own.

Now compare your results to the solution shown below.

Next let’s find the probability that a randomly chosen data value is less than 2.9. Recall that this probability is equivalent to the probability of the z-score being less than -1.75 in the standard normal distribution. So you will find the area of the left-tail region under the density curve.

Follow these steps once again:

1. Open the [normal probability applet](http://byuimath.com/apps/normprob.html).
2. Shade in the left tail in the applet.
3. Enter the z-score of -1.75 in the box on the edge of the shaded region and press enter.
4. The area of the shaded region will be shown in the area box at the top. The area for this problem is 0.0401. Thus, the probability of a randomly chosen z-score being less than -1.75 is 0.0401.

### Summary

In this section of Lesson 2, you learned about about the properties of normal distributions and how to calculate z-scores and normal probabilities.

In the next section, you will learn about sampling distributions and the Central Limit Theorem.

### Supplemental Resource

[Lesson 2.10 – Inference: Normal Distribution](https://cdnapisec.kaltura.com/index.php/extwidget/preview/partner_id/1157612/uiconf_id/42438192/entry_id/1_o0miilbn/embed/dynamic)

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