

Math 108X

Table of Contents

Unit 1	1
Lesson 1: Preparing for Success	3
Lesson 2: Introduction to The Process	17
Lesson 3: Logical Reasoning	31
Lesson 4: Tools for Building Better Budgets and More	45
Lesson 5: Summarizing Data	65
Unit 2	93
Lesson 6: Visualizing Data	95
Lesson 7: Functions in Excel	131
Lesson 8: Functions and Their Graphs	167
Lesson 9: Change Over Time	199
Unit 3	221
Lesson 10: Making Predictions From Data	223
Lesson 11: Solving Systems of Equations	257
Lesson 12: Probability and Confidence Intervals	283
Lesson 13: Applying the Quantitative Reasoning Process	315

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Unit 1

Lesson 1: Preparing for Success
Lesson 2: Introduction to The Process
Lesson 3: Logical Reasoning
Lesson 4: Tools for Building Better Budgets and More
Lesson 5: Summarizing Data



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Lesson 1: Preparing for Success

Opening Video

In a BYU-Idaho devotional President Russel M. Nelson shared his thoughts about the potential of BYU-Idaho students. As you watch this four-minute clip, pay attention to what President Nelson says BYU-Idaho students can accomplish.

Russell M. Nelson “The Lord Uses the Unlikely to Accomplish the Impossible”

[Russell M. Nelson “The Lord Uses the Unlikely to Accomplish the Impossible” Transcript](#)

Introduction

As mentioned by President Nelson, “the Lord has more in mind for you than you have in mind for yourself.” In this course, Math 108X: Mathematical Tools for the Real World, we will learn tools that can help you be prepared to make informed decisions. As you make informed decisions you will be able to do as promised by President Nelson and “accomplish the impossible.”

Tips for Success

- *Read the Textbook* This online textbook has been prepared as a resource to help you succeed in this class. Please take the time to carefully read the assigned reading each week.
 - *Work through the Textbook* As you read the textbook, realize that you can't read a math textbook the same way you do a novel. As you complete the reading assignments, read with a paper and pencil and Excel nearby. As you read, take notes on important principles and concepts. When you encounter an example or interactive problem in the text, work the example out on paper and make sure you could repeat the example again on your own. When you are given instructions for how to do something in Excel, open Excel on your computer and follow along with the instructions.
 - *How Did I Do?* The online textbook, homework assignments, and Excel case studies include a "How Did I Do" button or tab that allows you to check your work. Use these tools appropriately. If you don't get a problem correct the first time, take time to check your work. Go back and look at the reading material and see if there are concepts you need to review.
 - *Work with Your Group* Whether you are an online student working with a group online or a campus student working with a group in class, work together as a group. You will benefit as you serve others by taking time to explain principles you understand. And you will learn from others as they explain principles you are struggling to learn. Be an active and willing participating in group activities and assignments.
 - *Notice Applications* As you work on readings, homework, case studies, and exams always consider how you could apply the principles you are learning to your own life.
-

The goal of this course is to help you use mathematical and statistical tools to make informed decisions. Life is full of opportunities to make decisions that impact our lives.

President Henry B. Eyring, Second Counselor in the First Presidency, said "The Lord knows both what He will need you to do and what you will need to know. He is kind and He is all-knowing. So you can with confidence expect that He has prepared opportunities for you to learn in preparation for the service you will give. You will not recognize those opportunities perfectly....But when you put the spiritual things first in your life, you will be blessed to feel directed toward certain learning, and you will be motivated to work harder." ¹

This course is an opportunity for you to learn to make informed decisions in your life so that you can be of better service to the Lord.

Algebraic Tools

Let's review some algebraic tools and properties that will be helpful this semester. We will review manipulating fractions, the order of operations, and properties of exponents.

Manipulating Fractions

Fractions commonly occur in mathematical situations. Remember that a fraction is made up of one whole number divided by another. The number on the top of the fraction is called the *numerator*. The number on the bottom of the fraction is called the *denominator*.

We will review the rules we follow when adding, subtracting, multiplying and dividing fractions. These examples are brief and are just intended to be a quick review of principles you have learned before.

Adding Fractions

In order to add fractions, you first find a common denominator. Then you add the numerator and keep the common denominator of the fraction the same. Let's look at some examples.

Example 1

Add

$$\frac{3}{4} + \frac{2}{3}.$$

Solution:

First, we find a common denominator. In this case, the denominators of the two fractions are 4 and 3. We need to find a denominator that is divisible by both 4 and 3. The smallest number that can be evenly divided by 4 and 3 is 12, so our common denominator is 12. We multiply both fractions by 1, written as a number divided by itself, so that we can get the common denominator of 12:

$$\begin{aligned}\frac{3}{4} + \frac{2}{3} &= \frac{3}{4} \cdot \frac{3}{3} + \frac{2}{3} \cdot \frac{4}{4} \\ &= \frac{9}{12} + \frac{8}{12} \\ &= \frac{9+8}{12} \\ &= \frac{17}{12}\end{aligned}$$

Notice that after we found a common denominator of 12, we added the numerators of the fractions, but kept the common denominator of 12 on the bottom of the fraction.

Example 2

Add

$$\frac{17}{9} + \frac{8}{3}.$$

Solution:

First, we find a common denominator. In this case, the denominators of the two fractions are 9 and 3. We need to find a denominator that is divisible by both 9 and 3. The smallest number that can be evenly divided by 9 and 3 is 9. So the common denominator is 9. We multiply the second fraction by 1, written as a number divided by itself, so that we get the common denominator of 9:

$$\begin{aligned}\frac{17}{9} + \frac{8}{3} &= \frac{17}{9} + \frac{8}{3} \cdot \frac{3}{3} \\ &= \frac{17}{9} + \frac{24}{9} \\ &= \frac{17 + 24}{9} \\ &= \frac{41}{9}\end{aligned}$$

Notice that after we found the common denominator of 9, we added the numerators of the fractions, but kept the common denominator of 9 on the bottom of the fraction.

Subtracting Fractions

Subtraction of fractions is very similar to adding fractions. We find a common denominator then subtract the numerators while keeping the denominator constant.

Example 3

Simplify:

$$\frac{3}{8} - \frac{13}{12}$$

Solution:

For this problem, we notice that the smallest number that is divisible by both 8 and 12 is 24. So we use a common denominator of 24.

$$\begin{aligned}\frac{3}{8} - \frac{13}{12} &= \frac{3}{8} \cdot \frac{3}{3} - \frac{13}{12} \cdot \frac{2}{2} \\ &= \frac{9}{24} - \frac{26}{24} \\ &= \frac{9 - 26}{24} \\ &= \frac{-17}{24}\end{aligned}$$

Example 4

Simplify:

$$\frac{5}{7} - \frac{5}{3}$$

Solution:

For this problem, we notice that the smallest number that is divisible by both 7 and 3 is 21. So we use a common denominator of 21.

$$\begin{aligned}\frac{5}{7} - \frac{5}{3} &= \frac{5}{7} \cdot \frac{3}{3} - \frac{5}{3} \cdot \frac{7}{7} \\ &= \frac{15}{21} - \frac{35}{21} \\ &= \frac{15 - 35}{21} \\ &= \frac{-20}{21}\end{aligned}$$

Multiplying Fractions

To multiply fractions, we just multiply the numerators and multiply the denominators.

Example 5

Multiply the fractions

$$\frac{2}{5} \text{ and } \frac{-4}{3}.$$

Solution:

We multiply the fractions by multiplying the numerators and multiplying the denominators:

$$\begin{aligned}\frac{2}{5} \cdot \frac{-4}{3} &= \frac{2 \cdot (-4)}{5 \cdot 3} \\ &= \frac{-8}{15}\end{aligned}$$

Dividing Fractions

To divide fractions, we *invert and multiply*. This means that we take the second fraction, flip it upside down, and then multiply the two resulting fractions.

Example 6

Do the following:

$$\frac{3}{4} \div \frac{6}{5}.$$

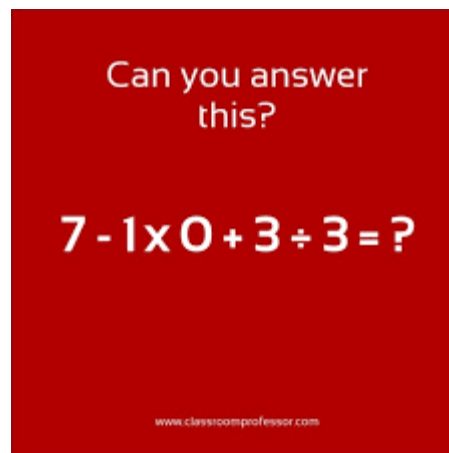
Solution:

We perform the division by inverting the second fraction and then multiplying.

$$\begin{aligned}\frac{3}{4} \div \frac{6}{5} &= \frac{3}{4} \cdot \frac{5}{6} \\ &= \frac{3 \cdot 5}{4 \cdot 6} \\ &= \frac{15}{24} \\ &= \frac{5}{8}\end{aligned}$$

Order of Operations

Have you ever come across a question like this on Facebook or other Social Media?



The comments on this type of social media posts typically shows great disagreement on the answer. The correct answer relies on the order of operations. The order of operations is a rule that helps us know the order in which to apply mathematical operations when simplifying an expression. The order of operations tells us to perform operations in the following order:

Step 1: Carry out operations enclosed in parentheses.

Step 2: Carry out operations involving exponents.

Step 3: Carry out any multiplication or division, in the order in which they appear from left to right.

Step 4: Carry out any addition or subtraction, in the order in which they appear from left to right.

The acronym P.E.M.D.A.S. is often used to describe the order of operations. P.E.M.D.A.S stands for parentheses, exponents, multiplication and division, addition and subtraction. Even though the M appears before the D and the A appears before the S, it is essential to remember that multiplication and division are carried out in the order they appear from left to right. It is similarly important to carry out addition and subtraction in the order they appear from left to right. The acronym P.E.M.D.A.S. might be better listed as P.E.(M.D.)(A.S.) with the parentheses reminding us that multiplication and division occur together, as do addition and subtraction.

Examples With Integers

Example 7

Apply the order of operations to the Facebook meme posted above.

$$7 - 1 \times 0 + 3 \div 3$$

Solution:

Steps 1 and 2 don't apply because there are no parentheses or exponents. So we start with Step 3 and perform all multiplication and division in the order in which they appear from left to right. Because $1 \times 0 = 0$ and $3 \div 3 = 1$, we can rewrite the equation as

$$7 - 0 + 1$$

We now go to Step 4 and carry out the addition and subtraction in the order in which they appear. This gives the answer:

$$7 - 0 + 1 = 7 + 1 = 8$$

Using the order of operations, we can clearly see that the answer is 8.

Example 8

Use the order of operations to simplify the following:

$$3(4 - 1)^2 - 10 \div 2 + 3$$

Solution:

In Step 1 we simplify the expression in the parentheses:

$$3(3)^2 - 10 \div 2 + 3$$

Then Step 2 tells us to apply any exponents:

$$3 \times 9 - 10 \div 2 + 3$$

We then go to Step 3 and apply multiplication and division in order from left to right:

$$27 - 5 + 3$$

Finally, at Step 4 we carry out the addition and subtraction in order from left to right:

$$25$$

Examples with Variables

Example 9

When using the order of operations in algebra, you often see variables in the expressions that you will need to simplify. Simplify the following expression:

$$3x - 4(x - 3) + 2x$$

Solution:

Step 1, we start by simplifying the expression in parentheses. In this case, $x + 3$ is already in the simplest form, so we can't simplify it further. Step 2 doesn't apply because there are no exponents. Step 3, perform multiplication (there is no division). We can multiply the 4 by the $x + 3$ using the distributive property to get:

$$3x - 4x + 12 + 2x$$

Step 4, addition and subtraction, but we can only combine like terms. This gives us $x + 12$ as the answer.

Example 10

Often, instead of using the \div symbol, we write expressions as fractions. If possible, simplify the following expression:

$$\frac{3x - 4}{3}$$

Solution:

It is important to realize that when an expression is written as a fraction, there are implied parentheses on the numerator and the denominator of the fraction. So this expression is actually

$$\frac{(3x - 4)}{(3)}$$

or

$$(3x - 4) \div (3)$$

This means that we have to subtract 4 from the $3x$ *before* we can divide by 3 (because Step 1 tells us to perform the operation in parentheses first, including implied parentheses). Because $3x - 4$ cannot be simplified, the entire expression is as simple as possible. Note that the 3 in the numerator and the 3 in the denominator can't "cancel" because the order of operations with the implied parentheses in the fraction bar tells us we must subtract the 4 before we divide.

Practice: Order of Operations

Problem 1

Apply the order of operations to simplify the following expression:

$$7 - 3(3 - 7)^2 + 3$$

How Did I Do?

$$-38$$

Problem 2

Apply the order of operations to simplify the following expression.

$$4(8 \div 4 - 5)^2 - 2$$

How Did I Do?

34

Problem 3

Apply the order of operations to simplify the following expression.

$$6(x - 2) + 8 - 2(3x + 2)$$

How Did I Do?

-8

Lesson Checklist

By the end of this lesson, you should be able to:

- Summarize behaviors that lead to student success.
 - Perform mathematical operations with fractions (addition, subtraction, multiplication, and division).
 - Apply the order of operations.
 - Use the rules of exponents.
-

¹ Henry B. Eyring, "[Education for Real Life](#)," Ensign, Oct. 2002, 18-19



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https://books.byui.edu/math_108x/lesson_1_preparing_for_success.

Lesson 2: Introduction to The Process

Opening Story

Lesson 2 - Opening Story
[L02 Opening Story Transcript](#)

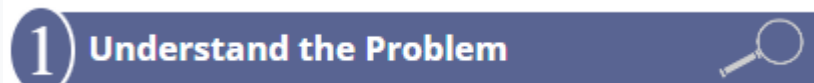
The Quantitative Reasoning Process

As you saw in the introductory video, Lucas and Amanda are facing a complicated decision that impacts their family life and finances now while they are going to school as well as their future. Should they take out loans to cover their expenses and finish school more quickly? Or should they have Lucas take another part-time job to stay debt free, but cause school to take an extra three to four semesters?

This scenario is a good example of a real-world problem where mathematics can be used to help make an informed decision. In this course we will be learning how the Quantitative Reasoning Process can help us make informed decisions in many different aspects of our lives.



The Quantitative Reasoning Process is a tool for making better, more informed decisions. It gives us a systematic way to include mathematical calculations in the decision-making process. Click on each banner below to see a description of that step of the Quantitative Reasoning Process.



The Quantitative Reasoning Process is used to make decisions. Start by carefully considering the situation and making sure you understand the problem and know what decision you need to make.

2 Identify Variables & Assumptions

Identify all the key aspects of the problem. What variables are important in the problem? What assumptions do you need to make to simplify the problem?

3 Apply Quantitative Tools

Use quantitative tools such as computation, algebra, statistics, and/or graphics to gain a better understanding of the situation.

4 Make an Informed Decision

Use the results of your quantitative calculations and analysis to help you make an informed decision about the situation.

5 Evaluate Your Reasoning

Look back at every step of the process. Review and evaluate your reasoning. Reflect on your answer. Does your answer make sense? If not, go back and make necessary changes.

In order to better understand how the Quantitative Reasoning Process works, let's use the process, step-by-step, to help Lucas and Amanda make an informed decision about their financial situation.

1 Understand the Problem

Lucas and Amanda have identified two possible plans for Lucas to complete school and need to decide which one is financially better for their family.

Part-Time Job Option: Lucas could take on additional part-time work while going to school.

Student Loan Option: They could use student loans to cover their expenses.

Lucas and Amanda have already determined that they value their education. Amanda has already graduated and they want to make an informed decision about the best way for Lucas to finish school. They want to follow the advice President Dieter F. Uchtdorf gave:

For members of the Church, education is not merely a good idea – it’s a commandment. We are to learn ‘of things both in heaven and in the earth, and under the earth; things which have been, things which are, things which must shortly come to pass; things which are at home, things which are abroad.’ Joseph Smith loved learning even though he had few opportunities for formal education...Joseph taught the Saints that knowledge was a necessary part of our mortal journey, for ‘man is saved no faster than he [gains] knowledge,’ and that ‘whatever principle of intelligence we attain...in this life, it will rise with us in the resurrection.’ During challenging times, it is even more important to learn. The Prophet Joseph taught, ‘Knowledge does away with darkness, [anxiety], and doubt; for these cannot exist where knowledge is.’¹

2 Identify Variables & Assumptions

Identify Key Variables

Now that we have an understanding of the real-world problem Lucas and Amanda are facing, we need to identify the variables in their problem.

*A **variable** is an element of a situation that has the potential to vary or change in value.*

To identify the key variables, we ask ourselves which elements of the situation have the potential to vary or change in value. In Lucas and Amanda’s situation, here are some variables they should consider:

- Tuition costs
- Living expenses
- Lucas' salary after graduation
- Student loan amount
- Student loan cost
- Annual interest rate
- Wages from part-time employment.

If we consider each of these variables, we realize they are elements of the situation that could change. For example, tuition prices could go up before Lucas graduates and their living expenses could go up or down. Lucas's salary after graduation will depend on the job he gets. Lucas and Amanda could take the maximum loan amount available or they could decide to take a smaller amount. The costs of the loan and the interest rate will vary based on the economy. Lucas's wages from part-time employment could change if Lucas gets a raise or he loses his job.

All of these variables are important for Lucas and Amanda to consider when making their decision. Lucas and Amanda will have to decide which variables are the most important.

Making Assumptions

Lucas and Amanda's situation involves several different variables that could take on different values. This makes the problem more complicated because there are so many different quantities that could change. In order to simplify any situation and make it more manageable, we must make some assumptions.

*An **assumption** is an idea or concept that is accepted as true.*

Often, our assumptions are related to our variables (that is why assumptions and variables are listed in the same step of the Quantitative Reasoning Process). We frequently assume that a variable will behave in a certain way. For example, Lucas and Amanda could assume that their living expenses will remain fairly constant for the time Lucas needs to finish school. This assumption will simplify their calculations and allow them to focus their attention on other aspects of the situation.

In addition to making assumptions about variables, we also make general assumptions about situations. For example, Lucas and Amanda will need to assume that they have explored all possible options for scholarships and grants that could help them pay for school. If additional scholarship or grant money were available, that might impact their decision about taking out student loans.

These are not the only assumptions they need to make.

Practice: Assumptions

Problem 1

Lucas and Amanda need to make some assumptions in order to make their decision. Based on the information provided in the introductory video, decide if the following assumption is appropriate or not.

Assumption: **Lucas' wages from his part-time job will stay the same.**

How Did I Do?

This is an appropriate assumption. They will use Lucas's current wages in their calculation. If his wages change, it might change their decision.

Assumption: **They will get an annual interest rate of 5% on their loans.**

How Did I Do?

This is an appropriate assumption. They need to know what rate they will get on their student loans, so they can do their calculations. 5% is a reasonable guess at what their interest rate will be.

Assumption: **Amanda will inherit her rich uncle's entire estate.**

How Did I Do?

This is not an appropriate assumption. This is probably unlikely to happen and not something they should count on, so it is not an appropriate assumption.

Assumption: **Lucas will be hired quickly after graduation.**

How Did I Do?

This is an appropriate assumption. They are basing their calculations on Lucas being hired immediately after graduation. If it takes longer, their conclusion might not be valid.

Assumption: **Lucas and Amanda need semiannual vacations.**

How Did I Do?

This is not an appropriate assumption. They are trying to avoid debt. Taking semiannual vacations would not be a good use of their financial resources.

The assumptions are important to understand because if one of these assumptions is false it has the potential to change the outcome of Lucas and Amanda's decision about part-time work or student loans. Let's look at what would happen if some of these assumptions turn out not to be true.

- Lucas and Amanda assumed that their expenses would remain fairly constant until Lucas finishes school. They will be basing all their calculations on this assumption. What if their expenses increase significantly over time? Then the calculations they use to make their decision might be inaccurate and they might make the wrong decision because it was based on a faulty assumption. For this assumption to be true, Lucas and Amanda will need to live frugally until Lucas graduates and until any loans are paid off. "Wherefore, do not spend money for that which is of no worth, nor your labor for that which cannot satisfy."²
- Lucas and Amanda assumed that Lucas will have good grades with either option. But what if Lucas's grades go down when he starts another part-time job and he ends up having to repeat some classes? This would increase the time it takes Lucas to graduate.

It is important to understand that if an assumption is faulty, the decision based on that assumption can be a poor decision. However, it is also important to remember that making assumptions is essential to decision making.

3 Apply Quantitative Tools

When we have identified the key variables and made assumptions, we are ready to use algebraic, statistical, and computational tools to help us make a decision. During this semester we will learn many of these quantitative tools that help us make informed decisions. The goal of this lesson is to help you learn the Quantitative Reasoning Process, so we will not focus as much on the quantitative tools as we will in later lessons. Many of the tools we refer to in this lesson will be discussed in more detail later in the semester.

Estimating School and Living Expenses

First, let's estimate how much Lucas and Amanda will need to pay for school and living expenses. BYU-Idaho has created an estimate of how much these costs should be for a BYU-Idaho student living in Rexburg. We will use the costs they provide for married campus students on the [Tuition and Cost of Attendance](#) website:

Item	Cost
LDS Tuition (Semester)	\$2,328
Loan Fees	\$24
Books and Supplies	\$275
Transportation	\$1162

Item	Cost
Personal Expenses (Including health insurance, clothing, cell phone, etc.)	\$1386
Housing and Food	\$5502
Average Cost Per Semester	\$10,677

We assume Lucas and Amanda and their two children, Sophia and Emmy³, will have expenses of \$10,677 per semester. Because there are three semesters in a year, we will multiply this value by 3 to find their living expenses for one year.

$$\$10,677 * 3 = \$32,031$$

Part-time Work and Scholarship Income

We need to estimate how much Lucas and Amanda make through their part-time work. We know that Amanda is doing some part-time tutoring to earn extra money and Lucas is working at a part-time job. Amanda works about 10 hours per week and charges \$15/hour for tutoring. Lucas works part-time on campus for 17 hours per week and makes \$12.50 per hour. Lucas also receives a \$1000 scholarship each semester.

Practice: Estimating Income

Problem 2

What will Lucas's weekly income be from part time work?

How Did I Do?

\$212.50

Hint: To find Lucas's weekly income, multiply the number of hours he works per week by his salary per hour.

What will Amanda's weekly income be from part time work?

How Did I Do?

\$150

Hint: To find Amanda's weekly income, multiply the number of hours she works per week by her salary per hour.

What will their combined weekly income be from part time work?

How Did I Do?

\$362.50

Hint: To find their combined weekly income, add Lucas' weekly income to Amanda's weekly income.

If they each work 42 weeks in a year, what will their combined annual income be from part-time work?

How Did I Do?

\$15,225

Hint: To find their combined annual income, multiply their combined weekly income by 42.

If Lucas attends school 3 semesters a year, what will his annual scholarship income be?

How Did I Do?

\$3000

Hint: To find Lucas's annual scholarship income, multiply his semester scholarship amount by 3.

What will be their combined annual income be from part-time work and scholarships?

How Did I Do?

\$18,225

Loan Payment

In the video, we hear Lucas tell Amanda that they qualified for \$6,200 per semester in loan money. However, Amanda replied by saying she thinks they could get by on a lot less. We can find how much money they need to borrow by subtracting their wages from part-time work and scholarships from their total annual living expenses.

$$\text{Annual Living Expenses} - (\text{Annual Wages and Scholarships}) = \$32,031 - \$18,225 = \$13,806$$

From this calculation, we see that they are short by about \$13,800 per year. If they take out student loans, Lucas wants to graduate as soon as possible so he will apply for Fast Grad and go to school all three semesters. This means they would need to borrow about \$4600 per semester.

If they take out a loan, Lucas will be able to graduate in 2 years (6 semesters). So when they graduate they will have a total loan of

$$\$4600 * 6 = \$27,600$$

Anytime we take out a loan, we have to pay the banking institution for the use of their money. The money we pay to the bank in addition to the amount we borrowed is called interest. One of our assumptions is that they could get an interest rate of 5%.

Lucas and Amanda also realize that if they don't take out a student loan, it will take them two additional years to graduate. But, when they do graduate, they would be debt free. So, if they want to be in the same situation by taking out student loans, they want to pay off the loan in 2 years.

Now that we know they will have a loan amount of \$27,600, an interest rate of 5%, and a loan term of 2 years, we can use the following online loan calculator to determine their monthly payment on the loan.

Online Loan Calculator

Their monthly loan payment is about \$1210. Calculate the amount to be paid to the bank each year by multiplying this monthly payment by 12 months.

$$\$1210 * 12 = \$14,520$$

How much do they need to make?

Now the question is whether or not Lucas and Amanda can afford to make the monthly loan payment. This depends on Lucas's take-home salary after graduation. Let's figure out exactly how much Lucas' take-home pay would need to be per year in order to be able to pay for their living expenses and make their loan payment. Keep in mind that this is just his take-home pay. His total salary would need to be higher in order to allow for paying taxes.

Total annual expense = Living expense + Loan payment

$$= \$32,031 + \$14,520 = \$46,551$$

This information could also be shown in the following graph.



4 Make an Informed Decision



Now that Lucas and Amanda have modeled the situation, they need to interpret the information they have gathered. They conclude that if they take out student loans and then repay them in the first two years, Lucas's salary after graduation would need to have a take-home amount of at least \$46,551. If his take-home amount is less than \$46,551, they could not repay the loan in two years. If his take-home amount is more than \$46,551, they could repay the loan and still have some money left.

What do you think Lucas and Amanda should do?

Please think carefully about your answer to this question. On your Lesson 2 Prep Quiz you will be asked to answer this question and justify your answer.

Lucas and Amanda will need to look carefully at the potential careers that correspond to Lucas's major. They will need to evaluate salaries and the cost of living in different locations as well as other family concerns. But now that they have used the Quantitative Reasoning Process they have a well-grounded foundation upon which to base their financial decisions.

As they make their decision they should also consider the following advice of President Clark Gilbert:

"Your academic stewardship at BYU-Idaho includes your time to graduation. Did you know that your probability of being an engaged future employee nearly doubles when you graduate on time and with modest college debt? Work hard in your classes. Take your education seriously. Never steal from the Lord or yourself by cheating in a class. Be a profitable servant, and return with increase the talents He has given you in this remarkable faculty and rich academic environment" [4](#)

5 Evaluate Your Reasoning



Once Lucas and Amanda have made a decision, it is important that they reflect on and evaluate their decision. They should ask themselves questions like

- Does this decision make sense?
- Are my assumptions reasonable?
- Are there any consequences of this decision I haven't considered?
- What types of events could happen that would violate my assumptions? (Perhaps an unplanned illness or the loss of a job.) Do I have a plan to deal with these events?
- Have I prayed about the decision?

Why is it so important that Lucas and Amanda make a careful, informed decision in this situation? Heber J. Grant answers this question:

"If there is any one thing that will bring peace and contentment into the human heart, and into the family, it is to live within our means. And if there is any one thing that is grinding and discouraging and disheartening, it is to have debts and obligations that one cannot meet."¹

Conclusion

The Quantitative Reasoning Process gives us a way to carefully consider difficult choices and to make informed decisions. Through the Quantitative Reasoning Process we identify the real-world situation, identify the assumptions and key variables, apply necessary quantitative tools, and then use our results to make informed decision.



The financial situation presented in this lesson is just one example of how the Quantitative Reasoning Process can help us make informed decisions. During this semester as we learn to make informed decisions in all aspects of our lives, we will be more prepared to serve others in our homes, churches, and communities.

Lesson Checklist

At the end of this lesson, you should be able to do the following:

- Identify the steps of the Quantitative Reasoning Process.
 - Identify assumptions
 - Identify key variables
 - Perform arithmetic to solve a real-world problem
 - Make and justify an informed decision.
 - Reflect and evaluate a decision.
-

¹ Dieter F. Uchtdorf, "[Two Principles for Any Economy](#)", Ensign, November 2009

² [2 Nephi 9:51](#)

³ Sophia and Emmy are named after famous female mathematicians: [Sophie Germain](#), [Sofia Kovalevskaya](#), and [Emmy Noether](#)

⁴ Clark Gilbert, "[Children of Promise](#)", September 13, 2016 Devotional

⁵ President Heber J. Grant, "[Gospel Standards](#)", comp. G. Homer Durham [1941], page 111



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Lesson 3: Logical Reasoning

Opening Story

As you watch this video, notice the different advertisements and media encountered by individuals in the video. Think about how often you encounter advertisements and media that try to influence the choices you make.

Lesson 3 - Opening Story

[L03-Opening Story Transcript](#)

Making Choices

Every day we make many choices. We choose which products to buy, which candidate will get our vote, how we will take care of our health, what to do with our money, and even what we will believe.

To make good decisions using quantitative reasoning we need to be familiar with the tools of the Quantitative Reasoning Process. We also need to understand some of the pitfalls that can misdirect us when we try to reason through decisions.

Logical Reasoning is the process of analyzing and evaluating arguments in order to draw valid conclusions.

Logical reasoning is key to making good decisions in all aspects of our lives. Last week, we introduced the Quantitative Reasoning model. Recall that the purpose of the model was to help you make informed decisions that are based on sound reasoning with quantitative evidence.



People frequently use logical reasoning to try to influence our decisions. When logical reasoning is used to support a specific conclusion, we call it a logical argument. Sometimes we use logical arguments to persuade someone to make a choice with a particular outcome. Logical arguments present reasons for accepting a conclusion.

Propositions and Conditional Statements

Logical arguments are created by combining together one or more propositions and conditional statements.

Propositions

*A **proposition** is a statement that is either true or false.*

The following statements are all propositions:

"The sun is the center of our solar system."

"Mammals have hair."

"Snakes are mammals."

"Caleb got an A on the exam."

Notice that each of these statements is either true or false. "The sun is the center of our solar system" is an example of a true proposition, while "Snakes are mammals" is an example of a false proposition. Both are propositions because they are statements that are either true or false.

Only statements can be propositions. For example, "Is it raining?" is not a proposition because it is a question instead of a statement. "Hey you! Come here!" is not a proposition because it is a command and has no truth value.

Conditional Statements

Conditional statements are a particular type of proposition.

*A **conditional statement** is a proposition with an if-then structure.*

The following statements are all conditional statements:

"If the basketball team wins this game, then they will go to the state championship."

"If you take out student loans, then you will have to repay them after you graduate."

"If ye love me, keep my commandments."

Conditional statements are made up of two parts: the "if" part and the "then" part.

*The "if" part of a conditional statement is called an **assumption**.*

*The "then" part of a conditional statement is called a **conclusion**.*

Some propositions don't use the if-then wording, but can be reworded as a conditional statement. These propositions are called **implied conditional statements**. Key words like "all", "whenever", "always", "never", "every time", and "none" help us identify implied conditional statements.

Example 1

"All dogs are mammals" is an implied conditional statement. We could reword it as "If an animal is a dog, then it is a mammal." The assumption of this implied conditional statement is "If an animal is a dog" and the conclusion is "then it is a mammal."

Example 2

"You are tired in the morning whenever you stay out late" is an implied conditional statement because it could be reworded as "If you stay out late, then you will be tired in the morning." In this example the assumption is "If you stay out late" and the conclusion is "then you will be tired in the morning."

2 Identify Variables & Assumptions

Notice that the second step of the Quantitative Reasoning Process is to "identify variables and assumptions." Understanding that a conditional statement is made up of an assumption and a conclusion helps us apply the Quantitative Reasoning Process correctly. For example, in the last lesson we saw Lucas and Amanda make the assumption that their expenses would stay fairly constant until Lucas finished school. The conclusions and decisions that we made last week were based on this assumption. We could write this as a conditional statement:

"If Lucas and Amanda's expenses stay fairly constant until Lucas finishes school, then their decision about student loans will be appropriate."

When making decisions it is important to identify the assumptions we are using. If Lucas and Amanda's health insurance suddenly increased, their expenses would not remain constant. This would then affect the decision they made using the Quantitative Reasoning Process.

Conditional statements show up in a variety of mediums including technical mediums such as computer codes and financial equations, and message-based mediums such as the scriptures, advertisements, and political messages. In message-based mediums, assumptions are not always stated directly. Sometimes it is assumed that everyone believes the same thing.

Think about the time when people believed the world was flat. That assumption led to logical thinking such as: If the world is flat, then it must have an edge. If there is an edge, then we could fall off the edge of the world. If we try to sail across the ocean, we will fall off the edge of the world.



Truth Values of Conditional Statements

Every proposition has a truth value. Since conditional statements are one type of proposition, this means every conditional statement is either true or false.

Consider the following conditional statement:

“If Anthony takes this pill, then his headache will go away.”

Is this conditional statement true or false?

Without having more information there isn't really a clear answer. The only way we can *know* if the statement is true or false is to have Anthony take the pill and see whether or not his headache goes away. If Anthony doesn't take the pill, then the best we can do is to *assume* the statement is true.

This is always the case. In order to determine the truth value of a conditional statement, we need to know the truth value of the assumption and the truth value of the conclusion.

In this example, there are four different possibilities:

- Anthony took the pill and his headache went away.
- Anthony took the pill and his headache did not go away.
- Anthony did not take the pill and his headache went away.
- Anthony did not take the pill and his headache did not go away.

Only one of these possibilities would make the conditional statement false. If Anthony took the pill and his headache did not go away, then the conditional statement was false.

The conditional statement can only be true if Anthony took the pill and his headache went away.

If Anthony didn't take the pill, then we have no way of knowing if his headache would have gone away. In this situation we follow the same “innocent until proven guilty” strategy that is used in a court of law. Because Anthony didn't take the pill we have no way of knowing if it would have made his headache go away. Therefore, we give the claim the benefit of the doubt and assume the conditional statement is true.

This example demonstrates three important principles about the truth value of conditional statements:

- A conditional statement is FALSE when the assumption is true and the conclusion is false.
- A conditional statement is TRUE when the assumption is true and the conclusion is true.
- A conditional statement is *assumed* to be TRUE when the assumption is false. (The conclusion could be either true or false.) Because the assumption is false, there is no way to tell whether the conditional statement is true, so we give the benefit of the doubt and assume the conditional statement to be true.

Conditional statements are called *conditional* because their truth value depends on the truth of the assumption. The assumption tells us the *conditions* that have to be true for the conclusion to be guaranteed to be true.

In a true conditional statement, if the assumption is true then the conclusion is guaranteed to always be true.

It is important to understand that the Quantitative Reasoning Process will lead to a conclusion that is based on a conditional statement. When we make a decision using the Quantitative Reasoning Process, it is only guaranteed to be a good decision if the assumptions we used are found to be true.

If our assumptions are false, then our conclusion may or may not be accurate. **Anytime we make an informed decision using quantitative reasoning, it is important to be aware that our conclusion is only guaranteed to be true if the assumptions we made are accurate and true.**

Let's look at a few other examples of conditional statements.

Example 3

Under what conditions would the following conditional statement be false?

"If the basketball team wins this game, then they will go to the state championship."

Solution: As stated in the principle given above, a conditional statement is false if the assumption is true and the conclusion is false. In this example that would mean that the basketball team wins this game (so the assumption is true) but they don't get to go to the state championship (the conclusion is false).

Example 4

Using the same conditional statement from example 1, let's say that the basketball team loses the game, but they still end up getting to go to the state championship. Is the conditional statement true or false?

Solution: The conditional statement as a whole is assumed to be true. In this case, the assumption of the conditional statement is false (they didn't win the game), so the conclusion may or may not be true (they might get to go to the state championship and they might not). Either way, the conditional statement is assumed to be **true** because the assumption is false. We don't know what would have happened if they had won the game, so we give the benefit of the doubt to the original conditional statement and assume the statement to be true.

Practice: Truth Values

Problem 1

Consider the following conditional statements and scenarios. Is each conditional statement true or false?

Conditional statement: If you get a dog, then your daughter will feed it every day.

Scenario: You get a dog, but your daughter only feeds it a few times a week (you have to feed it the rest of the time).

How Did I Do?

Since you got a dog but your daughter didn't feed it every day, the conditional statement is **false**.

Conditional statement: If you buy a new car, then you will get better gas mileage.

Scenario: You don't buy a new car and your gas mileage stays the same.

How Did I Do?

Since you didn't buy a new car, the conditional statement is **assumed to be true**.

Conditional statement: If you exercise for 30 minutes a day, then you will lose weight.

Scenario: You exercise every day, but because you gain muscle mass you actually end up gaining weight.

How Did I Do?

Since you exercised daily and gained weight, the conditional statement is **false**.

Conditional Statements in the Scriptures

Looking for logical structures can help us better understand all types of logical arguments, including those found in the scriptures. Many of the conditional statements found in the scriptures are promises. If we do our part to make the assumption true, the conclusion is guaranteed to follow.

Here are some examples:

Example 5

Then said Jesus to those Jews which believed on him, if ye continue in my word, then are ye my disciples indeed;¹

The assumption in this scripture is “if you continue in my word” and the conclusion is “then are ye my disciples indeed.” Because we know this scripture is true, this tells us if we continue to read, study, and follow the words of the Savior, then we are guaranteed to be His disciples.

Example 6

If ye love me, keep my commandments.²

The assumption in this scripture is “if ye love me” and the conclusion is “keep my commandments”. Because we know this scripture is true, we know if we love the Savior, we will keep his commandments.

Example 7

Then if our hearts have been hardened, yea, if we have hardened our hearts against the word, insomuch that it has not been found in us, then will our state be awful, for then we shall be condemned.³

The assumption in this scripture is “if we have hardened our hearts against the word” and the conclusion is “then will our state be awful, for then we shall be condemned.” Because we know this scripture is true, we know if we harden our hearts against the word, we are guaranteed to be condemned.

Example 8

In addition to finding conditional statements in the scriptures, we often see them in the words of the prophets. Consider the following statement made by President Gordon B. Hinckley:

“If we will give...service [to others], our days will be filled with joy and gladness.”⁴

The assumption in this statement is “if we will give service to others” and the conclusion is “our days will be filled with joy and gladness.” This promise from a prophet guarantees that we will have days filled with joy and gladness if we give service to others. Those who do not choose to serve others may or may not find that same joy and gladness. It is possible they will, but it is only guaranteed to those who give service to others.

Activity

Find a scripture or gospel quote that uses a conditional statement. Conduct a similar analysis and see what you learn about that quote by considering the logical structure of the argument. You will be asked to share this scripture in your preparation homework.

Strength of Arguments

As we have seen so far in this lesson logical reasoning is an essential part of the Quantitative Reasoning Process. It is essential that we clearly understand the assumptions we are making and realize that our conclusions are only guaranteed to be accurate if our assumptions are true. Additionally, it is important to understand persuasive logical arguments we are presented with in the news, social media, and the internet.

In the opening video for this lesson you saw several ads and commercials that used faulty reasoning. Logical arguments that use faulty reasoning are very common. We call this type of argument logical fallacies.

*A **logical fallacy** is an argument that is based on an error in reasoning.*

In particular, we will discuss four types of logical fallacies. There are many others, but these four are some of the most common:

- Improper generalization
- Appeal to emotion
- Personal attacks
- Alternative explanations

Improper Generalization

The weight loss ad from the introductory video is a great example of an argument that includes the improper generalization fallacy.



CALL 555-0925

Lose 50 lbs
FAST
without dieting!

Others have done it and
YOU CAN TOO!

The advertisement implies that because this program has worked for others, it will work for you too. The ad is not just congratulating the individual on their weight loss. It is inviting us to join them in weight loss through this program.

*The **improper generalization** fallacy refers to an argument that concludes that because one individual or group experiences a certain result, then everyone should experience the similar results.*

The weight loss program may have worked for the individuals referred to in the ad, but that doesn't mean it will work for everyone. The advertisers are trying to get you to spend your money on their program.

Appeal to Emotion

Another common fallacy is an appeal to emotion

*The **appeal to emotion** fallacy is an argument that claims something is true because of an emotional reason unrelated to the argument.*

The animal shelter ad shown in the introductory video is a good example of this.



This ad includes a picture of a dog and a tagline that says: “Adopt today! A pet is waiting for you.” The image and the words combine to provide an emotional appeal to encourage pet adoption. Note that the advertisers are trying to influence your actions by appealing to your emotions.

Another example of an appeal to emotion is shown in the following ad for Michelin tires.



Including a baby in this advertisement connects to your emotions by appealing to your desire to protect children. Rather than telling you why Michelin tires are better than other tires, it implies that Michelin tires will help you keep your baby safe. Note, however, that it gives no evidence for this claim.

Personal Attacks

In the introductory video you heard a political ad promoting Debbie Williams as the mayor of Garden City. This ad is an example of a personal attack. The argument presented did not provide any logical reasons to vote for Debbie Williams; it just attacked Bryce Phillips, the current mayor of Garden City. The attacks were of a personal nature rather than attacking his policies.

A personal attack fallacy is an argument that is based on attacking the person making the argument, rather than critiquing the argument on its own merits.

Here is another example of a political ad that uses a personal attack.



This ad is from a State Senator election in Montana in 2014. Notice that, among other things, the ad claims “Tonya Shellnutt opposes holding rapists and abusers accountable for their crimes.” This is clearly an exaggeration that portrays Ms. Shellnutt in a poor light. It is unlikely that Ms. Shellnutt wants rapists and abusers to have no consequences for their actions, and equally unlikely that she would like Montana women and families to be in danger. The ad attacks Ms. Shellnutt personally because of her position on a particular piece of legislation, instead of addressing the actual law to which it refers.

Alternative Explanations

Did you know that more drownings occur in months where ice cream consumption is high? This must mean that ice cream consumption causes drowning, right?

Rather than making that conclusion, we should consider whether there is an alternative explanation.

*The **alternative explanation** fallacy refers to an argument where a conclusion is made without adequately considering other explanations that might explain the observed effect.*

In this example, we should consider the weather. People tend to eat ice cream more often in warmer weather. And people go swimming more when the weather is warmer. So the increase in drowning deaths is most likely due to an increase in the number of people swimming during warmer weather. It has nothing to do with ice cream.

We saw another example in the introductory video where the newspaper headline claimed: “Soda a Day Leads to Heart Attacks in Men.”⁵ Researchers found a correlation between soda drinking and heart attacks. But just knowing there is a correlation does not mean the researchers can conclude that drinking a soda every day causes the heart attack. There may be other alternative explanations.

Another historical example relates to smoking and lung cancer. Prior to the 1900s, lung cancer was very rare. However, as smoking became more popular, doctors noticed an increase in the occurrence of lung cancer. In the 1920s and 1930s, doctors started noticing a correlation between smoking and lung cancer. However many alternative explanations were suggested, especially by the tobacco industry. In the 1960s, only about one-third of US doctors believed that lung cancer was caused by smoking. Before concluding that smoking causes lung cancer, doctors had to rule out any possible alternative explanations. This took many carefully designed scientific studies over many years. Some of those studies looked at the prevalence of smoking in different areas of the world and compared it to the prevalence of lung

cancer in that area. In this example, researchers were eventually able to eliminate alternative explanations and show that most cases of lung cancer are indeed caused by smoking.

Some references:

- <http://tobaccocontrol.bmj.com/content/21/2/87.full>
- <https://www.gapminder.org/videos/lung-cancer-statistics/>

Conclusion

This lesson highlights the importance of understanding logical arguments when making decisions. In the Quantitative Reasoning Process we make assumptions. It is important to keep in mind that the validity of the decisions we make are conditional. If the assumptions are true, it is guaranteed that our conclusions are true.

We also need to be aware of logical fallacies when we examine logical arguments. Logical fallacies such as improper generalizations, appeals to emotion, personal attacks, and alternative explanations can lead to invalid conclusions.

Lesson Checklist

By the end of this lesson you should be able to do the following:

- Identify conditional statements.
- Identify scriptural conditional statements
- Identify the assumptions and conclusions of a conditional statement
- Determine the truth of a conditional statement.
- Identify key assumptions used in a given model of a real-world situation.
- Identify potential flaws in quantitative reasoning, including
 - improper generalization.
 - alternative explanations.
 - appeals to emotion.
 - personal attacks.

¹ [John 8:31](#)

² [John 14:15](#)

³ [Alma 12:13](#)

⁴ Gordon B. Hinkley. [Giving Ourselves to the Service of the Lord](#). Ensign, March 1987

⁵ CBS News. [Soda a Day May Lead to Heart Attacks in Men](#). July 25, 2013.



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Lesson 4: Tools for Building Better Budgets and More

Opening Story

Lesson 4 - Opening Story

[L04 Opening Story Transcript](#)

Budgeting

As we saw in the video, different people have different approaches to budgeting. Budgeting is an important tool for maintaining financial health. In order to make any informed decision involving our finances we need to have a budget. Consider the council given by President Thomas S. Monson:

*"Perhaps no counsel has been repeated more often than how to manage wisely our income...Too many in the Church have failed to avoid unnecessary debt. They have little, if any, financial reserve. The solution is to budget, to live within our means, and to save some for the future."*¹

President Spencer W. Kimball said,

*"Every family should have a budget...We have to know approximately what we may receive and we certainly must know what we are going to spend."*²

In this lesson we will be learning some computation tools that can help us build a budget. These tools will allow us to carefully consider the income we receive and what expenses we have. However, these tools will be helpful for more than just budgeting.



Step 3 in the Quantitative Reasoning Process is to apply quantitative tools. Applying quantitative tools allows us to do calculations that help make an informed decision. Some common quantitative tools include percentages and units connected to numbers. Most of the numbers we encounter outside of the classroom come in connection with words

and units rather than in disconnected math equations. It is important to be able translate English words into math symbols to build equations.

English-to-Math Translations

Understanding how to translate English words into mathematical equations is essential to problem solving. Some common English-to-math translations are found in the following table.

English Word	Math Symbol	English example	Math Example
sum of	+	The sum of seven and nine	$7 + 9$
difference of	-	The difference of thirteen and six	$13 - 6$
product of	x or \cdot	The product of five and two	5×2
quotient of	\div or /	The quotient of eight and four	$8 \div 4$
is (are, was, were, will be)	=	Sixteen is the sum of 7 and 9	$16 = 7 + 9$
of	x or \cdot	Two-thirds of 99	$\frac{2}{3} \times 99$
per	\div or /	Ten miles per hour	$\frac{10 \text{ miles}}{\text{hour}}$
percent	divide by 100 ("per" = divide; "cent" = 100)	Nine percent	$\frac{9}{100}$
a percentage more than	x (1 + percentage written as a decimal)	Twenty percent more than thirteen	$13 \times (1 + 0.20)$
a percentage less than	x (1 - percentage written as a decimal)	Twenty percent less than thirteen	$13 \times (1 - 0.20)$

Let's look at some examples of converting English words to mathematical symbols.

English-to-Math Examples

Example 1

Translate the English words "The sales tax is 6% of the original price" to math symbols.

Solution:

The key words in this sentence are "is", "%", and "of". By changing "is" to =, "%" to division by 100, and "of" to multiplication we get:

$$\text{The sales tax} = \frac{6}{100} \times \text{the original price}$$

$$\text{The sales tax} = \frac{6}{100} \times \text{the original price.}$$

Example 2

Translate the English words "The mortgage on my new home is 10% more than the mortgage on my old home" to math symbols.

Solution:

The key words in the sentence are "is" and "10% more than." Using the information in the table, we change the sentence to:

$$\text{new mortgage} = \text{old mortgage} \times (1 + 0.10)$$

$$\text{new mortgage} = \text{old mortgage} \times (1 + 0.10).$$

Example 3

Translate the English words "The sum of my power bill and my phone bill is eight percent of my income" to math symbols.

Solution:

The key words in the sentence are "sum of," "is," "percent," and "of." Using the information in the table we change the sentence to:

$$\text{power bill} + \text{phone bill} = \frac{8}{100} \times \text{income}$$

Once we understand how to translate English words to mathematical symbols, we can use the mathematical equations to do helpful calculations. Here are some examples:

Solving Examples

Example 4

The sales tax is 6% of the original price. The original price is \$10.00. Find the sales tax.

Solution:

First we convert the English sentence to math symbols (see Example 1):

$$\text{sales tax} = \frac{6}{100} \times \text{original price}$$

Since we know the original price is \$10 the equation gives us a way to find the sales tax.

$$\text{sales tax} = \frac{6}{100} \times \$10$$

Now change the fraction to a decimal:

$$\text{sales tax} = 0.06 \times \$10$$

Finally, multiply out the right side of the equation

$$\text{sales tax} = \$0.60$$

Example 5

The sales tax is 6% of the original price. We know the sales tax is \$0.90. Find the original price.

Solution:

Like the previous problem, we start by converting the English sentence to math symbols:

$$\text{sales tax} = \frac{6}{100} \times \text{original price}$$

Since we know sales tax is \$0.90, we can solve for the original price.

$$\$0.90 = 0.06 \times \text{original price}$$

Divide both sides of the equation by 0.06 to isolate the "original price."

$$\frac{\$0.90}{0.06} = \text{original price}$$

Divide the left side of the equation:

$$\$15.00 = \text{original price}$$

Example 6

After sales tax, the final price of a sweater is 5% more than the price on the price tag. The price tag on a sweater is \$35, what is the final price of the sweater?

Solution:

"The final price is 5% more than the price tag." This sentence translates to the mathematical equation:

$$\text{final price} = \text{price tag} \times (1 + 0.05)$$

Since the price tag is \$35, this gives us the equation we need to find the final price:

$$\begin{aligned}\text{final price} &= \$35 \times (1.05) \\ &= \$36.75\end{aligned}$$

Example 7

After sales tax, the final price of a sweater is 5% more than the price on the price tag. If the final price was \$15.25, find the price on the price tag.

Solution:

Using the same equation as in Example 6:

$$\text{final price} = \text{price tag} \times (1 + 0.05)$$

Since the final price was \$15.25, we can substitute the value for the final price and solve for the price tag:

$$\$15.25 = 1.05 \times \text{price tag}$$

$$\frac{\$15.25}{1.05} = \text{price tag}$$

$$\$14.52 = \text{price tag}$$

Example 8

Sometimes people purchase items that are on sale to keep their expenses within their planned budget.

ONE DAY SALE! SALE ENDS TONIGHT!

Take an Additional
30% OFF
SITEWIDE!
NO MINIMUMS. NO EXCLUSIONS.

Suppose we need to purchase a new printer with an original purchase price of \$64.99. The sales price is 30% less than the original price of \$64.99. Find the sales price and the dollar amount saved.

This example shows there can be more than one way to solve a problem.

Solution 1

This sentence translates to the following equation:

$$\begin{aligned}\text{sales price} &= (1 - 0.30) \times \text{original price} \\ &= 0.70 \times \$64.99 \\ &= \$45.49\end{aligned}$$

The total dollars saved would be the difference between the original price and the sales price.

$$\text{dollars saved} = \text{original price} - \text{sales price}$$

Substituting the dollar amounts that we know into the math equation yields the following:

$$\begin{aligned}\text{dollars saved} &= \$64.99 - \$45.49 \\ &= \$19.50\end{aligned}$$

Solution 2

We could also say that the savings will be 30% of the original cost. The words in this sentence translate to:

$$\begin{aligned}\text{savings} &= 0.30 \times \text{original cost} \\ &= 0.30 \times \$64.99 \\ &= \$19.50\end{aligned}$$

The sales price will be the original price minus the dollars saved.

$$\begin{aligned}\text{sales price} &= \$64.99 - \$19.50 \\ &= \$45.49\end{aligned}$$

Practice: English to Math Translations

Problem 1

"30 percent of the number of girls in the preschool is the sum of the number of boys and the number of teachers."

Arrange the choices below in the right order needed to translate the above English sentence to math symbols.

1. +
2. number of girls
3. number of boys
4. 0.30
5. number of teachers
6. =
7. x

How Did I Do?

0.30 x number of girls = number of boys + number of teachers.

Problem 2

"The price of a new pair of shoes is 9 percent more than the price of a new shirt. If the new shoes cost \$24, what is the cost of the new shirt?"

How Did I Do?

\$22.02

Hint: Translate the English to the math sentence: shoes = shirt \times (1+0.09).

Problem 3

A grocery store is offering Roma tomatoes. The sale price is thirty percent less than the original price. If the original price for one pound of tomatoes is \$1.24, how much will you save if you purchase one pound of tomatoes at the sale price?

How Did I Do?

\$0.37

Hint: Remember to calculate for the savings, not the sales price.

Units and Unit Conversions

The units connected to a number tell us what the number counts or measures. Consider the following examples: 10 doors, \$500, 80 mph, 10 K. The units are respectively doors, dollars, miles per hour (or miles/hour), and kilometers.

Practice: Units

Problem 4

Identify the units in this scenario: "Tuition and books are about \$2000 every semester."

- ☐ semesters
- ☐ dollars per semester
- ☐ number of books
- ☐ semesters per dollar
- ☐ dollars

Problem 5

Identify the units in this scenario: "The exchange rate to convert from American dollars to Mexican pesos."

- ☐ pesos
- ☐ dollars per peso
- ☐ pesos per dollar
- ☐ dollars
- ☐ pesos times dollars

Problem 6

Identify the units in this scenario: "Every second, the dam releases 198 cubic meters of water into the river."

How Did I Do?

cubic meters per second or cubic meters/second

Problem 7

Identify the units in this scenario: "The cost to carpet a 12 foot by 10 foot room is \$325."

How Did I Do?

dollars per square foot or dollars/square foot

Sometimes we need to convert an amount of something with one unit to an equivalent amount expressed with different units. We want to change the way we express things so that the actual amount or quantity stays the same but the units change.

Example 9

The race car drove 85,800 feet in 5 minutes. Find how fast it was traveling in miles per hour.

Solution:

We will need to change the units, but the actual distance and time the car traveled should not change. We will just be expressing them in different units. We are going to use the multiplicative identity property to change the units. The multiplicative identity property tells us that if you multiply a quantity by 1, the quantity does not change.

We know that 60 minutes = 1 hour. Even though the numbers are different and the units are different the quantity of time is exactly the same. If we stack this equality relationship as a fraction, we get

$$\frac{60 \text{ minutes}}{1 \text{ hour}} = 1$$

We can even swap the numerator and denominator to get

$$\frac{1 \text{ hour}}{60 \text{ minutes}} = 1$$

Now we will use this relationship, called a unit conversion rate, to change our units from minutes to hours.

$$\frac{85,800 \text{ feet}}{5 \text{ minutes}} \times \frac{60 \text{ minutes}}{1 \text{ hour}}$$

When we multiply two fractions we multiply the two numerators and multiply the two denominators to get the product of the fractions.

Now we have

$$\frac{85,800 \text{ feet} \times 60 \text{ minutes}}{5 \text{ minutes} \times 1 \text{ hour}}$$

We have the units "minutes"/"minutes" which is equal to 1. We can now cancel the unit "minutes" in the numerator and in the denominator because multiplying by 1 does not change the quantity of time.

$$\frac{85,800 \text{ feet} \times 60}{5 \times 1 \text{ hour}} = \frac{5,148,000 \text{ feet}}{5 \text{ hours}} = \frac{1,029,600 \text{ feet}}{1 \text{ hour}}$$

We have our time unit correct so we only need to change our distance unit from feet to miles. There are 5280 feet in one mile so we will either use 1 mile/5280 feet or 5280 feet/1 mile as our unit conversion tool. Because "feet" is in the numerator of the fraction we want to change, we will want "feet" in the denominator of the unit conversion tool. This way we can have "feet"/"feet" and we can cancel the unit "feet."

$$\begin{aligned} &= \frac{1,029,600 \text{ feet}}{1 \text{ hour}} \times \frac{1 \text{ mile}}{5280 \text{ feet}} = \frac{1,029,600 \text{ feet} \times 1 \text{ mile}}{1 \text{ hour} \times 5280 \text{ feet}} \\ &= \frac{1,029,600 \text{ miles}}{5280 \text{ hours}} \\ &= \frac{195 \text{ miles}}{1 \text{ hour}} \end{aligned}$$

Thus the race car was traveling at a speed of 195 miles per hour.

Practice: Unit Conversions

Problem 8

The school bus traveled at 25 miles per hour. Choose all of the correct unit conversion rates needed to be able to find the speed of the school bus in feet per second.

Options:

1. 5280 feet/1 mile
2. 60 minutes/1 hour
3. 1 minute/60 seconds
4. 1 mile/5280 feet
5. 1 hour/60 minutes
6. 60 seconds/1 minute

How Did I Do?

5280 feet/1 mile, 1 hour/60 minutes, and 1 minute/60 seconds.

Problem 9

The price of gasoline in Italy is 1.45 Euros per liter. Choose all of the correct unit conversion rates needed to be able to find the price of gasoline in Italy in US dollars per gallon.

Options:

1. 60 minutes/1 hour
2. 1 gallon/3.79 liters
3. 1 Euro/1.09 US dollars
4. 3.79 liters/1 gallon
5. 60 seconds/1 minute
6. 1 US dollar/.92 Euros

How Did I Do?

1 US dollar/.92 Euros and 3.79 liters/1 gallon.

Budgeting

The purpose of budgeting is to help us make wise decisions about how to use the resources we have. In the following video Henry J. Eyring, former president of BYU-Idaho, provides some advice and council to BYU-Idaho students on using money wisely.

President Eyring Video

L04 - Budgeting - Eyring

[L04-Budgeting-Eyring Transcript](#)

We frequently use unit conversions when we build a budget. We want to know about money in different time frames. Usually we want a monthly budget, but sometimes it makes more sense to look at a yearly budget, or a weekly budget. College students sometimes plan their budgets around semesters. Budgeting is much more effective when we use the same time frame for all of our income and expenses. When we change the time frame for expenses or income it is called *prorating*.

An Example Budget

Let's look at how the tools we have learned in this lesson can be used as part of the Quantitative Reasoning Process.

1 Understand the Problem

Heidi is a BYU-Idaho student who has not been using a budget. On a recent visit to a restaurant, her card was declined when she tried to pay for her meal. Her roommate was kind and paid for her meal, but Heidi was embarrassed and wants to improve her finances. She has decided to create a budget.

2 Identify Variables & Assumptions

A budget is a mathematical model of our finances. It is an itemized plan for how we will use our money. In order to create and use an effective budget we need to know two important things:

1. The amount of money we have **coming in**, and
2. The amount of money we have **going out**.

These are our key variables. We typically refer to the money we have coming in as income and the money we spend as expenses. If a monthly time frame is used, the difference between monthly income and monthly expenses is called *net monthly cash flow*.

Heidi will also need to make assumptions about her income and her expenses. In this case she will be making the following assumptions:

- Her only income will be from her part time job.
- Her pay rate at her part time job will stay the same.
- Her expenses will be fairly constant each month.

3 Apply Quantitative Tools

Heidi needs to consider each of her two key variables: how much money is she spending and how much money is coming in.

Heidi's income

Heidi works for 20 hours each week. Her job pays \$11.90 per hour. Heidi uses the following unit conversion to compute her total monthly income.

$$\frac{\$11.90}{1 \text{ hour of work}} \times \frac{20 \text{ hours of work}}{1 \text{ week}} \times \frac{4 \text{ weeks}}{1 \text{ month}}$$

Notice that the “hours of work” unit and the “weeks” unit will divide to be 1, so the units will be left as dollars (\$) per month. When Heidi multiplied the top of the fractions and the bottom of the fractions, she got

\$952 per month.

Heidi's expenses

Heidi pays 10% of her income for tithing each month. Additionally, based on her past spending she estimates that she spends about \$60 dollars a week for groceries, \$1620 a year on car repairs, \$500 a month on rent, and every other Saturday she spends \$25 for dinner and a movie with friends. We will use the following table to summarize her monthly expenses.

Heidi's Expenses	Time Conversions	Monthly Expense
Tithing	$0.10 \times \$952 \text{ per month}$	\$95.20/month
Groceries	$\$60/\text{week} \times 4 \text{ weeks}/1 \text{ month}$	\$240/month
Car Repairs	$\$1620/\text{year} \times 1 \text{ year}/12 \text{ months}$	\$135/month
Rent	No time conversion needed	\$500/month
Entertainment	$\$25/2 \text{ Saturdays} \times 4 \text{ Saturdays}/1 \text{ month}$	\$50/month
	Total Monthly Expenses	\$1020.20/month

4 Make an Informed Decision

After doing her calculations, Heidi realized that she was spending more money than she was making (her net monthly cash flow is $\$952 - \$1020.20 = -\$68.20$). This helped her understand why her card was declined at the restaurant. Based on her calculations she realized that she either needs to increase her income or decrease her expenses.

After considering her options, Heidi realized that she couldn't get more hours at work and didn't have time to work another job. So her best option would be to cut her expenses. She decided to decrease the amount she spent on

groceries per week to \$50 and to cut her entertainment spending down to \$20 per month. When she redid the calculations, she got the following result:

Heidi's Expenses	Time Conversions	Monthly Expense
Tithing	$0.10 \times \$952$ per month	\$95.20/month
Groceries	$\$50/\text{week} \times 4 \text{ weeks}/1 \text{ month}$	\$200/month
Car Repairs	$\$1620/\text{year} \times 1 \text{ year}/12 \text{ months}$	\$135/month
Rent	No time conversion needed	\$500/month
Entertainment	No time conversion needed	\$20/month
	Total Monthly Expenses	\$950.20/month

Heidi realized that this would lead to her spending being less than her earnings which would put her in a better financial situation. In 2 Nephi 9:51, we are counseled about the choices we make with our money, "Wherefore, do not spend money for that which is of no worth, nor your labor for that which cannot satisfy." ³ Once we have used the computational tools to understand and calculate our expenses and income, we are in a good position to interpret the information and make an informed decision about how we want to spend our money. We can set budgeting goals and then track our spending to see if we are meeting our goals. President N. Eldon Tanner of the Quorum of the Twelve (1962-1982) stated, "I'm convinced that it is not the amount of money that an individual earns that brings peace of mind, as much as having control of your money." ⁴ President Heber J. Grant said, "If there is any one thing that will bring peace, it is to live within our means. If there is any one thing that is grinding, it is to have debts and obligations that one cannot meet." ⁵

5 Evaluate Your Reasoning

Now that Heidi has made a decision about her budget, it is important that she take some time to reflect and evaluate her decision. She should ask if her budget is reasonable. Will she be able to stick to the smaller entertainment budget? She also should evaluate her assumptions and make sure they are appropriate and reasonable.

It will also be helpful for Heidi to reflect and evaluate her budget after using it for a few months. Having experience living by the budget will help her make sure that the decisions she made are effective. It is likely she will find areas where the budget needs to be adjusted.

Practice: Budgeting

Problem 10

The Garcia family takes home 42,000 dollars a year. They donate 12 percent of their take-home pay in tithing and other charitable donations. They spend 180 dollars a week for groceries, 5,700 dollars a year for car payments, and 60 dollars a week on gas and repairs. Their mortgage payment is 1,260 dollars a month. They spend 3,600 dollars a year traveling to visit family and 125 dollars each month for entertainment.

Prorate the Garcia's income and expenses to create a model for their current monthly budget. Complete the spreadsheet below:

Garcia Family Monthly Budget	
Category	Budget
Tithing and Donations	
Groceries	
Car Payments	
Gas and Car Repairs	
Mortgage	
Travel	
Entertainment	

How Did I Do?

Garcia Family Monthly Budget	
Category	Budget
Tithing and donations	\$420
Groceries	\$720
Car Payments	\$475

Garcia Family Monthly Budget	
Gas and car repairs	\$240
Mortgage	\$1,260
Travel	\$300
Entertainment	\$125

Problem 11

The Garcia family takes home \$42,000 a year. Compute their monthly income and total monthly expenses.

How Did I Do?

Monthly income answer: \$3,500

Monthly expense answer: \$3,540

Hint: Divide the total annual income by the number of months in the year to find the monthly income and sum the monthly expenses for each category to find the monthly expenses.

What would you suggest to help the Garcia family?

Conclusion

Are you using a budget? If not, we encourage you to apply the Quantitative Reasoning Process to your personal situation. If so, now might be a good time to reevaluate your budget and determine if there are improvements that can be made. Living on a budget can help improve your financial situation, no matter how much money you have. As Barbara B. Smith, former General Relief Society President said, "Living on a budget is not a chore. It need not even be a deprivation. Budgeting should be a great learning experience." [6](#)

Lesson Checklist

By the end of this lesson you should be able to do the following:

- Understand the mathematical meaning of keywords such as per, of, is, more than, and less than.
- Identify the units involved in solving a problem.
- Use appropriate units and unit conversions to solve a problem.
- Solve problems using percentages.
- Create a monthly budget by using unit conversions to prorate costs.

Optional Resources

The following articles discuss a study mentioned by President Henry J. Eyring in the video included in this week's lesson:

- [Happiness is Love – and \\$75,000](#)
- [Income Buys Happiness Differently Based on Where you Live](#)
- [High Income Improves Evaluation of Life But Not Emotional Well-Being](#)

¹ Thomas S. Monson. [Guiding Principles of Personal and Family Welfare](#). Ensign Magazine, Sept. 1986

² Spencer W. Kimball. As quoted in [One for the Money](#)

³ [2 Nephi 9:51](#)

⁴ N. Eldon Tanner. [Constancy Amid Change](#). General Conference, October 1979

⁵ President Heber J. Grant. [Gospel Standards](#), comp. G. Homer Durham [1941], page 111

⁶ Sister Barbara B. Smith, General Relief Society President. [Reach for the Stars](#). General conference, April 1981.



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Lesson 5: Summarizing Data

Opening Story

Lesson 5 - Opening Story
[L5 Opening Story Transcript](#)

Summarizing Data

Information surrounds us. In the introductory video, we heard a conversation between Lucas, Amanda, and their doctor. Even in a simple conversation like this one, a lot of information is exchanged. We need to be able to make sense of the information we hear. Knowing how to make sense of information is a highly valued skill in today's workplace. It is also an important trait of the wise consumer, the informed citizen, and the educated individual. Making sense of information is usually called *summarizing data*. We summarize data every day in many ways, but the goal is always to give an improved understanding of a situation.

One example of summarizing data was seen in the introductory video when Amanda stated, "The doctor has been gone a really long time." Lucas replied, "I wouldn't worry about it. The longest we've ever had to wait is 15 minutes." This comment is a concise summary of all of their previous visits to the doctor's office. It allowed Lucas to comfort Amanda by referencing their past experience in a concise way.

Practice: Summarizing Data

Problem 1

Choose all of the statements from the introductory video that summarized information.

- Amanda says, "The doctor has been gone a really long time."
- Lucas says, "I wouldn't worry about it."
- Lucas says, "The longest we've ever had to wait is 15 minutes."
- Amanda asks, "What are the chances you think he'll come sooner than that?"
- Lucas says, "He's usually pretty consistent."

[Hint: There are two correct answers.]

How Did I Do?

The statements that summarized information are:

- Lucas says, "The longest we've ever had to wait is 15 minutes."
- Lucas says, "He's usually pretty consistent."

Problem 2

Choose all of the statements from the Introductory video that summarized information.

- The doctor says, "Sorry to keep you waiting."
- The doctor says, "Everything looks good."
- The doctor says, "I was reviewing the nurse's report on the way over."
- The doctor asks, "As you can see, she is the fourth percentile for girls her age."
- The doctor says, "On average, infants her age fall within this range." (Points at chart.)

[Hint: There are three correct answers.]

How Did I Do?

The statements that summarized information are:

- The doctor says, "Everything looks good."
- The doctor asks, "As you can see, she is the fourth percentile for girls her age."
- The doctor says, "On average, infants her age fall within this range." (Points at chart.)

Problem 3

Choose all of the statements from the introductory video that summarized information.

- Amanda says, "Hmm. She hasn't been eating very well."
- The doctor says, "I saw that in the report. And it's something to keep an eye on."
- The doctor says, "But it's actually not that unusual."
- The doctor says, "I don't think we need to be concerned, unless it worsens."
- The doctor says, "We'll watch her progress closely."

[Hint: There are two correct answers.]

How Did I Do?

The statements that summarized information are:

- Amanda says, "Hmm. She hasn't been eating very well."
- The doctor says, "But it's actually not that unusual."

In this lesson, we will learn some of the most commonly used tools for summarizing data. We will also learn how to use how to use Excel to help us do these calculations.

Tools for Summarizing Data

Basic statistical calculations can help us summarize information, especially when that information consists of numbers. We will be discussing how the minimum, maximum, mean, median, mode, standard deviation, and percentiles can help us summarize information.

Minimum and Maximum

First, let's look at the minimum and maximum. The minimum and maximum are common ways of summarizing the most extreme values that have occurred in a given scenario.

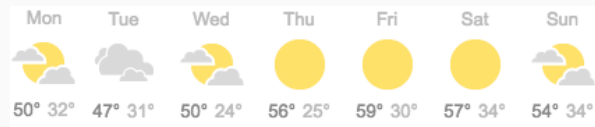
*The **minimum** value in a list of observations is the smallest value that occurs.*

*The **maximum** value in a list of observations is the largest value that occurs.*

The minimum and maximum help us see what values are possible, since we would expect all the other observations to be between these two extremes.

Example 1

Look at any weather report and you will typically see the high and low temperatures reported. Here is an example of a typical weekly weather forecast. Find the minimum and maximum temperatures for the week and explain what they tell us.



Solution:

We see the high and low temperature for each day listed. If you compare the predicted high temperatures for each day of the week, you will see the largest (or highest) temperature of 59° Fahrenheit is predicted to occur on Friday. The smallest (or lowest) temperature of 24° Fahrenheit is predicted to occur on Wednesday. Therefore, the maximum temperature is 59° F and the minimum temperature is 24°F. We would expect all other temperatures during this given week to be between 24°F and 59°F.

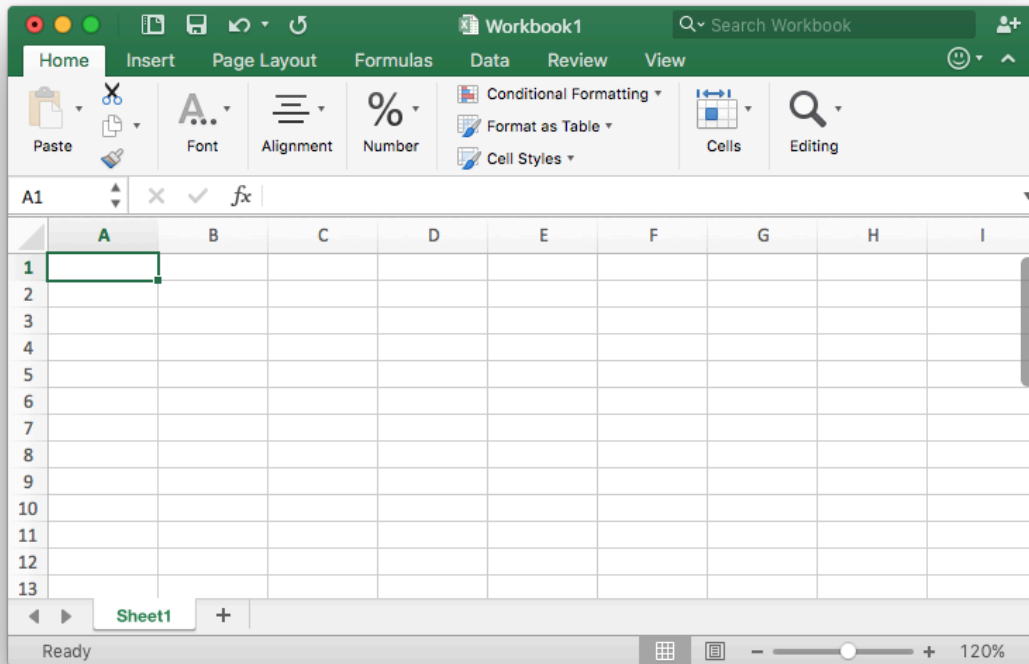
Locating the maximum and minimum values can always be done by hand. However, if we have a lot of values to look at it can be more practical to let Excel find the maximum and minimum for us.

Excel Instructions - Maximum and Minimum

Follow these steps to learn how to calculate the maximum and minimum in Excel:

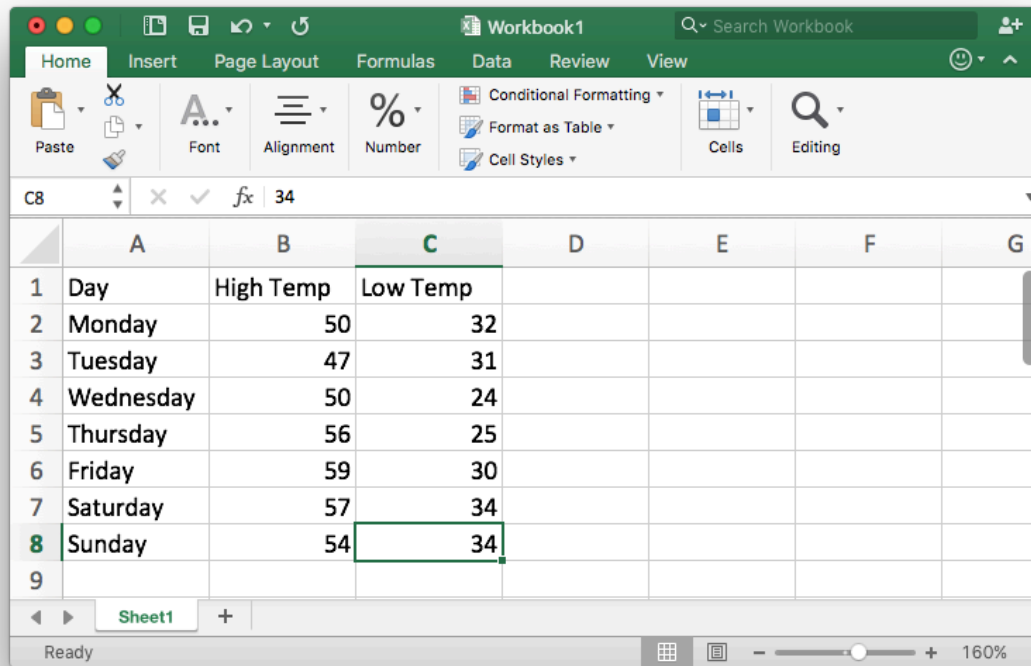
Step 1

Open a "New Workbook" in Excel.



Step 2

Type in the data for which you want to calculate the minimum or maximum. For this example, we will use the predicted high and low temperatures shown in the typical weather forecast depicted in the example above.

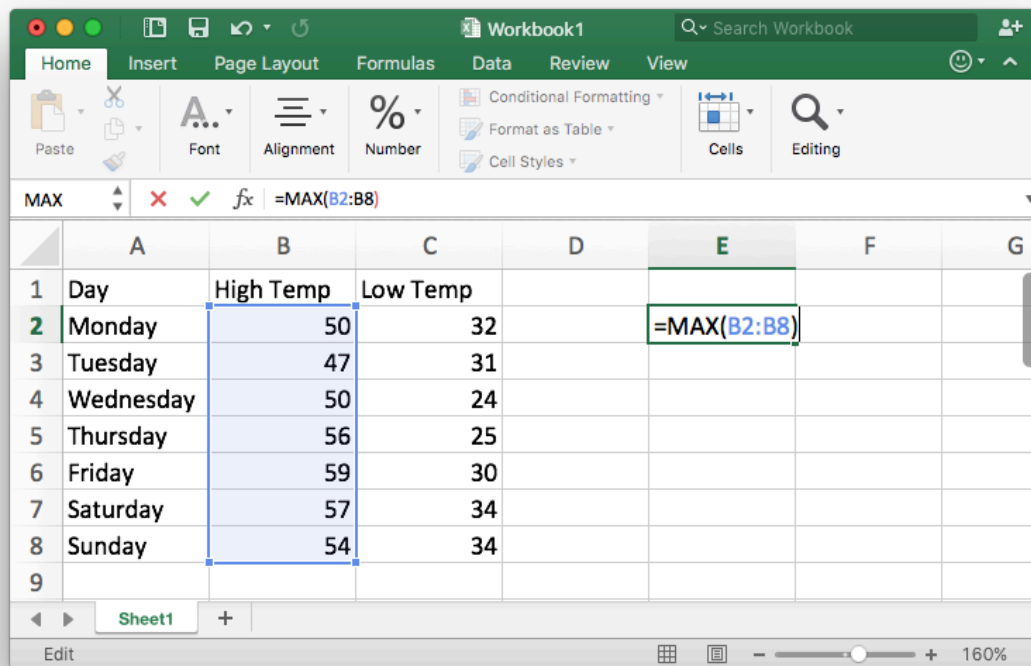


Step 3

In any blank cell of the document

- type the command `=MAX(`
- use your mouse to highlight the data you are interested in
- and finish typing the closing parentheses `)`

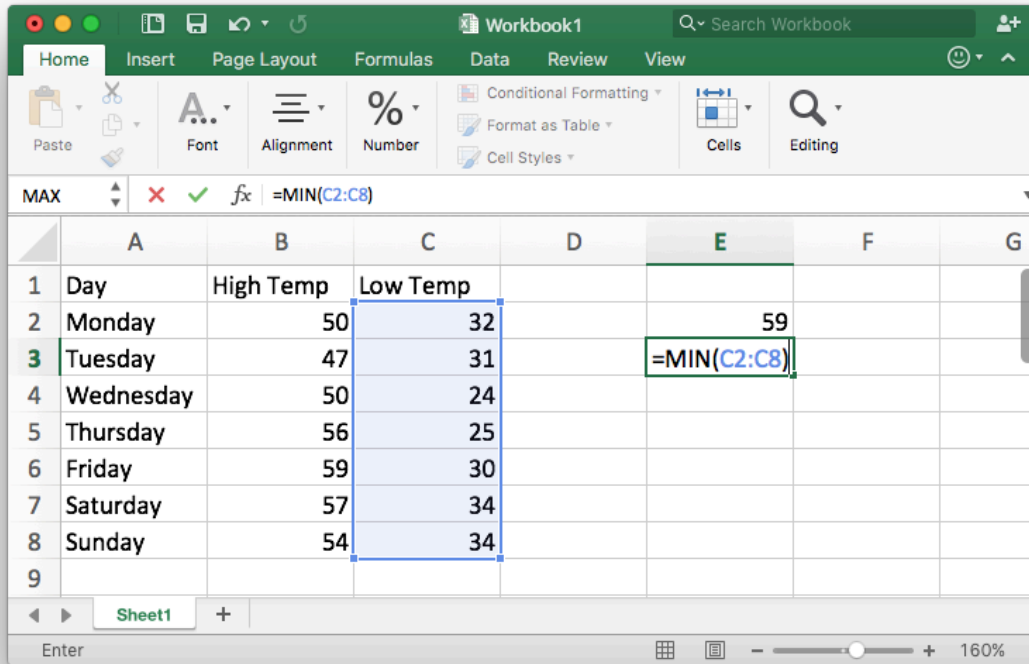
For this example, this gives: `=MAX(B2:B8)`



Step 4

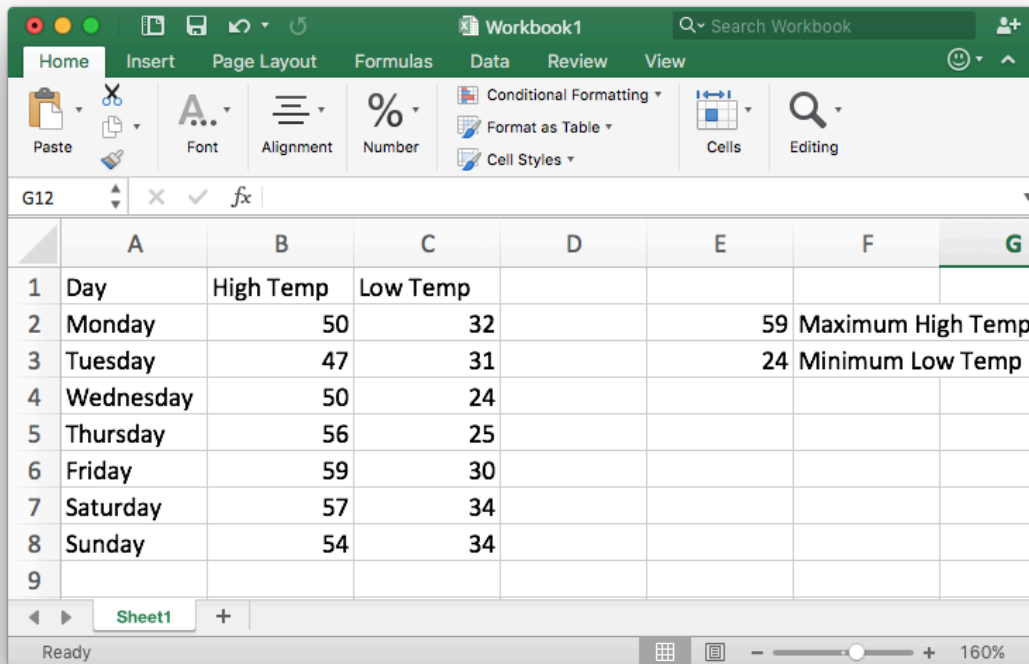
Press **Enter** (or **return**) and the cursor will move to the next line with the result for the maximum of 59°F now showing.

Repeat the above process using the `=MIN()` command to obtain the minimum.



Step 5

Labeling these values is optional, but helpful for remembering what they are.



Mean, Median, and Mode

The mean, median, and mode are ways of summarizing the most typical values that have occurred in a given scenario. They are often called *measures of center* because they give a good feel for the values that are “right in the center” of all of the observed values.

*The **mean** is more commonly referred to as the average. It is calculated by adding up all of the values and dividing by the total number of values. It is good for summarizing the center of values that are generally very similar to each other.*

*The **median** is the middle number in a list of numbers that is sorted from smallest to largest. Thus, fifty percent of the values are lower than the median and fifty percent of the values are larger than the median. It is good for summarizing the center of values where most of the values are similar but some are dramatically higher (or lower) than the rest.*

*The **mode** is the value that occurs most often in a list of numbers. It is good for summarizing data that only has a few distinct values that are possible.*

Example 2

How long does a human pregnancy last? The common answer is 9 months, but we know that not every pregnancy is the same length. Should you use the mean, median, or mode to describe the length of human pregnancies? Why?

Solution:

The *mean* (or *average*) is a good estimate of the length of pregnancies because most pregnancies last about the same amount of time.

The mean length of a pregnancy is 266 days from conception (or 38 weeks). This mean could be calculated by taking the number of days every pregnancy lasts and dividing by the total number of pregnancies. The mean, or average, is a good estimate of the length of a pregnancy because two-thirds of pregnancies are within 2 weeks of this estimate. In other words, the lengths of most pregnancies are very close to 266 days even though a rare few are much longer or much shorter than that.

(Note that sometimes the last menstrual period rather than date of conception is used to predict a pregnancy. In this case, the mean is two weeks longer, i.e., 280 days, or 40 weeks.)

Example 3

What is a typical household annual income? We know that not every household has the same income, so we need to summarize this data in order to understand it better. Should you use the mean, median, or mode to describe household income? Why?

Solution:

The *median* is often used to summarize incomes for a given region. For example, a 2013 news headline stated, "Worldwide, Median Household Income About \$10,000."¹ This is because while most people earn around the same amount of money each month, with some earning more than others, there are yet others who earn extraordinarily more money than the rest of us. The median income is reported in order to keep the extremely high incomes from skewing the results. Thus, the median income gives a better feel than would the mean for what most people actually make.

To demonstrate further, a survey of 183 BYU-Idaho students² showed that the median hourly wage of students was \$9.02 an hour. The mean hourly wage was \$10.81, but was skewed by a student who stated they were earning \$100.00 an hour! The median is a better representation in this case of the typical hourly wage of BYU-Idaho students. Remember, 50% of students will earn less than the median wage and 50% will earn more than the median wage.

Example 4

If you have ever tried to find a certain size of clothing at a store you may have noticed that some sizes are more commonly sold than others. But not all clothing sizes are the same size, so we can summarize this information to get a better idea of what clothing sizes are available. Should you use the mean, median, or mode to describe available clothing sizes? Why?

Solution:

This is an example of the *mode*, or the most commonly occurring clothing sizes. The mode is a good value to use because there are only a few sizes possible.

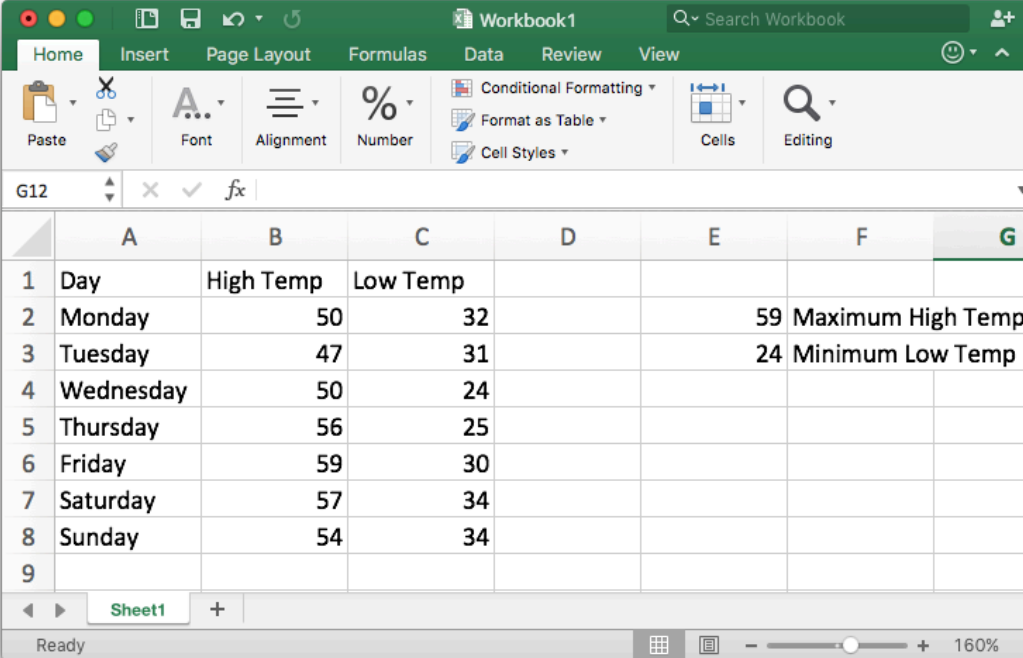
To demonstrate further, some data from college students in the U.S. [3](#) shows that the modal shoe size (most commonly occurring shoe size) for females is size 8. The modal size for males is size 11. If you go to a shoe store you will probably find a better selection in these sizes than you would in less common sizes.

Excel Instructions - Mean, Median, and Mode

Follow these steps to calculate the mean, median, or mode in Excel:

Step 1

Enter the data into Excel. For this example, we will continue using the weather data from the previous Excel Instructions where we found the maximum and minimum values.



The screenshot shows the Microsoft Excel interface with a workbook titled 'Workbook1'. The 'Home' tab is selected in the ribbon. The formula bar shows 'G12'. The spreadsheet contains the following data:

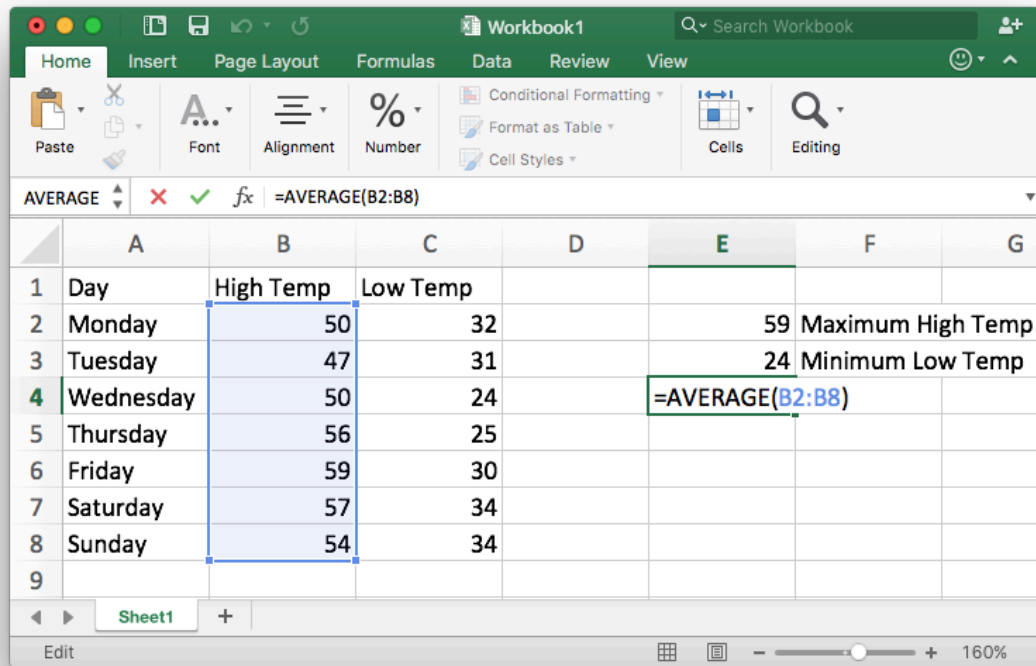
	A	B	C	D	E	F	G
1	Day	High Temp	Low Temp				
2	Monday	50	32		59	Maximum High Temp	
3	Tuesday	47	31		24	Minimum Low Temp	
4	Wednesday	50	24				
5	Thursday	56	25				
6	Friday	59	30				
7	Saturday	57	34				
8	Sunday	54	34				
9							

The status bar at the bottom indicates 'Ready' and a zoom level of 160%.

Step 2

To compute the **mean**:

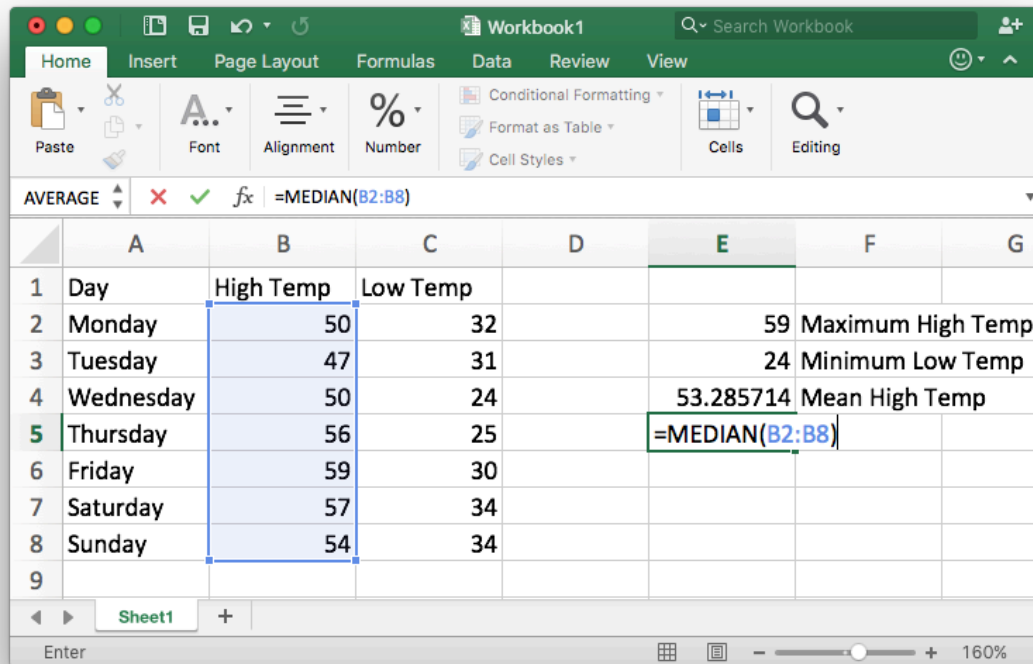
- type the command `=AVERAGE(` in a blank cell
- highlight the data you are interested in
- finish by typing the closing parentheses `)` and pressing **Enter** or **return**.



Step 3

To compute the **median**:

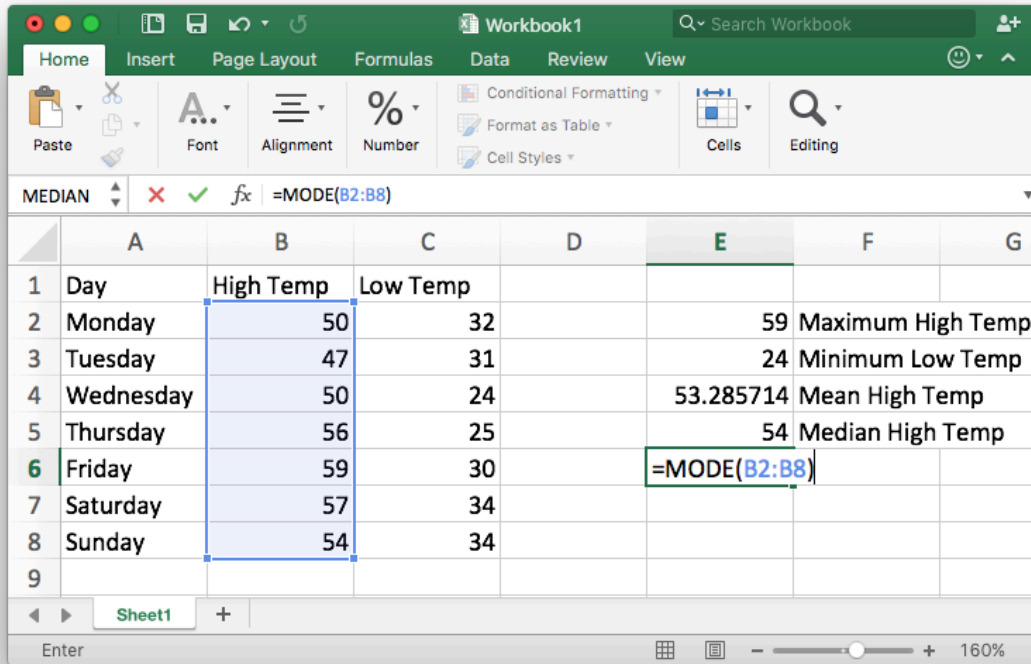
- type the command `=MEDIAN(` in a blank cell
- highlight the data you are interested in
- finish by typing the closing parentheses `)` and pressing **Enter** or **return**.



Step 4

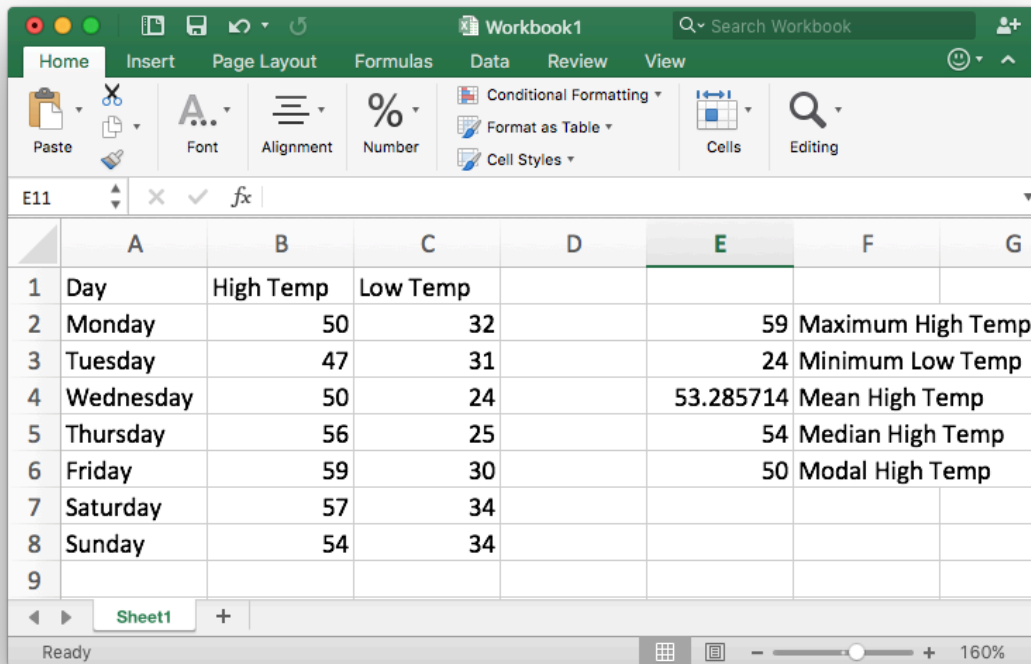
To compute the **mode**:

- type the command `=MODE()` in a blank cell
- highlight the data you are interested in
- finish by typing the closing parentheses `)` and pressing **Enter** or **return**.
- Note that Excel just finds one mode and there might be more than one.



Step 5

The final result is as follows.



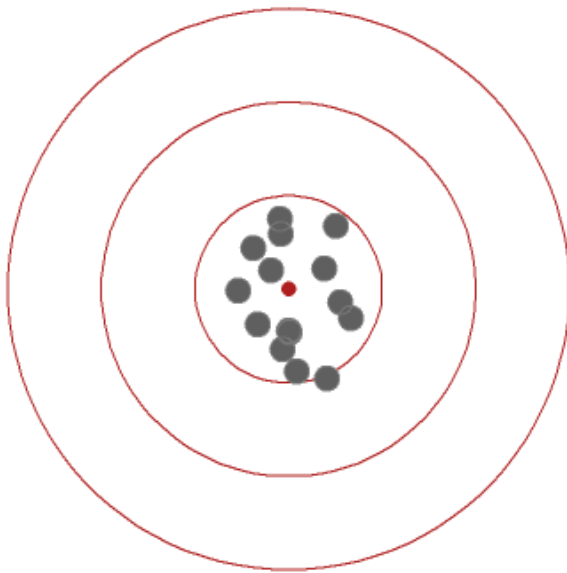
Standard Deviation

The standard deviation helps us know how close to the mean all the values in a data set are. This is helpful because it tells us if all the points in a data set are close together or if they are very spread out.

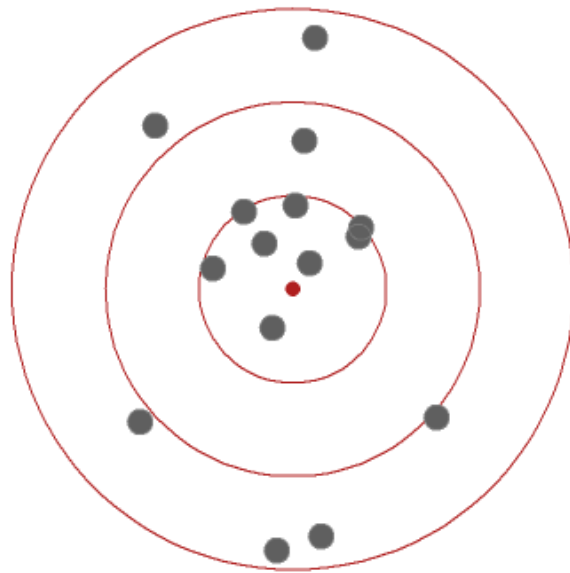
*The **standard deviation** is a summary of how much values vary from the mean of a set of data. When the values are all very close to the mean, the standard deviation is small. When the values vary dramatically from the mean, the standard deviation is large.*

The following diagram demonstrates the idea of standard deviation. In each picture, the red dot in the center represents the mean. Notice in the diagram representing a small standard deviation most of the points are quite close to the mean. In the diagram representing a large standard deviation we can see the points are quite spread out and not as close to the mean.

Small Standard Deviation



Large Standard Deviation



Example 5

Consider the following two sets of data: a) all the daily high temperatures in a given year in Honolulu, Hawaii, or b) all the daily high temperatures in the same year in Rexburg, Idaho. Which set of data would have a larger standard deviation? Explain.

Solution:

We know that the temperature in Hawaii is pretty similar all year long. But the temperature in Idaho varies a lot. In the summer the high temperature in Rexburg might be 95°F while in the winter the high temperature might only be -10°F .

For example, during 2016, the average annual temperature in Honolulu, Hawaii was 84.09°F and the standard deviation was 2.97°F . The warmest days of the year were July 14 and August 12, 2016. On both of those days the high temperature was 91°F . The coldest day of the year was December 30th when the high temperature was 75°F . [4](#)

During 2016, the average annual temperature in Rexburg, Idaho was 57.33°F with a standard deviation of 22°F . The warmest day of the year was August 2, 2016 when the high temperature was 95°F . The coldest day of the year, however, was on December 17th when the high temperature was only 4°F . [5](#)

If we compare the standard deviations for Honolulu and Rexburg, we see that Honolulu's standard deviation of 2.97°F is much smaller than Rexburg's standard deviation of 22°F . This tells us the temperatures in Honolulu are usually much closer to the average when compared to the temperatures in Rexburg that vary a lot from the average.

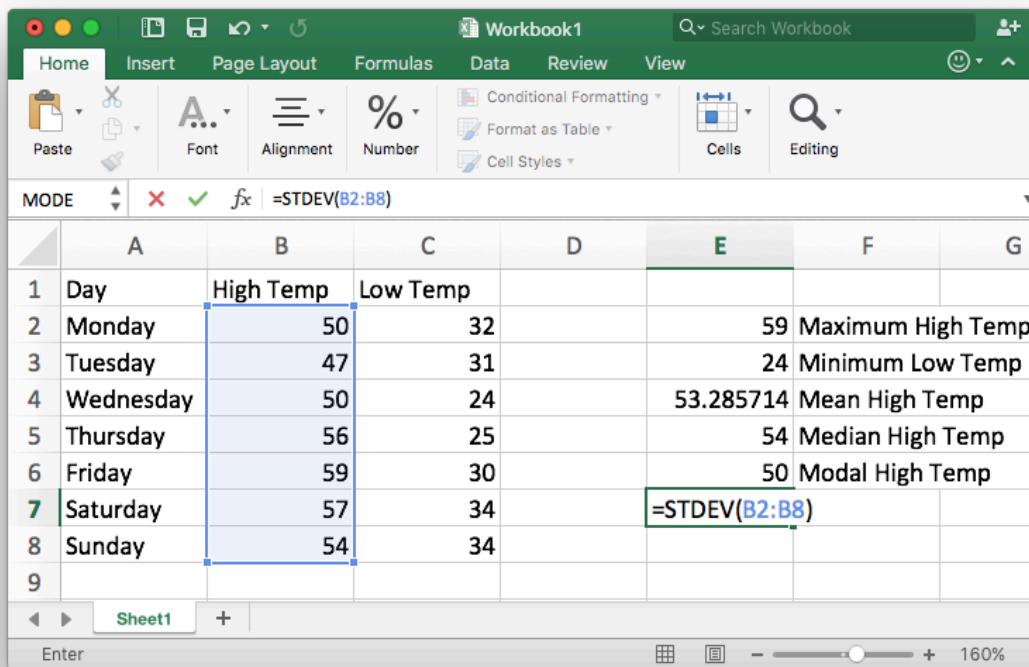
Excel Instructions - Standard Deviation

To compute the standard deviation in Excel:

Step 1

Open your data in Excel,

- type the command `=STDEV(` in a blank cell
- highlight the data you are interested in
- finish by typing the closing parentheses `)` and pressing `Enter` or `return`.



Step 2

The result is as follows.

The screenshot shows an Excel spreadsheet with the following data and formulas:

	A	B	C	D	E	F	G
1	Day	High Temp	Low Temp				
2	Monday	50	32		59	Maximum High Temp	
3	Tuesday	47	31		24	Minimum Low Temp	
4	Wednesday	50	24		53.285714	Mean High Temp	
5	Thursday	56	25		54	Median High Temp	
6	Friday	59	30		50	Modal High Temp	
7	Saturday	57	34		4.3861253	St. Dev. High Temp	
8	Sunday	54	34				
9							

The formula bar shows the formula for cell E7: `=STDEV(B2:B8)`. The status bar at the bottom indicates the spreadsheet is 'Ready' and the zoom level is 160%.

Percentiles

Percentiles help us see how a certain value compares to all the other values in a set of data.

*A **percentile** measures the percentage of values that are the same as or less than the specified value.*

They are typically used to measure the performance of an individual with respect to their peers.

Context is very important to consider when looking at percentiles. Being at the 5th percentile might be good, it might be bad, and it might not be significant at all. It really just depends on the situation. You should never consider a percentile without looking at the related context.

Example 6

Juan took a standardized test and then was told that he scored at the 82nd percentile. Interpret what the percentile means.

Solution:

Knowing that Juan's score was at the 82nd percentile helps us understand how he scored compared to the other students who took the standardized test. We know he scored as well or better than 82% of all the people who took the test. This means 18% of students who took the test scored better than Juan did. Juan's percentile score does not tell us his actual score on the exam. It only tells us what percentage of students he scored higher than (or the same as).

Example 7

In the introductory video we heard the doctor tell Lucas and Amanda that their daughter Emmy is in the 4th percentile for girls her age. Explain what this means.

Solution:

Since Emmy is in the 4th percentile we know that out of all the baby girls her age, only 4% weigh the same or less than Emmy does. That means 96% of the babies weight more than Emmy. We could also think of it as: 4 out of every 100 babies will be the same weight or smaller. The doctor noted: "It is something to keep an eye on, but it's actually not that unusual." In this case the context is very important, the doctor does not just use a percentile to make a decision about Emmy's care. He will also consider other indicators of her health.

Excel Instructions - Percentiles

To compute a percentile in Excel:

Step 1

Open your data in Excel,

- type the command `=PERCENTILE(` in a blank cell
- highlight the data you are interested in
- type a comma `,` and the desired percentile as a decimal like `0.9`
- finish by typing the closing parentheses `)` and pressing `Enter` or `return`

	A	B	C	D	E	F	G
1	Day	High Temp	Low Temp				
2	Monday	50	32		59	Maximum High Temp	
3	Tuesday	47	31		24	Minimum Low Temp	
4	Wednesday	50	24		53.285714	Mean High Temp	
5	Thursday	56	25		54	Median High Temp	
6	Friday	59	30		50	Modal High Temp	
7	Saturday	57	34		4.3861253	St. Dev. High Temp	
8	Sunday	54	34		=PERCENTILE(B2:B8, 0.9)		
9							

Step 2

The result is as follows.

Workbook1

Home Insert Page Layout Formulas Data Review View

Paste Font Alignment Number Conditional Formatting Format as Table Cells Editing

E8 \times \checkmark fx =PERCENTILE(B2:B8, 0.9)

	A	B	C	D	E	F	G
1	Day	High Temp	Low Temp				
2	Monday	50	32		59	Maximum High Temp	
3	Tuesday	47	31		24	Minimum Low Temp	
4	Wednesday	50	24		53.28571	Mean High Temp	
5	Thursday	56	25		54	Median High Temp	
6	Friday	59	30		50	Modal High Temp	
7	Saturday	57	34		4.386125	St. Dev. High Temp	
8	Sunday	54	34		57.8	90th Percentile High Temp	
9							

Sheet1 +

Ready 160%

Practice: Using Excel

Problem 4

The color of an individual's eyes can make a difference in how well they can detect a flickering light. If the light flickers fast enough, it looks like a steady light.

The following data shows how many flickers per second could be detected by individuals with brown, green, and blue eyes. The means for brown and green eyes have already been found.

What Excel formula you type in Box D10 to find the mean for blue eyes?

	A	B	C	D	E
1		Brown	Green	Blue	
2		26.8	26.4	25.7	
3		27.9	24.2	27.2	
4		23.7	28	29.9	
5		25	26.9	28.5	
6		26.3	29.1	29.4	
7		24.8		28.3	
8		25.7			
9		24.5			
10	Mean=	25.59	26.92		
11					

How Did I Do?

=average(D2:D7)

Problem 5

The color of an individual's eyes can make a difference in how well they can detect a flickering light. If the light flickers fast enough, it looks like a steady light. The following data shows how many flickers per second could be detected by individuals with brown, green, and blue eyes. Use the information in the Excel sheet to answer the following questions.

	A	B	C	D	E
1		Brown	Green	Blue	
2		26.8	26.4	25.7	
3		27.9	24.2	27.2	
4		23.7	28	29.9	
5		25	26.9	28.5	
6		26.3	29.1	29.4	
7		24.8		28.3	
8		25.7			
9		24.5			
10	Mean =	25.59	26.92	28.17	
11	StDev=	1.37	1.84	1.53	
12					

On average, which eye color can best see flickering light?

How Did I Do?

Blue

Because blue has the highest mean, we know that people with blue eyes can see flickering light best.

For which eye color are the measurements closest to the mean?

How Did I Do?

Brown

The standard deviation tells you how far from the deviation tells you how far measurements are from the mean. Brown eyes have the smallest standard deviation, so they are closest to the mean.

For which eye color do the measurements tend to be furthest from the mean?

How Did I Do?

Green

The standard deviation tells you how far from the deviation tells you how far measurements are from the mean. Green eyes have the largest standard deviation, so they are furthest from the mean.

Putting it All Together

Now that we know how to compute numbers that summarize data we can use these tools as part of the quantitative reasoning process to help us make an informed decision. Let's go back to Lucas and Amanda and their experience at the doctor in the introductory video. At the end of the video we heard the doctor say; "I don't think we need to be concerned unless it worsens. We'll watch her progress closely." (Note: This scenario is based on a real situation, but keep in mind that just having a low percentile for weight is not necessarily a cause for concern.)

1 Understand the Problem

Emmy hasn't been eating well and has spit up a lot after she eats. Lucas and Amanda's were concerned about Emmy's weight. They want to decide how to proceed to care for Emmy to ensure that she is healthy.

2 Identify Variables & Assumptions

The variables of interest in this situation include:

- How often Emmy spits up
- Emmy's weight
- Emmy's height
- The expense of doing medical tests

Lucas and Amanda need to make a few assumptions:

- Emmy's weight and height are an indication of her health
 - Emmy's weight and height are going to continue to be smaller than average
 - Emmy is going to continue to spit up often
-

3 Apply Quantitative Tools

The percentile is a useful statistical tool in this situation. Knowing that Emmy's weight is in the 4th percentile helps Lucas and Amanda understand how her weight compares to the weight of other babies her age. The chart the doctor showed is also a helpful tool because it gives them a visual tool to understand Emmy's weight. Lucas and Amanda understand that percentiles should always be considered in context. Just being small isn't necessarily a concern, but because Emmy is also not eating well and spitting up a lot they are wondering if something is wrong.

4 Make an Informed Decision

After discussing things further with the doctor, Lucas and Amanda decide not to do any medical tests at this time. The doctor wants to watch Emmy's progress, but he is not overly concerned. Lucas and Amanda are worried about doing medical tests because the tests are not covered by their health insurance and they would have to pay about \$100 each for the six tests the might apply. Based on their doctor's recommendation, they decide to wait and see if Emmy's weight improves.

5 Evaluate Your Reasoning



After carefully considering their decision, discussing it with the doctor, and praying about it, Lucas and Amanda are comfortable with their decision.

In two months they return to the doctor to follow up on Emmy's progress. At that visit they find that Emmy's weight has dropped to the 2nd percentile for baby girls her age. She is spitting up more often and has other signs of poor health. Based on this new information, they re-discuss their decision with the doctor and decide to go ahead and do the tests. The doctor helps them find a clinic where the tests can be done for less money than they would be at the hospital.

The tests come back and show that Emmy has a small health issue that is able to be fixed with medication. In a follow up visit a few months later Emmy's weight has increased to the 15th percentile for baby girls her age.

This example used the percentile as a tool to help understand Emmy's weight. In this case the doctor did the computation required and reported the percentile to Lucas and Amanda. In other cases we will find that Excel is a helpful tool for us to use as part of the Quantitative Reasoning Process when we want to find the minimum, maximum, mean, medium, mode, standard deviation, or a percentile.

Lesson Checklist

At the end of this lesson, you should be able to do the following:

- Use Excel to calculate the minimum, maximum, mean, median, mode, standard deviation, and a percentile.
 - Interpret the maximum and minimum as measures of the extreme values in a set of data.
 - Interpret the mean, median, and mode as measures of the center in a set of data.
 - Be able to determine whether to use the mean, median, or mode to find a measure of the center of a set of data.
 - Interpret the standard deviation as a measure of variability of a set of data.
 - Interpret percentiles as a way to compare an individual value to the rest of the data set.
 - Use the minimum, maximum, mean, median, mode, standard deviation, and percentiles as a quantitative tool in the Quantitative Reasoning Process.
-

¹ <http://www.gallup.com/poll/166211/worldwide-median-household-income-000.aspx>

² Brother Cromar's BYU-I Stats class survey

³ <http://newsfeed.time.com/2012/10/16/size-8-is-the-new-7-why-our-feet-are-getting-bigger/>

⁴ <http://www.ncdc.noaa.gov/>

⁵ <http://www.ncdc.noaa.gov/>



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Unit 2

Lesson 6: Visualizing Data
Lesson 7: Functions in Excel
Lesson 8: Functions and Their Graphs
Lesson 9: Change Over Time



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Access it online or download it at https://books.byui.edu/math_108x/unit_2.

Lesson 6: Visualizing Data

Opening Story

Lesson 6 - Opening Story

[L6 Opening Story Transcript](#)

Identifying Patterns

As we saw in the opening video, Craig's boss wanted him to create charts and graphs in order to help the team better understand their data. It can be hard to really understand what is going on by just looking at a list of numbers. Creating graphics such as charts and graphs to represent a set of data can make it much easier to understand. This is true because humans have an amazing ability to identify patterns in images. The following image is a great example. The tigers in the picture are hidden by the grass and the camouflaging stripes of the tigers. In spite of this, the human eye quickly locates the two tigers crouching in the grass.



Data visualization is the science of communicating important patterns from data through the use of graphics.

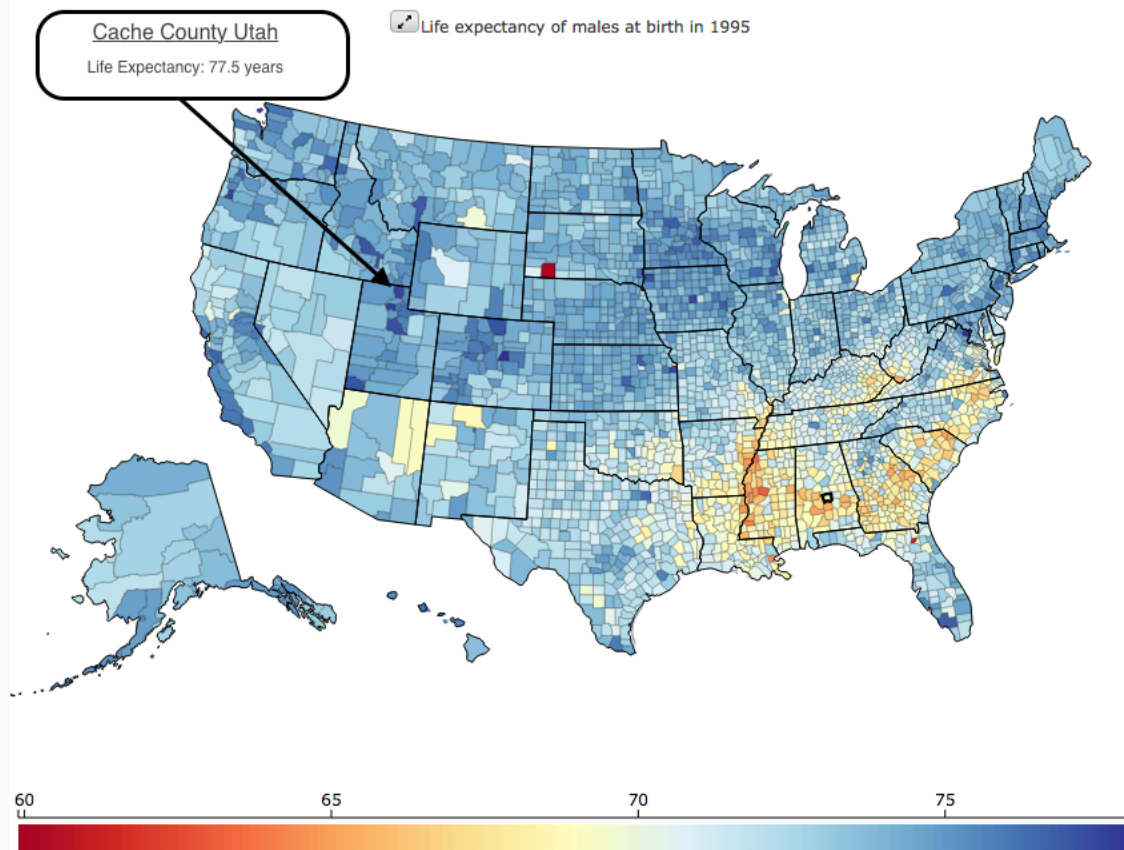
Figuratively speaking, the goal of a graphic is to help people quickly identify the “tiger in the grass.” In other words, the goal of a good graphic is to reveal important patterns in data.

Graphics can be very helpful as part of the Quantitative Reasoning Process. They provide a tool for examining data that enables a more powerful decision on the part of the user. To do this, graphics must communicate the truth data has to offer on a situation of interest.

Example 1

In 1995 a team of American researchers began a study on Alzheimer's disease.¹ Alzheimer's is a serious disease resulting in memory loss that most typically occurs in adults aged 65 and older. Individuals affected with the disease progressively lose the ability to function with normal daily tasks until they eventually die from the disease.

Before beginning their study, the researchers needed to find a geographic location where they could find enough elderly participants to carry out the study. Use the following graphic to find a location where the study could be carried out.



Solution:

The researchers need a population that is old enough and healthy enough to observe possible factors that contribute to the development of Alzheimer's disease. We notice the above graphic readily communicates which counties have the longest life expectancy by showing them in dark blue.

The graphic shows several counties showing dark blue. A detailed examination of each of the counties shows Cache County, Utah as having one of the longest life expectancies in the USA at 77.5 years (for males).

The researchers eventually selected Cache County Utah because, as they stated, "The elderly of Cache County have a longer life expectancy, higher educational attainment, and lower incidence of chronic disease (which can complicate the diagnosis of dementia) than other similar populations."²

Example 1 is a good illustration of how graphics can improve our ability to understand data. Looking at the graphic helps us get an overall impression of life expectancy data throughout the United States. If we didn't use a graphic, we would need to look at a list of life expectancies for each county in the United States. This would be a very long list of data and without using a graphic it would be much harder to get a good overall idea of what the data tells us.

Common Graphical Tools

While only the imagination limits what a good graphic can be, there are several types of graphics that are commonly used to communicate information. Learning how and when to use these common graphical tools will benefit you both as a student and as an employee.

Bar Charts

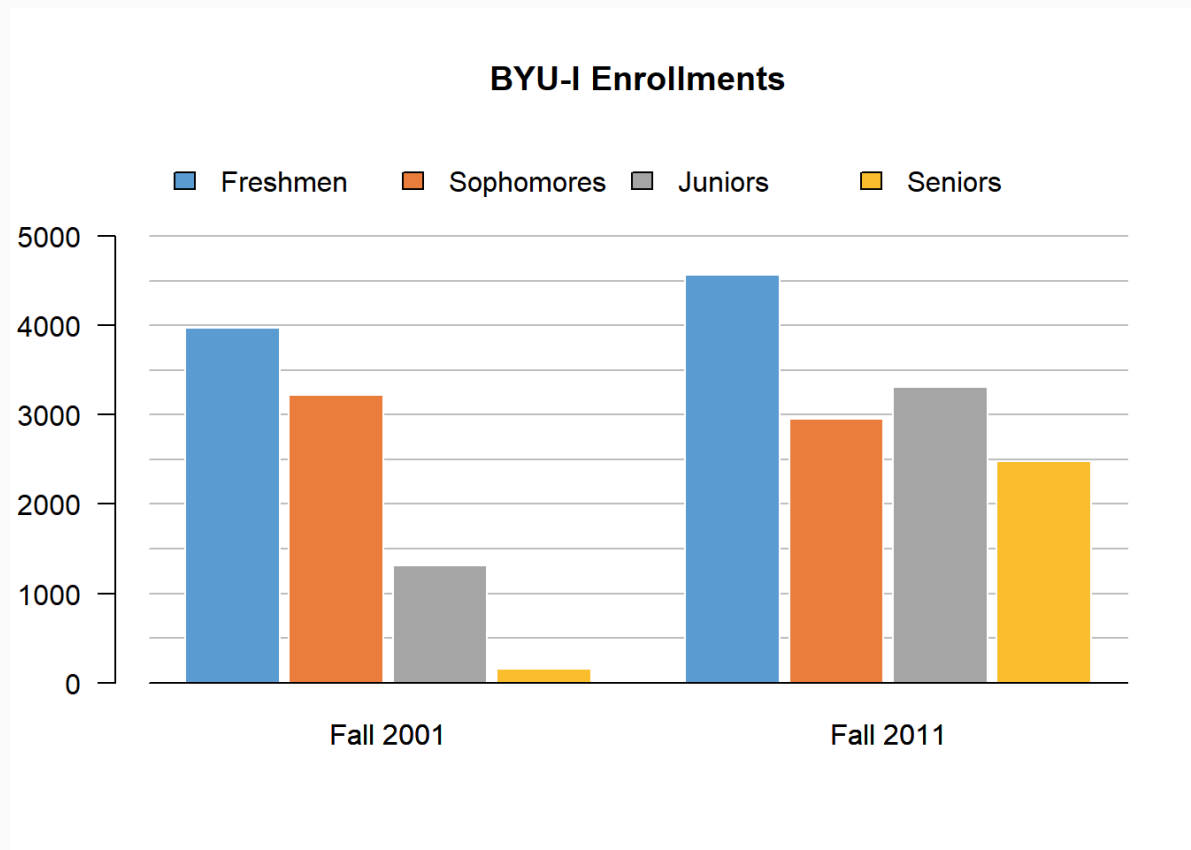
When data has several categories, the number of observations in each category can be visually summarized with a bar chart. What information does the chart provide?

Example 2

Use a graphic to compare the number of Freshman, Sophomores, Junior, and Seniors at BYU-Idaho in 2001 and 2011.

Solution:

A bar chart is a good graphic to use for this data because the data has four categories: freshman, sophomore, junior, and senior.

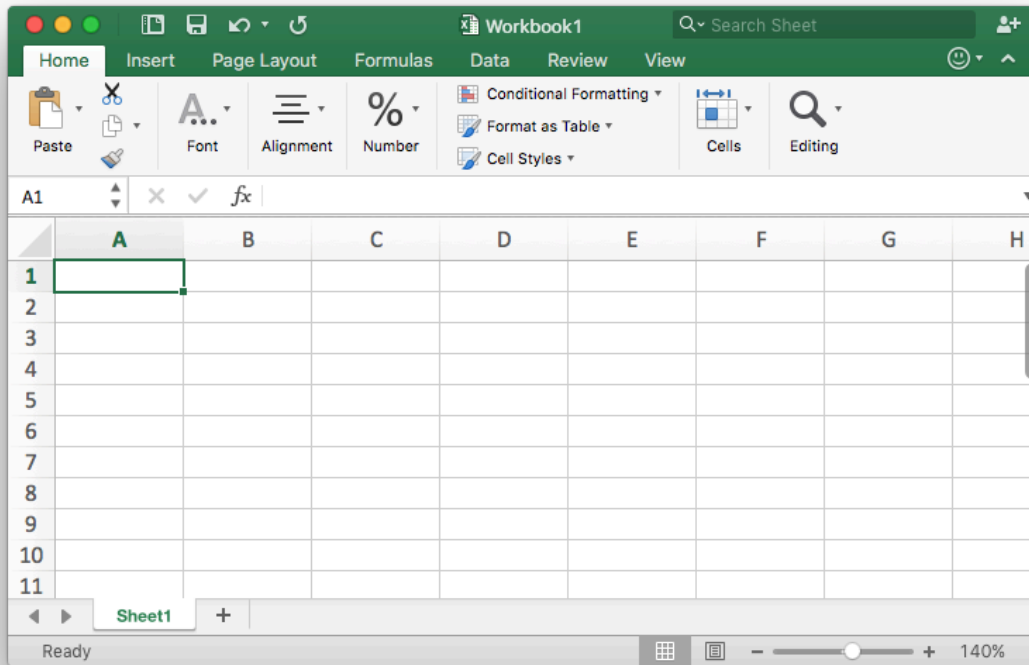


The graphic shows that there were relatively few Juniors and almost no Seniors during the Fall 2001 semester. This makes sense when you note that in the Fall of 2001, the two-year junior college known as Ricks College officially transitioned to become the four-year university BYU-Idaho. This graphic demonstrates how the number of Juniors and Seniors increased to be similar to the number of Sophomores just ten years later. Interestingly, in the Fall of 2011 the number of incoming Freshman was still much greater than any of the Sophomore, Junior, or Senior classes. This shows how a good graphic will often lead to new questions. In this case, the new question deals with why there are so many Freshman not continuing into their sophomore year at BYU-Idaho.

Excel Instructions - Bar Charts

Step 1

Open a "New Workbook" in Excel.



Step 2

Enter the data as shown in the screenshot below.

Workbook1

Search Sheet

Home Insert Page Layout Formulas Data Review View

Paste Font Alignment Number Conditional Formatting Format as Table Cell Styles Cells Editing

C5 2477

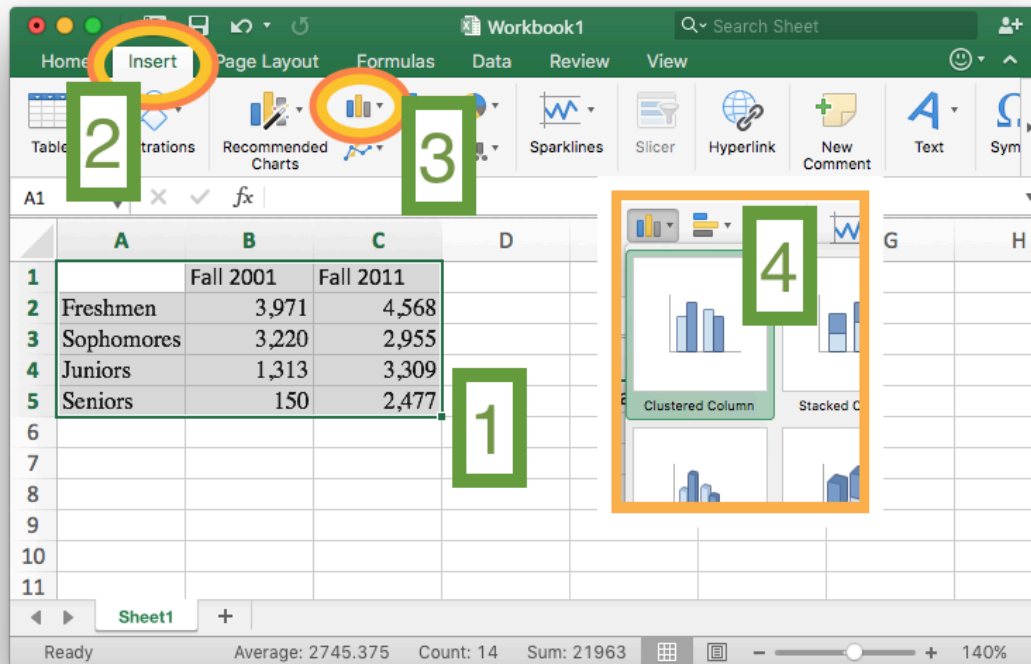
	A	B	C	D	E	F	G	H
1		Fall 2001	Fall 2011					
2	Freshmen	3,971	4,568					
3	Sophomores	3,220	2,955					
4	Juniors	1,313	3,309					
5	Seniors	150	2,477					
6								
7								
8								
9								
10								
11								

Sheet1

Ready 140%

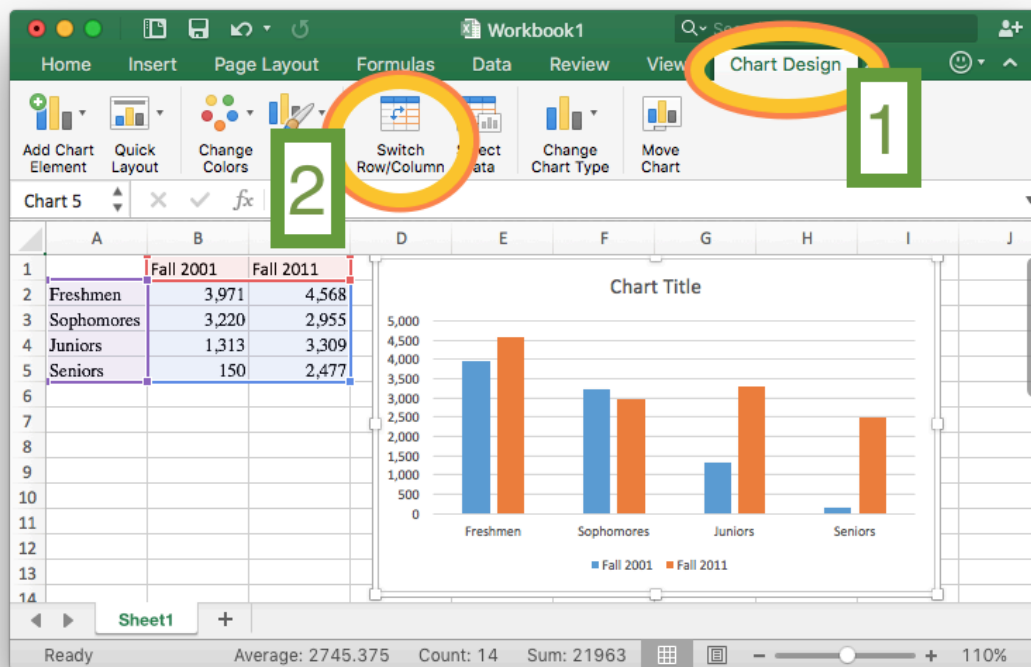
Step 3

- [1] Highlight the data table you created.
- [2] Select the "Insert" tab in the top menu.
- [3] Select the "Barplot" icon.
- [4] Select the "Clustered Column" icon.



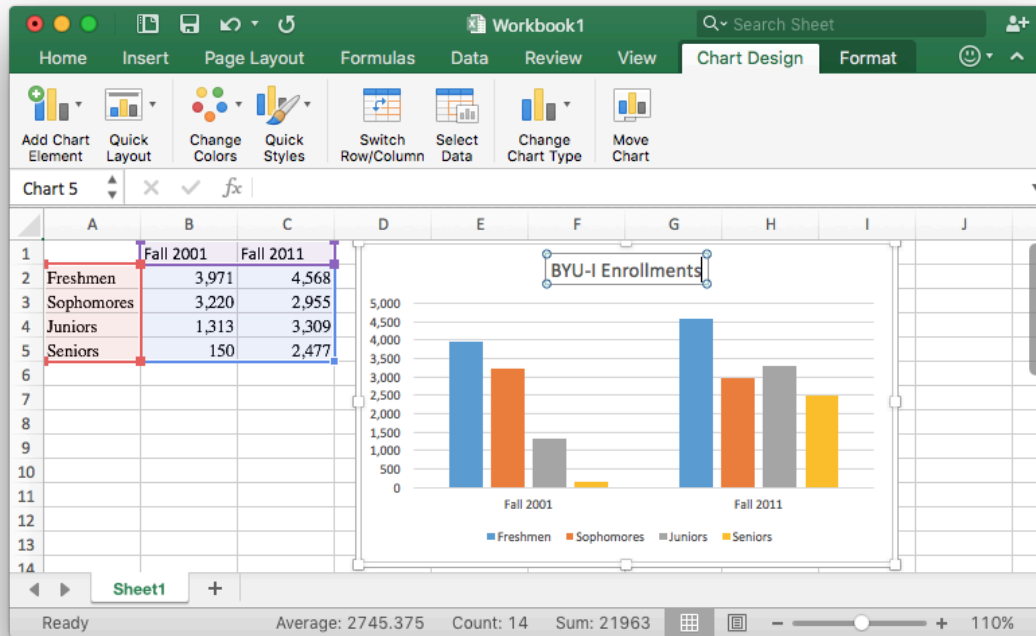
Step 4

Select the "Chart Design" tab in the top menu. - Select the "Switch Row/Column" icon.



Step 5

Give the chart a title.



Pie Charts

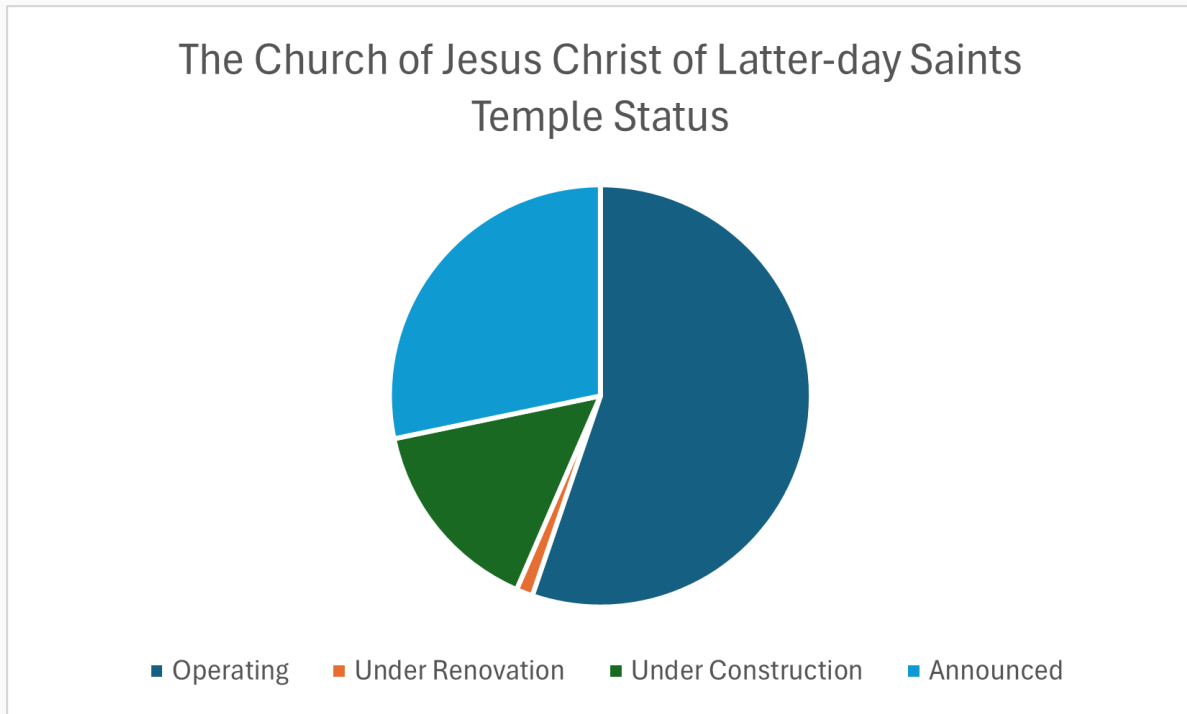
The pie chart is a sometimes useful alternative to the bar chart. Like the bar chart it is for displaying data that has several categories. Unlike the bar chart, the pie chart more readily displays the percentage of the whole “pie”, or data set, that belongs to each “slice”, or category, in the data.

Example 3

According to the [Temple List](#) of The Church of Jesus Christ of Latter-day Saints, there were 315 temples in the spring of 2023. The website indicates that there were “174 operating, 4 under renovation, 48 under construction, and 89 announced.” Create a pie chart that depicts this information.

Solution:

The following pie chart depicts this information and readily communicates that most temples are currently operating, but a very small percentage are under renovation.

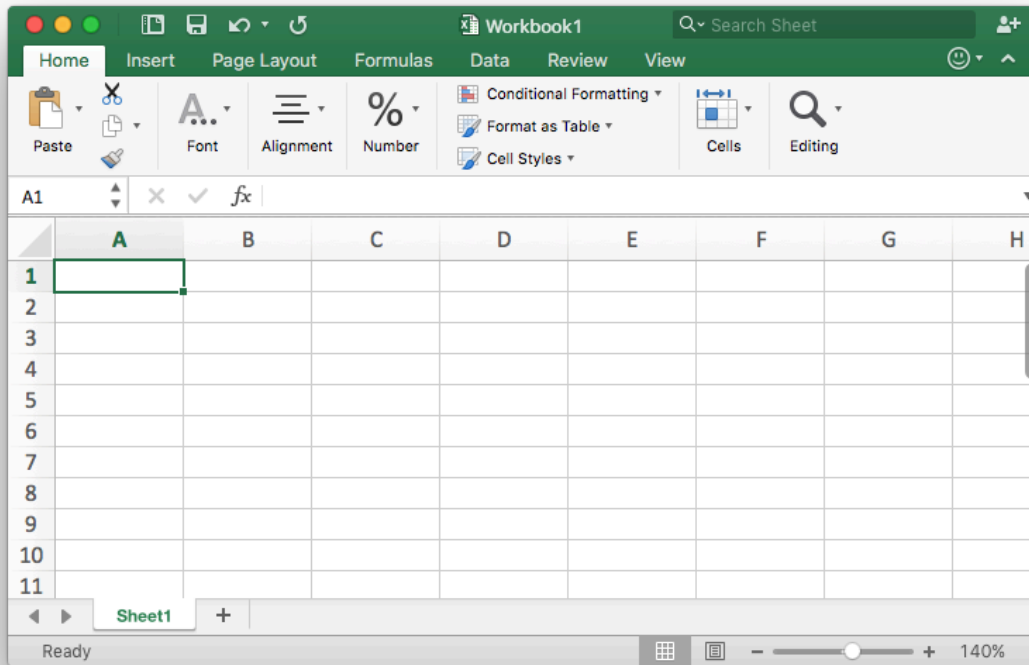


Excel Instructions - Pie Charts

To make the above pie chart:

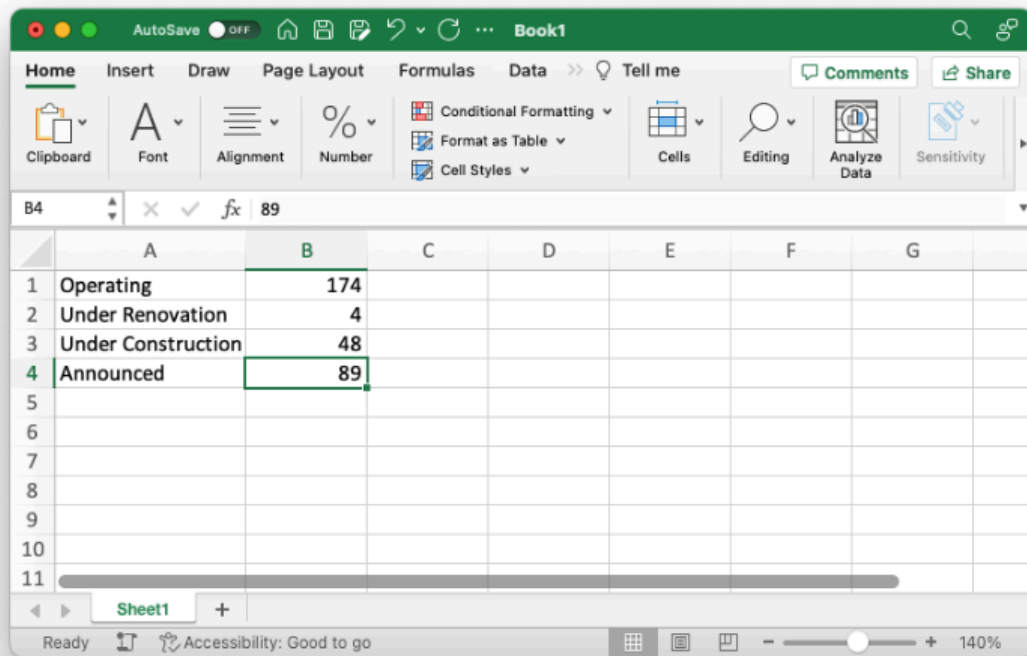
Step 1

Open a "New Workbook" in Excel.



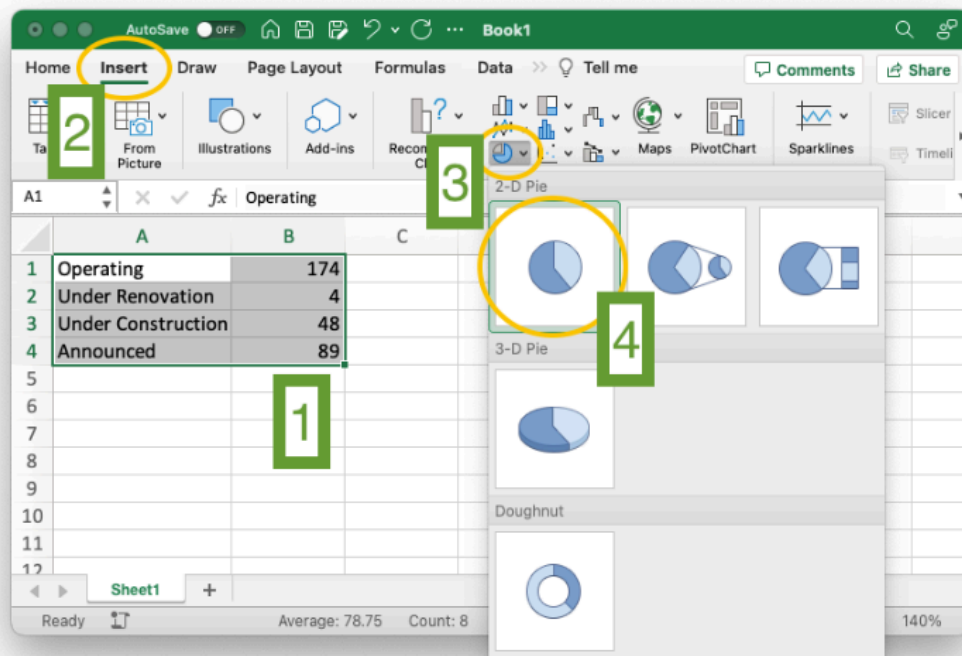
Step 2

Enter the data as given.



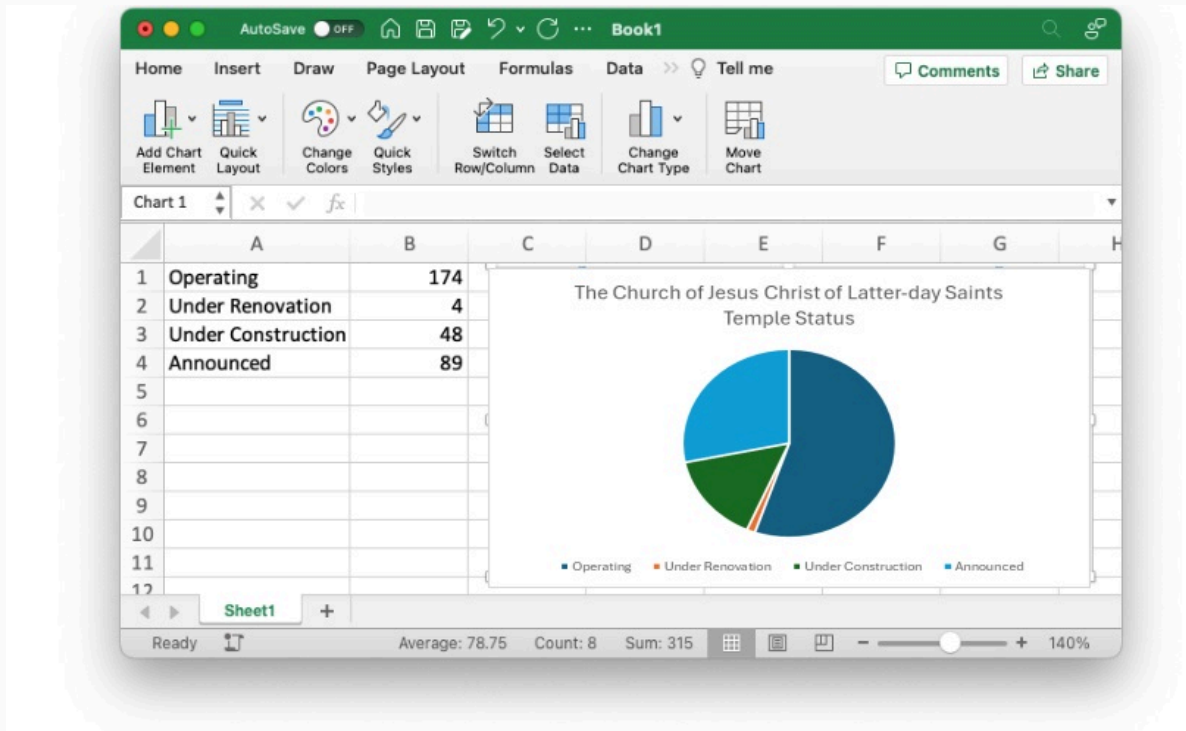
Step 3

- [1] Highlight the data table you created.
- [2] Select the "Insert" tab in the top menu.
- [3] Select the "Piechart" icon.
- [4] Select the "Pie" icon.



Step 4

Give the chart a title.



Time Series Plots

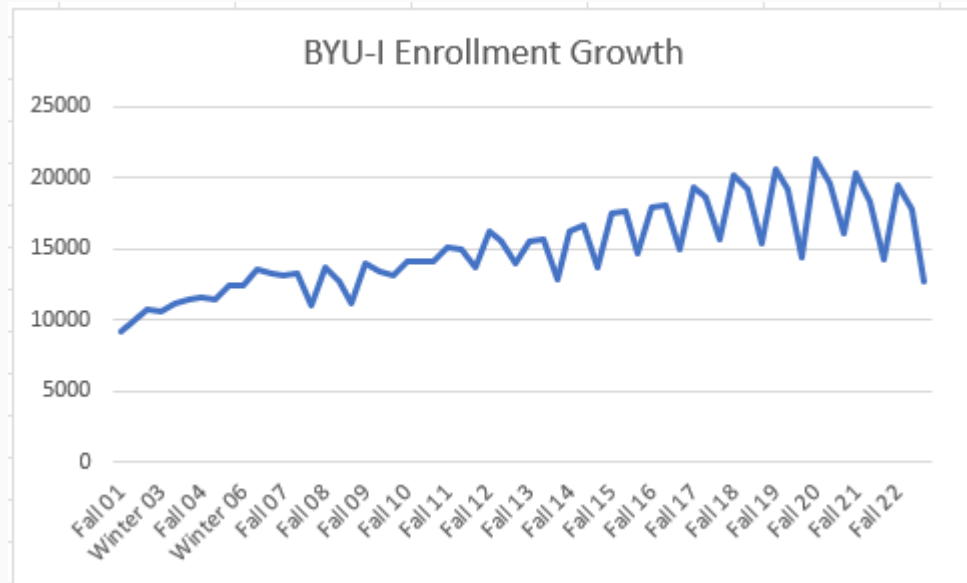
We are often interested in looking at patterns and trends in data over time. This is usually done with time series plots. These plots reveal important information about growth, decay, or repeating (cyclical) patterns.

Example 4

Create a graphic that shows the first 15 years of growth of the on-campus BYU-Idaho enrollments since BYU-Idaho officially began in the Fall semester of 2001.

Solution:

A time series plot is perfect in this situation because we want to see how the number of on-campus students at BYU-Idaho has changed over time. We will use the total of part-time and full-time students that were living on campus during that semester.



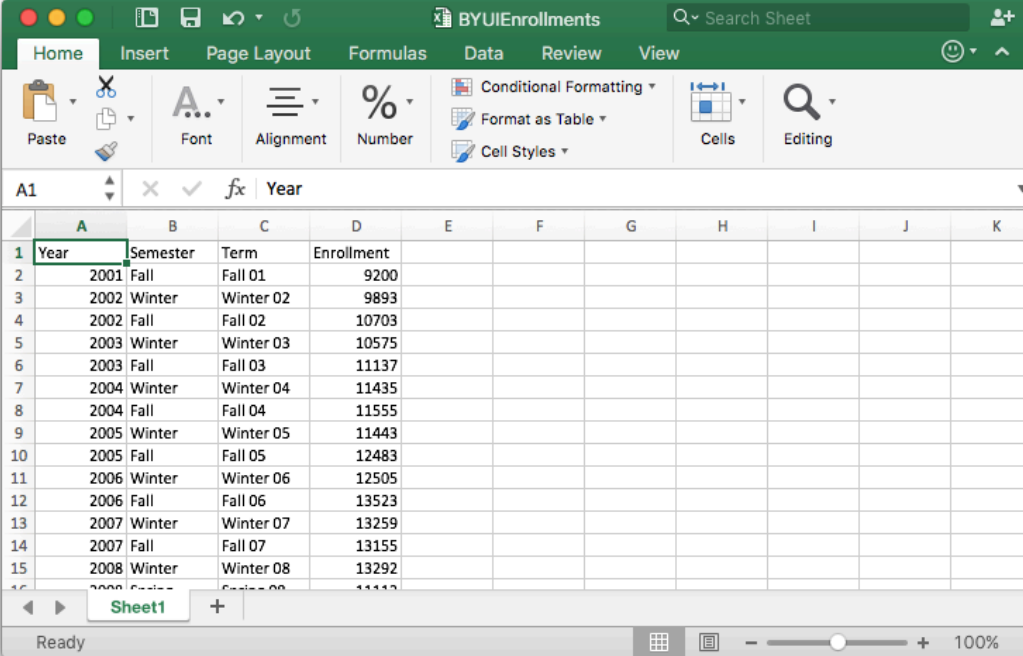
Notice in the graphic that the Spring semester is not officially part of the data until 2008. Also, note the cyclical pattern within each year that typically the Spring semester has the lowest enrollments of a given year. The overall growth of the university is also visible in the graph showing "a steady, upward course." [[A Steady, Upward Course](#)]

Excel Instructions - Time Series

To make the above time series plot:

Step 1

Download the [BYUIEnrollments.xlsx](#) data set and open it in Excel.



Year	Semester	Term	Enrollment
2001	Fall	Fall 01	9200
2002	Winter	Winter 02	9893
2002	Fall	Fall 02	10703
2003	Winter	Winter 03	10575
2003	Fall	Fall 03	11137
2004	Winter	Winter 04	11435
2004	Fall	Fall 04	11555
2005	Winter	Winter 05	11443
2005	Fall	Fall 05	12483
2006	Winter	Winter 06	12505
2006	Fall	Fall 06	13523
2007	Winter	Winter 07	13259
2007	Fall	Fall 07	13155
2008	Winter	Winter 08	13292
2008	Fall	Fall 08	11113

Step 2

Highlight the Term and Enrollment columns of the data set. (Be sure to highlight all the way to the bottom of the data that goes to row 61.)

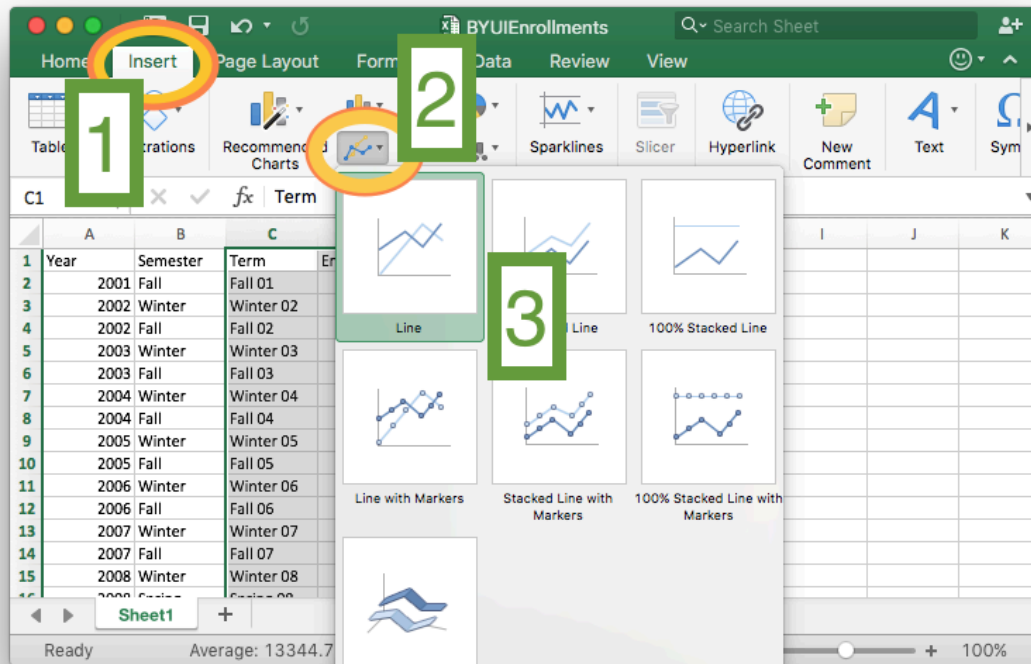
The screenshot shows the Microsoft Excel interface with the 'Insert' tab selected. The 'Line Chart' icon is highlighted in the 'Charts' group. The spreadsheet data is as follows:

	A	B	C	D	E	F	G	H	I	J	K
1	Year	Semester	Term	Enrollment							
2	2001	Fall	Fall 01	9200							
3	2002	Winter	Winter 02	9893							
4	2002	Fall	Fall 02	10703							
5	2003	Winter	Winter 03	10575							
6	2003	Fall	Fall 03	11137							
7	2004	Winter	Winter 04	11435							
8	2004	Fall	Fall 04	11555							
9	2005	Winter	Winter 05	11443							
10	2005	Fall	Fall 05	12483							
11	2006	Winter	Winter 06	12505							
12	2006	Fall	Fall 06	13523							
13	2007	Winter	Winter 07	13259							
14	2007	Fall	Fall 07	13155							
15	2008	Winter	Winter 08	13292							

Step 3

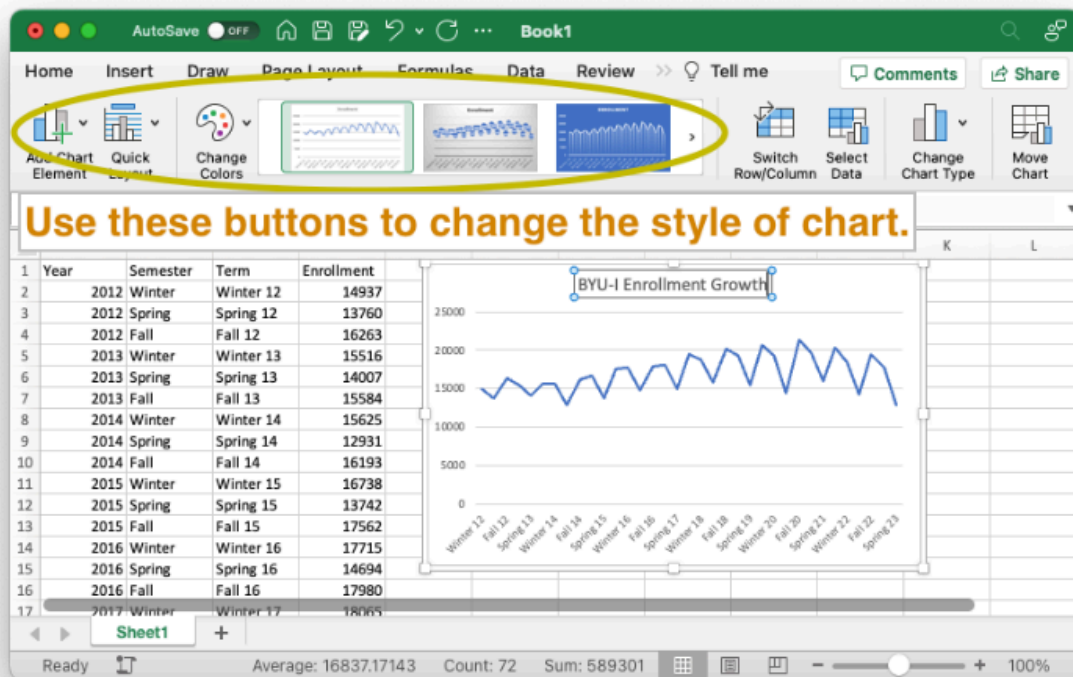
Then,

- [1] Select the "Insert" tab in the top menu.
- [2] Select the "Line Chart" icon.
- [3] Select the "Line" icon.



Step 4

Edit the chart title and change the styling of the chart if you want.



Scatter Plots

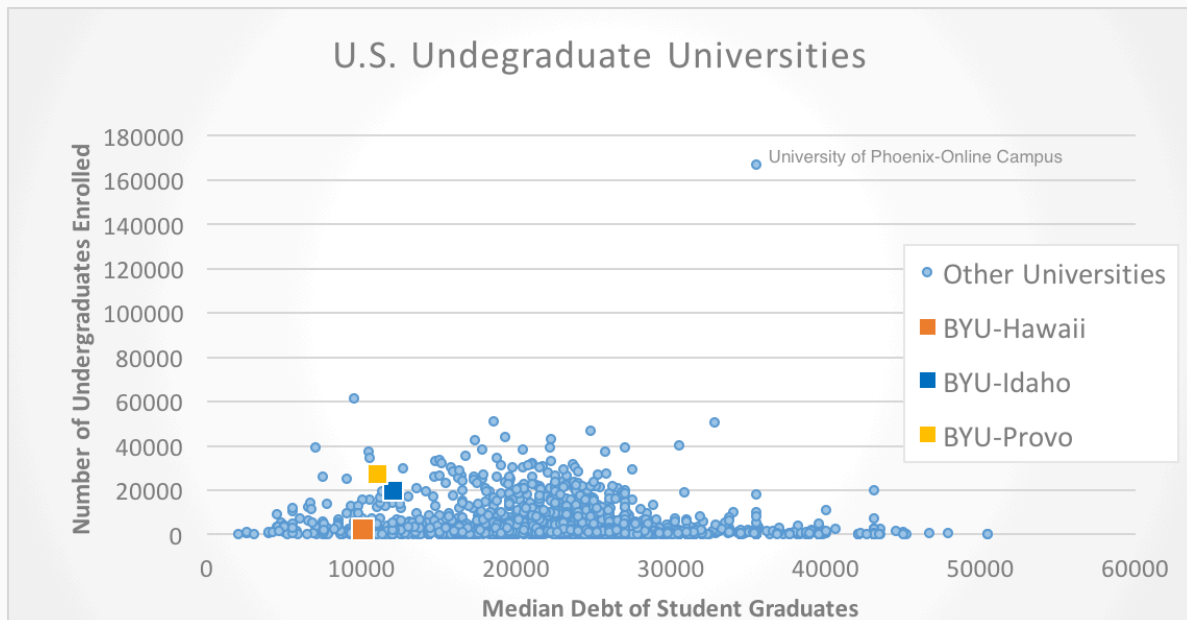
The scatter plot is similar to a time series plot, but it does not require time to be part of the plot. It provides a visual understanding of how two different measurements are related.

Example 5

The US Department of Education has put together a useful resource called the [College Scorecard](#) for college bound students to make important decisions about where to attend college. An important consideration for college students is the amount of student loan debt they may take on during college. Use the information provided by the College Scorecard to determine whether students who attend bigger universities end up with more debt.

Solution:

The scatter plot below represents the data from the College Scorecard website. Notice that the horizontal axis represents the median debt of student graduates. The vertical axis represents the number of undergraduates enrolled.



We want to use this graph to answer the question: Do students who attend bigger universities end up with more debt? If this were the case, we would expect to see the data points follow a linear pattern with large values on the horizontal axis corresponding to large values of the vertical axis. This is not the case. There really isn't a pattern to the data.

Since there is not a pattern in the data, we know there is not any correlation between the size of the university and the median debt load of its graduates. It is interesting to note that all three Brigham Young University schools are on the lower end of the median student loan debt spectrum. However, BYU-Idaho, has the highest median student loan debt of the three while being in the middle for the number of undergraduates enrolled.

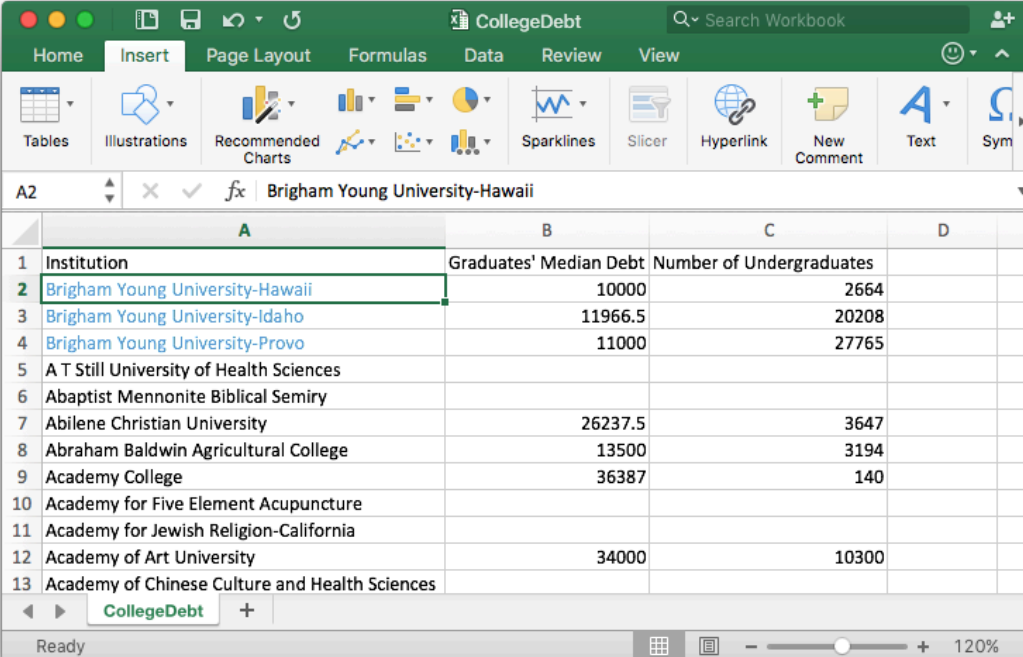
Clearly there are other factors that are not considered here that would provide better explanations of what brings on student debt.

Excel Instructions - Scatter Plots

To make the above scatter plot:

Step 1

Download the [CollegeDebt.xlsx](#) data set and open it in Excel.



	A	B	C	D
1	Institution	Graduates' Median Debt	Number of Undergraduates	
2	Brigham Young University-Hawaii	10000	2664	
3	Brigham Young University-Idaho	11966.5	20208	
4	Brigham Young University-Provo	11000	27765	
5	A T Still University of Health Sciences			
6	Abaptist Mennonite Biblical Semiry			
7	Abilene Christian University	26237.5	3647	
8	Abraham Baldwin Agricultural College	13500	3194	
9	Academy College	36387	140	
10	Academy for Five Element Acupuncture			
11	Academy for Jewish Religion-California			
12	Academy of Art University	34000	10300	
13	Academy of Chinese Culture and Health Sciences			

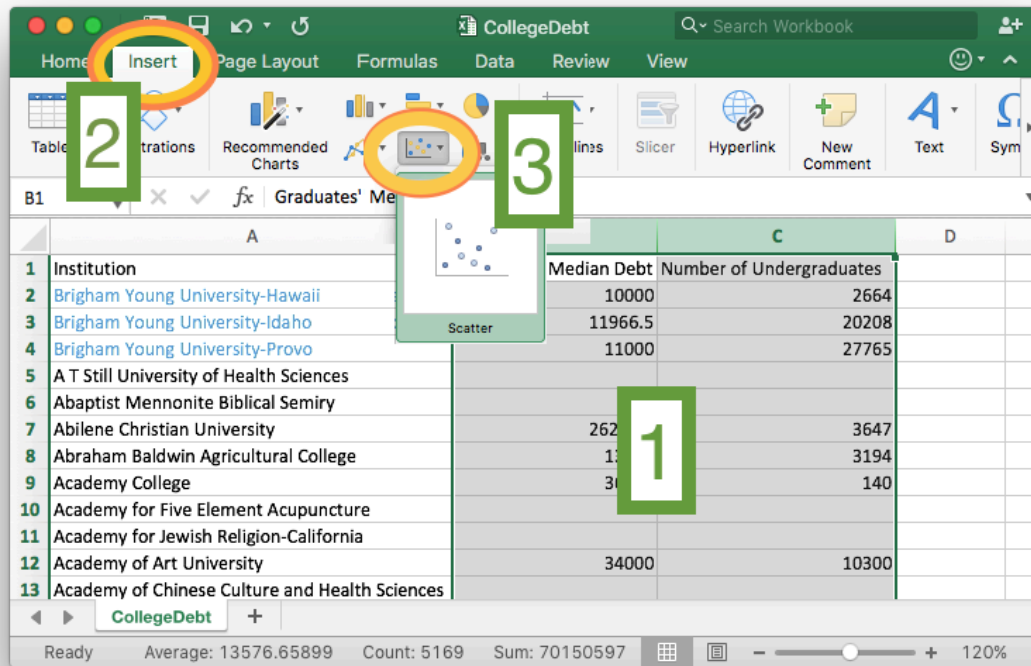
Step 2

Since there are 2,953 rows in this data set, highlighting the data is done as follows. - First, click on the B column label, which is above the Graduates' Median Debt entry. This will highlight the entire B column. - Second, hold down Shift and click on the C column label, which is above the Number of Undergraduates entry. This will expand the selection to include the C column.

Step 3

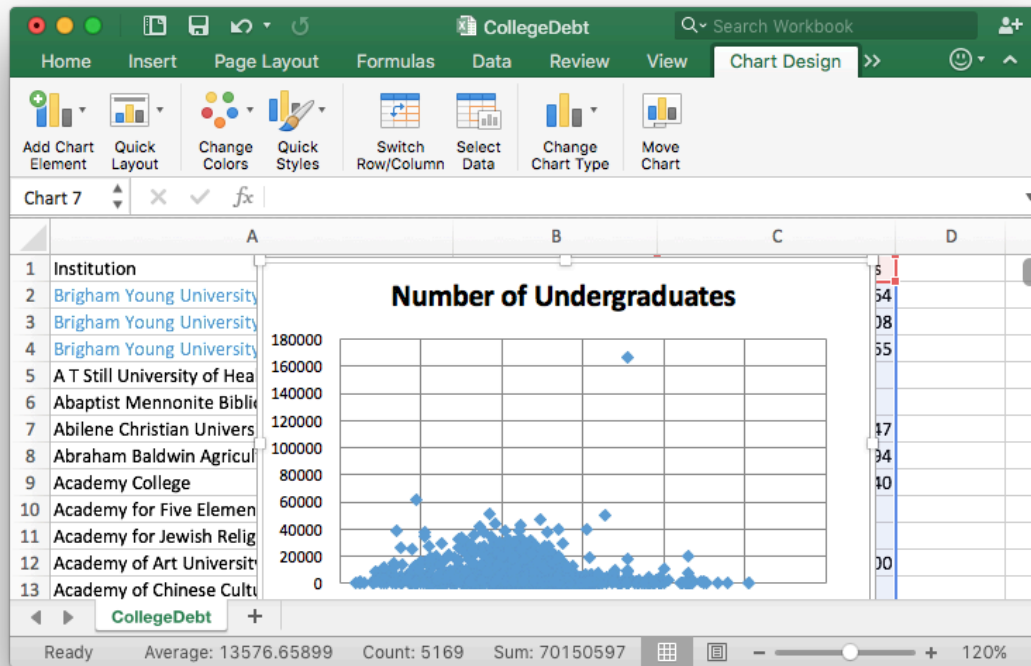
Once the data is selected [1], then

- [2] Select “Insert”.
- [3] Select the “Scatterplot” icon and the “Scatter” option.



Step 4

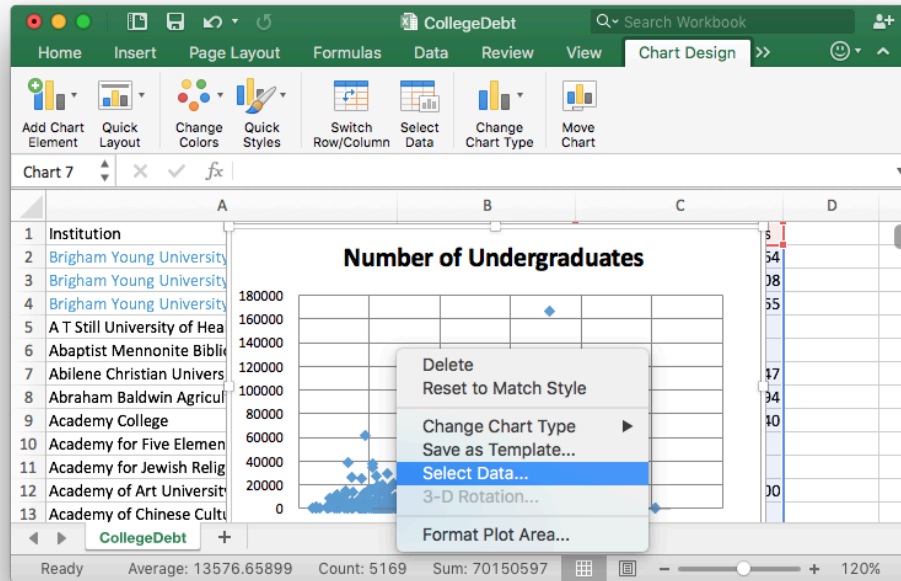
This is a scatter plot and it would be sufficient to stop here.



Step 5 [Bonus step: Click for instructions on how to label individual data points on the graph.]

Step 5A

To add the BYU colleges to the plot, right click on the plot and choose "Select Data..." from the menu.



Step 5B

A new window will appear entitled "Select Data Source."

Select Data Source

Range Details

Chart data range:

Legend entries (Series):

Number of Undergraduates

Name:

X values:

Y values:

Horizontal (Category) axis labels:

Hidden and Empty Cells

Show empty cells as:

☐ Show data in hidden rows and columns

Step 5C

Click the + sign and enter the following information exactly as shown.

Select Data Source

Range Details

Chart data range:

The Chart Data Range is too complex to be displayed. If a new Data Range is selected, it will replace all of the series on the Series Panel.

Legend entries (Series):

Number of Undergraduates
Series 2

Name:

X values:

Y values:

Horizontal (Category) axis labels:

Hidden and Empty Cells

Show empty cells as:

☐ Show data in hidden rows and columns

Step 5D

Note that the information for Brigham Young University-Hawaii has appeared. Now, click the + sign again and enter the following information exactly as shown.

Select Data Source

Range Details

Chart data range:

The Chart Data Range is too complex to be displayed. If a new Data Range is selected, it will replace all of the series on the Series Panel.

Legend entries (Series):

Number of Undergraduates	Name: <input type="text" value="=CollegeDebt!A3"/>
Brigham Young University-Hawaii	X values: <input type="text" value="=CollegeDebt!B3"/>
Series 3	Y values: <input type="text" value="=CollegeDebt!C3"/>

Horizontal (Category) axis labels:

Hidden and Empty Cells

Show empty cells as:

☐ Show data in hidden rows and columns

Step 5E

The information for Brigham Young University-Idaho has appeared. Again, click the + sign and enter the following information exactly as shown, then press OK.

Select Data Source

Range Details

Chart data range:

The Chart Data Range is too complex to be displayed. If a new Data Range is selected, it will replace all of the series on the Series Panel.

Legend entries (Series):

Number of Undergraduates	Name: <input type="text" value="=CollegeDebt!A4"/>
Brigham Young University-Hawaii	X values: <input type="text" value="=CollegeDebt!B4"/>
Brigham Young University-Idaho	Y values: <input type="text" value="=CollegeDebt!C4"/>
Series4	

Horizontal (Category) axis labels:

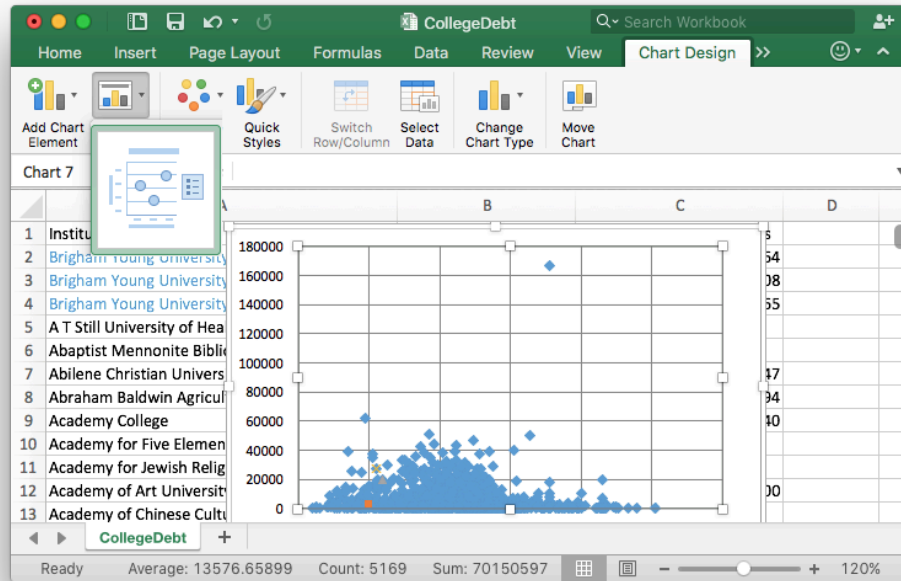
Hidden and Empty Cells

Show empty cells as:

☐ Show data in hidden rows and columns

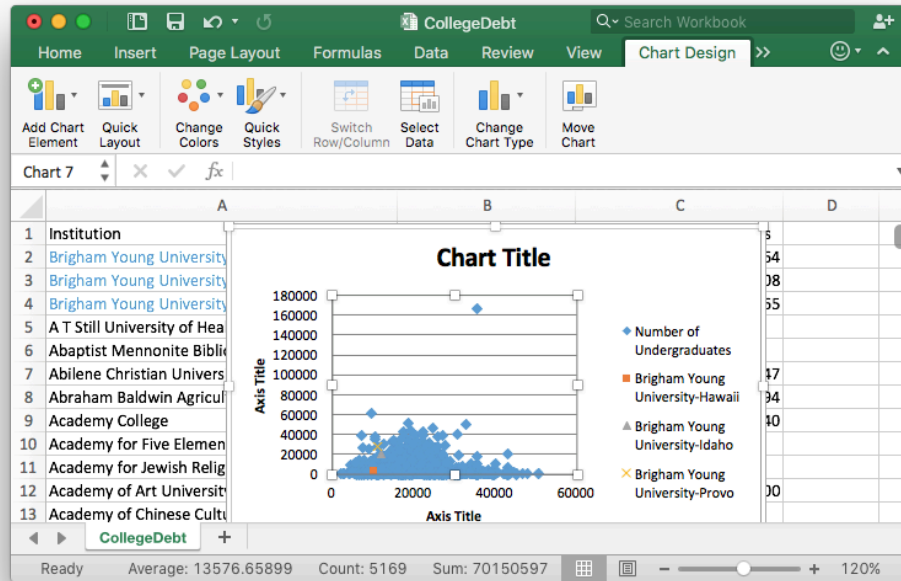
Step 5F

Finally, select the "Quick Layout" tab in the top left menu and select the first option.



Step 5G

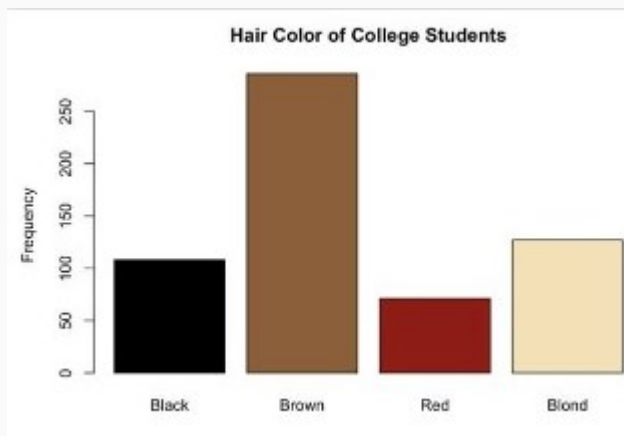
To complete the process use the "Quick Styles" menu to find a style you like, then double click on the different elements of the chart to complete the finishing touches.



Practice: Reading Graphs

Problem 1

What patterns do you notice in the following graphic?

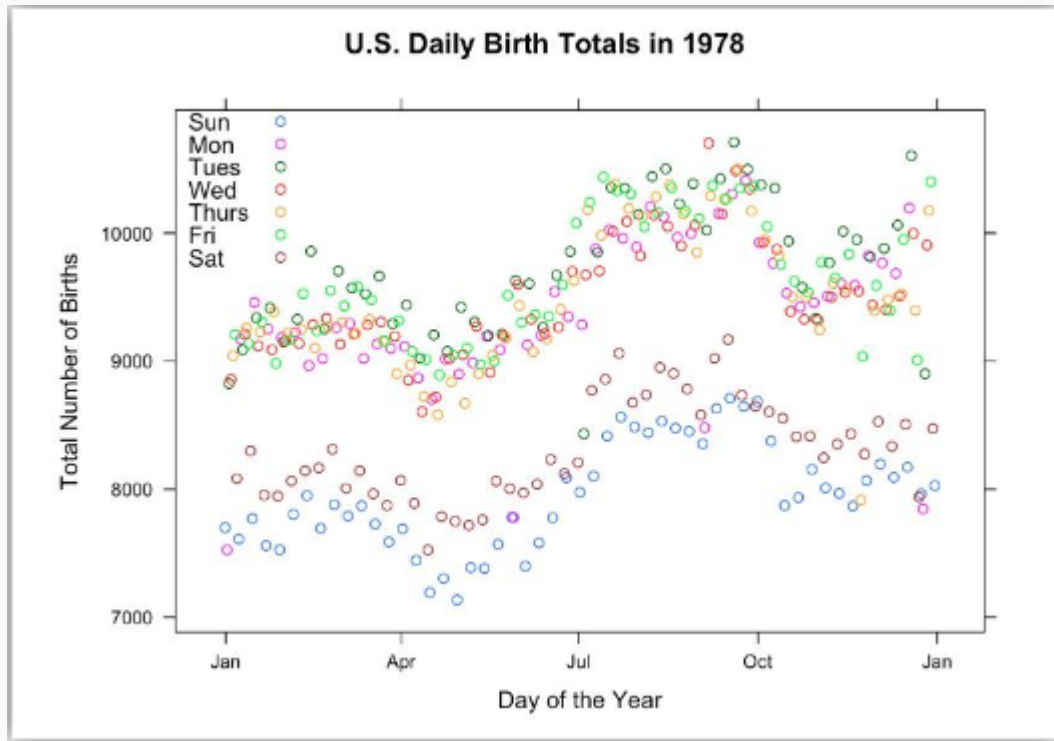


How Did I Do?

More college students have brown hair than other colors. The least common hair color is red. Black and blond are almost equal and combined do not total the number of college students with brown hair.

Problem 2

What patterns do you notice in the following graphic?



How Did I Do?

There are typically fewer births on Saturdays and Sundays than the other days of the week. July, August, September, and October show higher number of births than the other months of the year.

Bicycle Safety Example

Graphics are a helpful tool that can be used in Step 3 of the Quantitative Reasoning Process. Consider the following example.

1 Understand the Problem

Anita has three teenage children that ride their bikes to school every day. They ride 1.3 miles down a road that has a bike lane, but no sidewalk. The city wants to add a turning lane to the middle of the road. To make the turning lane fit in the space they have, the city will need to narrow the bike lane. Anita is concerned for the safety of her children and wants to look at some research on how the width of the bike lane affects safety.

2 Identify Variables & Assumptions

After looking at some studies, Anita found a research study that gave some data using the following variables:

- The distance from the cyclist to the center line on the road.
- The distance from the cyclist to a passing car.

In order to apply the data she found to the situation in her neighborhood, Anita has to make some important assumptions.

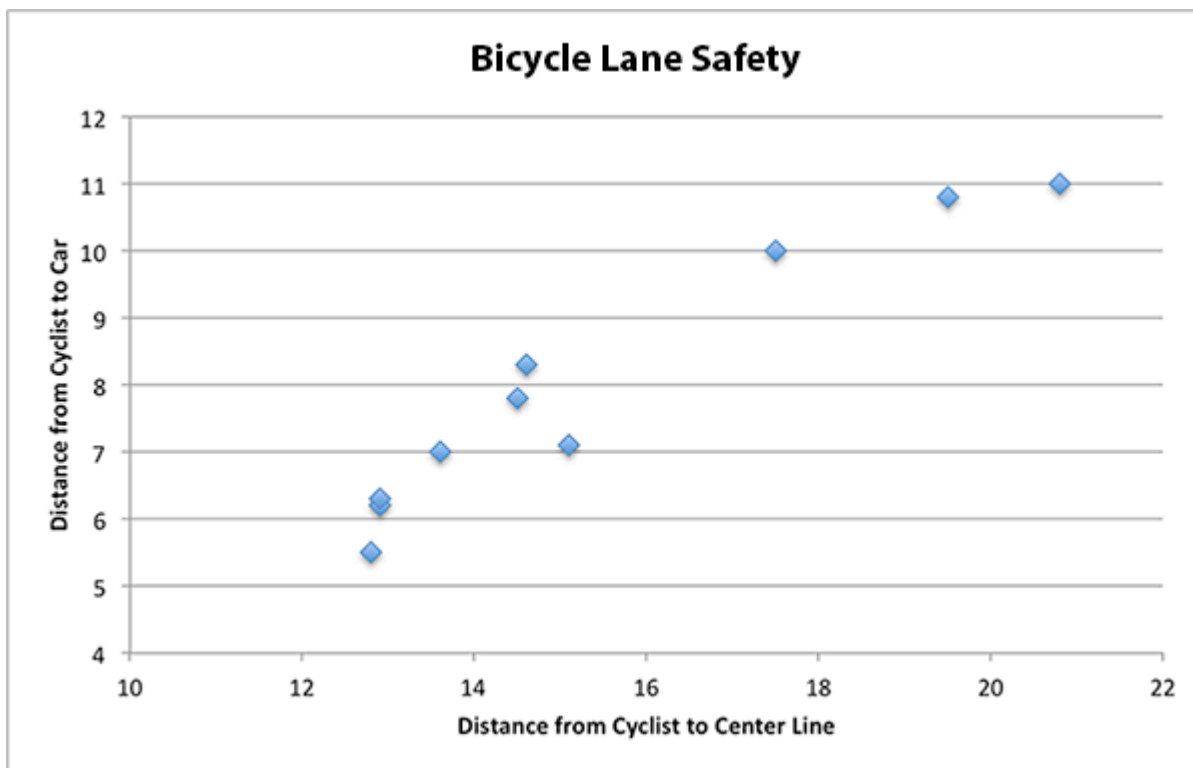
- She will assume that the closer a cyclist is to a passing car the less safe it is for the cyclist.
- She will assume that the narrower the bike lane is, the narrower the road must be. This means the distance from the center line on the road to the cyclist will be smaller.

3 Apply Quantitative Tools

The following data is given in the research study Anita found: [3](#)

Distance from Cyclist to Center Line	Distance from Cyclist to Car
12.8	5.5
12.9	6.2
12.9	6.3
13.6	7.0
14.5	7.8
14.6	8.3
15.1	7.1
17.5	10.0
19.5	10.8
20.8	11.0

Because this data relates two different variables, Anita decided to make a scatter plot of the data.



Because there is a pattern in this data, Anita was able to notice that when the distance from the cyclist to the center line is smaller, so is the distance from the cyclist to a passing car.

4 Make an Informed Decision



Anita was able to conclude that the scatter plot showed the road would be less safe if they narrowed the road to make room for the new sidewalk. This would have to decrease the distance from the cyclist to the center line and the graph shows this means the distance between the cyclist and a passing car would then be smaller. Anita's assumption is that this makes the road less safe for her children as they ride their bikes to school.

Based on the data, her graph, and her conclusion that the addition of a turning lane was going to make the road less safe, Anita decided to attend a city council meeting and express her concerns.

The city council listened respectfully to Anita's concerns. Anita's neighbors shared her concerns and also attended the meeting. As a result, the city decided to change their plans for making a narrow bicycle lane. This meant that they had to change their plans for the turning lane, but they were able to modify their plans in a way that still accommodated traffic.

5 Evaluate Your Reasoning



In this situation Anita's decision led to her desired outcome. As she reflected on the progress she made, she realized that having data to back up her concerns was helpful when she presented to the city council.

Another Example

Although Anita's story is fictional, similar situations are quite common in real-life. In December 2016, the city of Missoula, Montana considered reducing some two-lane streets to one lane with a bicycle lane.⁴ Some city residents were supportive of the change because it would improve safety for bicyclists and pedestrians, would slow traffic in a residential area, give more room for parking, and reduce crashes. The city has data on the amount of traffic on the street and found that a one-lane road could accommodate the current traffic without impacting vehicle flow.

Other city residents were opposed to the change, however. In particular, business owners were concerned about the impact reduced traffic might have on their businesses. Other opponents are concerned that the change is overly concerned about cyclists when cyclists are a very small portion of the traffic on the street.

The city council is taking comments from members of the community and concerned citizens could definitely use data and graphics, similar to the one used by Anita, to support their position.

Lesson Checklist

After completing this lesson you should be able to do the following:

- Create bar charts, pie charts, time series plots, and scatter plots of data in Excel.
 - Interpret the information provided in bar charts, pie charts, time series plots, and scatter plots.
 - Identify the type of chart (bar chart, pie chart, time series, scatter plot) that would be most appropriate for a given scenario.
 - Make a contextual conclusion from a given chart (bar chart, pie chart, time series, scatter plot).
-

¹ [The Utah Statesman, December 6, 2012 \(usu.edu\)](#). See article "USU Researchers assist in Alzheimer's breakthrough"

² [The Utah Statesman, December 6, 2012 \(usu.edu\)](#). See article "USU Researchers assist in Alzheimer's breakthrough"

³ <http://www.statsci.org/data/general/cyclist.html>

⁴ <http://www.missoulacurrent.com/government/2016/12/streets-single-lane/>



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Access it online or download it at https://books.byui.edu/math_108x/lesson_6_visualizing_data.

Lesson 7: Functions in Excel

Opening Story

Lesson 7 - Opening Story

[L7 Opening Story Transcript](#)

Introduction

Like Craig, there are many reasons why you may need to borrow money at some point in your lifetime. A loan can provide funds for you to purchase a home, buy a car, get an education, or start a small business without the delay that would be caused by saving the necessary money. However, planning ahead and saving money prior to a purchase can help us avoid borrowing money unnecessarily.

Our church leaders have counseled us to be wise in how we manage our personal finances and to avoid debt.

"Look to the condition of your finances. Discipline yourself in your purchases, avoiding debt to the extent you can. In most cases, you can avoid debt by managing your resources wisely. If you do incur debt, such as a reasonable amount in order to purchase a modest home or complete your education, work to repay it as quickly as possible and free yourself from bondage. When you have paid your debts and accumulated some savings, you will be prepared for financial storms that may come your way. You will have shelter for your family and peace in your heart."¹

The Quantitative Reasoning Process can be a great tool to help us follow this advice and manage our resources wisely. Contributing money to a savings account can help us acquire finances we need to make purchases. However, if you do need to borrow money, it is essential that you understand the calculations necessary to make an informed decision. In this lesson we will learn about functions and how they can be used as part of the Quantitative Reasoning Process to make informed decisions about savings and loans.



In the following video, Henry J. Eyring, former president of BYU-Idaho shares a personal experience he had in following the council of church leaders to get out of debt.

Functions

Mathematical functions are a useful mathematical tool. They provide a way to think about complex situations, such as savings and loans.

*A **function** is a rule that assigns the input value(s) to a single output value.*

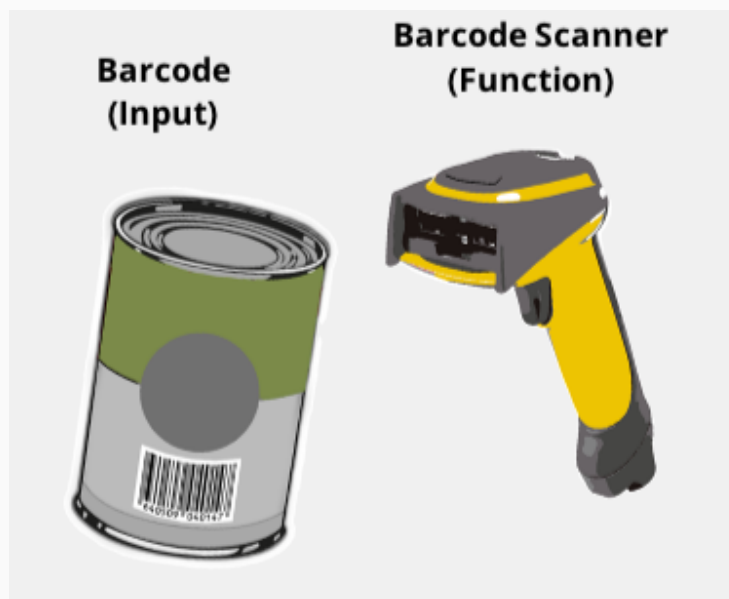
It is important to understand that a function deals with inputs and outputs. The function itself provides a way to determine which output corresponds with a given input. Here is an analogy to help explain functions:

Example 1

In some stores, items have barcodes. This is similar to the **input** of a function.



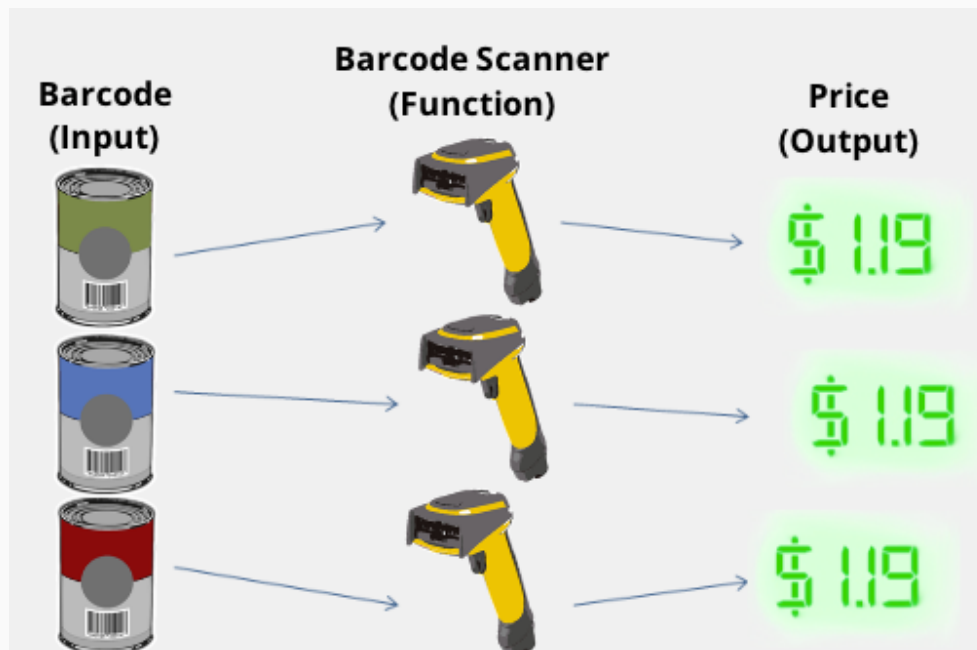
When you check out of the store, the barcode scanner is used to determine the price. The barcode scanner acts as a **function**.



The scanner reads the barcode and gives a price for the item. The price is the **output** of the function.



Notice that many different **inputs** can have the same **output**.



However, any given input has to correspond to a single output. Imagine the confusion of the scanner gave several possible prices for one barcode!



A function is a rule that assigns the input value or values to a single output value

Although functions always assign a single output once the input has been chosen, it is possible for a function to have several inputs. For example, the Body Mass Index (BMI) is used to determine if an individual has healthy body weight. The BMI is calculated using a function with two inputs. If you put your body weight and height into the function as inputs, the output is your BMI.

“This” is a function of “That”

In English, we often say that one thing is a function of something else. Or one variable depends on another variable. These phrases relate to the mathematical concept of a function. For example, suppose someone said one of the following:

- “How well you do on the test is a function of how much time you spend studying.”
- “Your starting salary when you graduate depends on your major.”
- “The price of a used car depends on the make and model of the car, as well as its age and condition.”
- “Your monthly cash flow is a function of your income and expenses.”
- “The amount you can borrow for the purchase of a home is a function of your net monthly income.”
- “Your Body Mass Index (BMI) depends on your height and weight.”
- “What you get out of sacrament meeting is a function of the amount of preparation you do.”

These statements all correspond to the mathematical definition of a function. Each situation includes at least one input and an output. The inputs and outputs of each of these statements are summarized in the following table:

Input(s)	Output
Time spent studying	How well you do on the test
Your college major	Starting salary after graduation
Make, model, age, and condition of car	Price of used car

Income and expenses	Monthly cash flow
Net monthly income	Amount you can borrow for a home
Height and weight	Body Mass Index (BMI)
Preparation for church	What you get out of sacrament meeting

Practice: “This” is a function of “That”

Problem 1

Complete the following sentence by filling in the blanks with the correct words.

A function is a _____ that assigns an _____ to a single _____.

Word choices:

- rule
- output value
- input value

How Did I Do?

A function is a **rule** that assigns an **input value** to a single **output value**.

Problem 2

Read the following scripture, which includes a “function” sentence. Then identify the *input* and *output* of the function.

“There is a law, irrevocably decreed in heaven before the foundations of this world, upon which all blessings are predicated—And when we obtain any blessing from God, it is by obedience to that law upon which it is predicated.” (Doctrine and Covenants 130:20–21)

Input: _____

Output: _____

Word choices:

- law
- obedience
- blessings
- obtain

How Did I Do?

Input: Obedience

Output: Blessings

Problem 3

Using the words below, write a “function” sentence that makes sense.

- Assignments
- Tests
- Final Grade
- Score
- Quizzes

How Did I Do?

Your final grade is a function of the scores you receive on your tests, assignments, and quizzes.

Function Notation

In addition to knowing the inputs and outputs, you need to know the rule associated with each function. In mathematics, we use function notation to write the rules for functions. For example:

$$f(x) = 5x - 2$$

This function is named f . The input is x and the rule to find the output is $5x - 2$. If we used an input of $x = 1$, we would get

$$f(1) = 5(1) - 2 = 3.$$

We write this as $f(1) = 3$. If we used an input of $x = 2$, we would get

$$f(2) = 5(2) - 2 = 8.$$

This would be written as $f(2) = 8$. And so on.

We could use this method to list some of the inputs and outputs for this function.

x	$f(x)$
1	3
2	8
3	13
4	18
5	23
6	28

From the table we see that $f(4) = 18$. This means if there is an input of 4, the output is 18.

Practice: Function Notation

Problem 4

For the function $f(x) = 3x - 1$, fill in the table below with the correct values of $f(x)$.

x	$f(x)$
1	<input type="text"/>
2	<input type="text"/>
3	<input type="text"/>
4	<input type="text"/>
5	<input type="text"/>
6	<input type="text"/>

How Did I Do?

x	$f(x)$
1	2
2	5
3	8
4	11
5	14
6	17

Examples of Functions

Example 1 - One Input

Assume you have a savings account that earns 0.95% interest each year (compounded daily). If you put \$500 into the account initially, the amount of money in your account will grow over time as interest is added to your account. A function for the amount of money in the account after t months is

$$f(t) = 500(1.000026027)^{30.4167t}$$

(Note: We will learn more about where this function comes from in Week 9. We included a lot of decimal places for this function because, with this particular function, if you round the decimals too much it introduces a lot of rounding error and you lose accuracy.)

Use this function to answer the following questions:

- What is the input of this function?
- What is the output of the function?
- How much money will you have in your account after 4 years and 2 months?

Solution:

The account balance is a function of how long the money has been in the account.

- The input is the number of months the money has been in the account.
- The output is the amount of money in the account.
- If we have had the money in the account for 4 years and 2 months (50 months total), we would use 50 as the input. The output then tells us the balance of the account:

$$f(50) = 500(1.000026027)^{(30.4167 \times 50)} = \$520.19$$

After 50 months, the account balance is \$520.19, which means we earned \$20.19 in interest over the four years and two months.

Example 2 - Two Inputs

The Body Mass Index (BMI) is a number often used to determine if an individual is a healthy weight. If w represents your weight (in pounds) and h represents your height (in inches), then the function for finding your BMI is:

$$f(w, h) = \frac{720w}{h^2}$$

Use this function to answer the following questions:

- What is the input of this function?
- What is the output of the function?
- What is the BMI of a person who weighs 125 pounds and is 63 inches tall?

Solution:

In this case, the BMI is a function of height and weight.

- The inputs of the function are height (in inches) and weight (in pounds).
- The output of the function is the individual's BMI.
- To find the BMI of a person who weighs 125 pounds and is 63 inches tall, we evaluate the function with 125 substituted in for w and 63 substituted in for h .

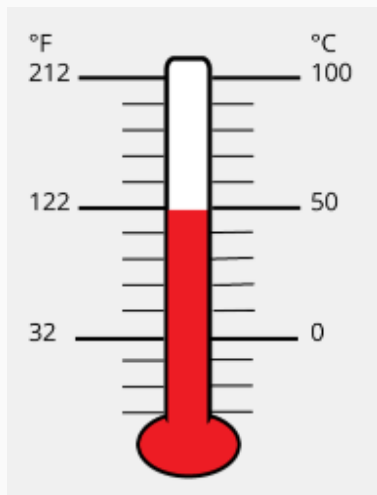
$$f(125, 63) = \frac{720 \times 125}{63^2} = 22.68$$

A healthy BMI is generally between 19 and 25, so this person would be considered to have a healthy body weight.

(Note: If body weight is measured in kilograms and height is measured in meters, the function is even simpler. In that case, $f(w, h) = \frac{w}{h^2}$.)

Practice: Check Your Understanding

Problem 5



The temperature in degrees Fahrenheit is a function of the temperature in degrees Celsius. The following function can be used to find the temperature in Fahrenheit if we are given the temperature in Celsius.

$$f(x) = \frac{5}{9}x + 32.$$

- What is the input of the function?
- What is the output of the function?
- If it is 17 degrees Celsius, what is the temperature in degrees Fahrenheit?

How Did I Do?

- Input is degrees Celsius.
- Output is degrees Fahrenheit.
- 17 degrees Celsius is 62.6 degrees Fahrenheit.

Functions in Excel

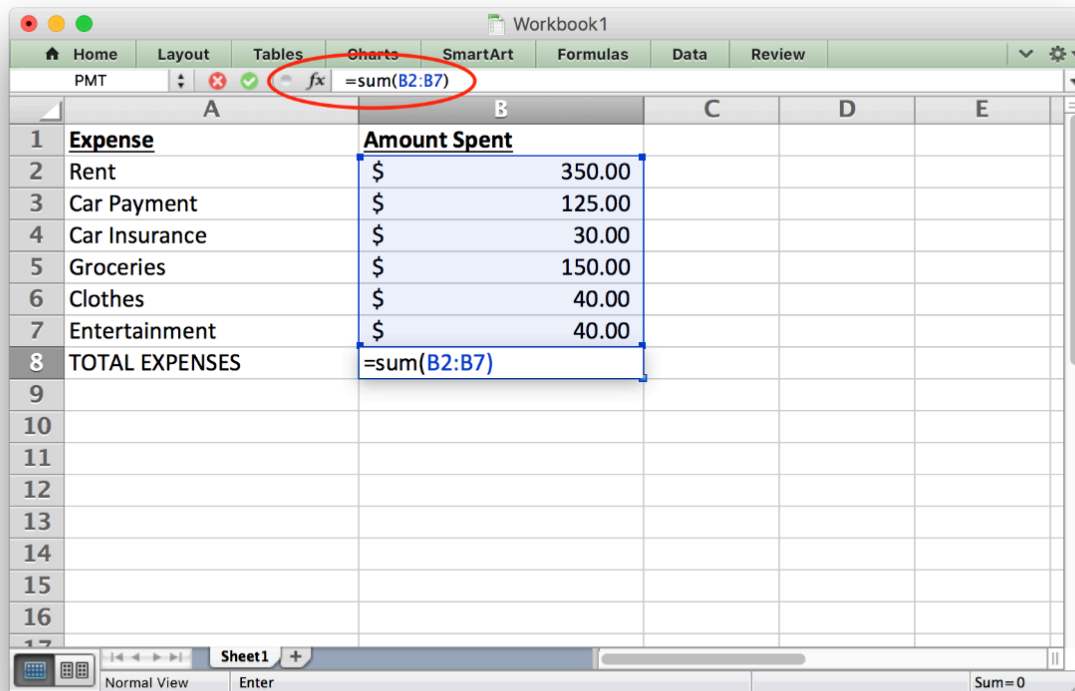
Throughout the semester we have been using Excel to do computations. Excel uses the function concept to carry out these calculations. Open Excel and create a spreadsheet that matches the spreadsheet shown in the following screenshot:

	A	B	C	D	E
1	Expense	Amount Spent			
2	Rent	\$ 350.00			
3	Car Payment	\$ 125.00			
4	Car Insurance	\$ 30.00			
5	Groceries	\$ 150.00			
6	Clothes	\$ 40.00			
7	Entertainment	\$ 40.00			
8					
9					
10					
11					
12					
13					
14					
15					
16					
17					

In this case we will create a function in Excel with several inputs and a single output. We want the inputs to be: rent, car payment, car insurance, groceries, clothes, and entertainment. We want the output to be Total Expenses.

We could say that the *Total Expenses* are a function of the amount spent on rent, car payment, car insurance, groceries, clothes, and entertainment.

We can create a function in Excel that will calculate the output for us. In this case, we will use the *Sum* function. Notice that Excel uses function notation similar to what we would use in mathematics. As shown in the red circle in the following image, we can see that the function $f(x)$ is equal to the sum of the entries in cells B2 through B7. The inputs of the function are \$350, \$125, \$30, \$150, \$40 and \$40. The rule used for this function is a sum, so we add these numbers together to get the output: \$735.00.



This semester we have already seen several other functions that are built into Excel. For example, we have computed the average, the minimum, the maximum, and the standard deviation. These are all examples of functions that are built into Excel.

One purpose in learning about functions is to help us better understand loan calculations. Excel has several built-in functions that will be beneficial to us. But in order to use them, we have to understand how to enter the inputs of the function into Excel so it can compute the output.

Some Special Excel Functions

There are a few functions in Excel that will be useful as we work on loan calculations. Some of these are functions we have seen before and some of them are new. Here is a summary of some helpful functions.

function	Syntax	Purpose
sum	=SUM(A1:A10)	Adds all the numbers in cells A1 through
average	= AVERAGE(A1:A10)	Finds the average of the numbers in cells
median	=MEDIAN(A1:A10)	Finds the median of the numbers in cells
mode	=MODE(A1:A10)	Finds the mode of the numbers in cells A
minimum	=MIN(A1:A10)	Finds the smallest value in cells A1 throu

maximum	= MAX(A1:A10)	Finds the largest value in cells A1 through A10.
standard deviation	= STDEV(A1:A10)	Finds the standard deviation of the numbers in cells A1 through A10.
percentile	=PERCENTILE(A1:A10,k)	Finds the k th percentile for the data in cells A1 through A10. Note that k must be a number between 0 and 1. For example, you would use $k = 0.20$ to find the 20th percentile.
percent rank	=PERCENTRANK.EXC(A1:A10,n)	Finds the percentile score for the data point in cell A1.
count	=COUNT(A1:A10)	Counts how many entries are in cells A1 through A10.
Payment	=PMT(rate, number of periods, loan amount)	Computes a monthly loan payment given the interest rate, the number of periods, and the current balance.
Future Value	=FV(rate, number of periods, payment made each period)	Computes the balance of an investment given the interest rate, the number of periods, and the amount invested each period.
if	=IF(condition, value if true, value if false)	Tests to see if a condition is true. If the condition is true, it gives the “value if true”. If it is false, it gives the “value if false”.
and	=AND(condition 1, conditions 2)	Checks to see if two conditions are both true. If both are true, it returns a value of true. If only one or none of the conditions are true, it returns a value of false.
or	=OR(condition 1, conditions 2)	Checks to see if two conditions are true. If either one or both are true, it returns a value of true. If neither condition is true, it returns a value of false.

Notice for each of these functions the syntax is the same. They always start with an equal sign, which tells Excel you want to use a function. They then have the name of the function, this tells Excel which rule to use for the function. They then have a list of inputs. This allows Excel to use the correct rule for the inputs you give and find the appropriate output. Notice how well this corresponds to our definition of a mathematical function:

A function is a rule that assigns each input value(s) to a unique output value.

Important Note: In Excel, positive and negative signs are used to distinguish between money you get and money you pay. Any money that figuratively leaves your wallet would be entered in Excel as a negative number. Any money figuratively entering your wallet would be entered in Excel as a positive number. Here are some examples:

Situation	Leave or Enter	Positive Negative
Monthly loan payment	The money “leaves your wallet” when you make your payment.	Negative
Loan principle	The bank gives you money you can “put in your wallet” when they give you a loan.	Positive

Monthly savings deposit	You take money “out of your wallet” and put in the bank.	Negative
Withdraw money from savings	You get the money from your bank account to “go in your wallet”.	Positive

Savings and Loans

When making a purchase, we generally have two options. We can save the money and pay at the time of purchase; or we can get a loan to pay for the item. For most purchases, it is wise to save the money first. For some very large purchases, such as a home, it is worthwhile to get a loan because it allows us to make the purchase without having to save a large amount of money.

The Future Value Function

The future value (FV) function can be very helpful when you want to find out how much money you will have in an investment (like a savings account or a retirement fund) after a certain amount of time. It computes the balance of the investment account given an interest rate, the number of periods, and the payment.

Example 3 – Savings Account

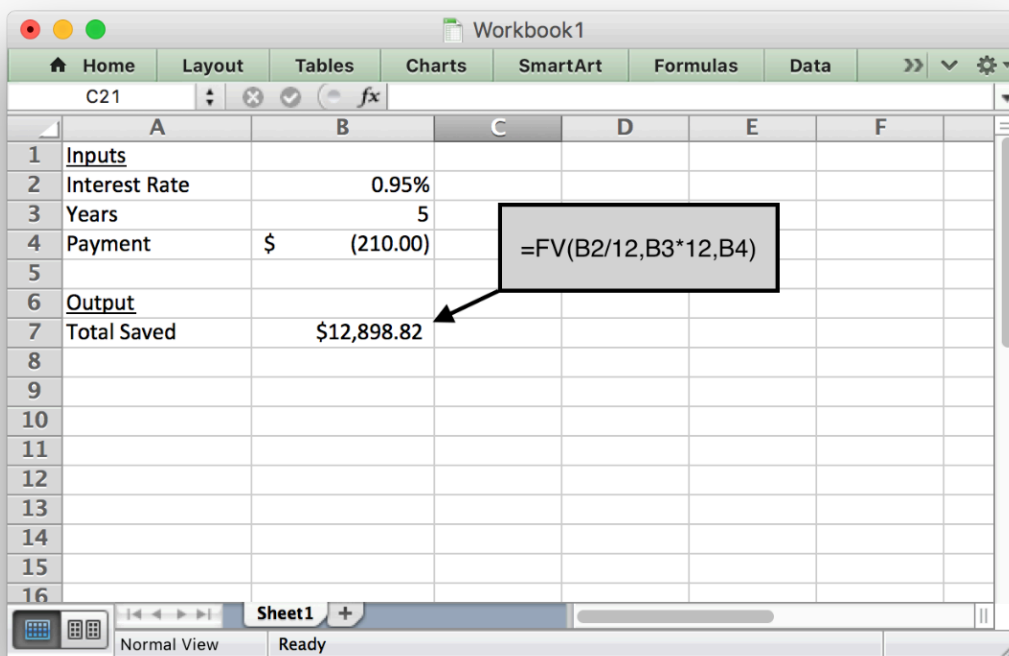
Anya is saving money for emergencies. After creating a budget, she realizes she can contribute \$210 to her savings account every month. Her savings account earns 0.95% interest annually. How much money will she have in her emergency savings account after 5 years?

Solution:

In this situation, Anya needs to know the interest rate, the number of periods, and the monthly payment in order to use the future value (FV) command in Excel.

Because she gives the monthly payment to the bank every month, it should be a negative number (Excel used parentheses to show a negative number in the image below).

After entering the information shown below, she finds that the balance of her savings account will be \$12,898.82 after she saves for 5 years.



The screenshot shows an Excel spreadsheet titled "Workbook1" with the following data:

	A	B	C	D	E	F
1	Inputs					
2	Interest Rate	0.95%				
3	Years	5				
4	Payment	\$ (210.00)				
5						
6	Output					
7	Total Saved	\$12,898.82				
8						
9						
10						
11						
12						
13						
14						
15						
16						

The formula bar shows the formula `=FV(B2/12,B3*12,B4)` entered in cell C7. An arrow points from the formula bar to cell C7, which contains the value \$12,898.82.

Example 4 – Retirement Savings

Doug is currently 42 years old and earns \$42,000 per year. He just decided to start saving for retirement. He starts contributing 8% of his salary to his 401K retirement account every month. His employer also contributes by paying 4% of Doug's salary to his 401K account each month. Assuming Doug's salary stays the same for the rest of his career and that he retires at age 65, how much money will Doug have when he retires if his 401K has an average annual interest rate of 5.2%?

Solution:

In order to use the future value (FV) function in Excel to find the future value of his 401K retirement account, Doug needs to know how much he and his employer contributes each month.

Doug contributes 8% of his salary: $0.08 \times \$42,000 = \3360

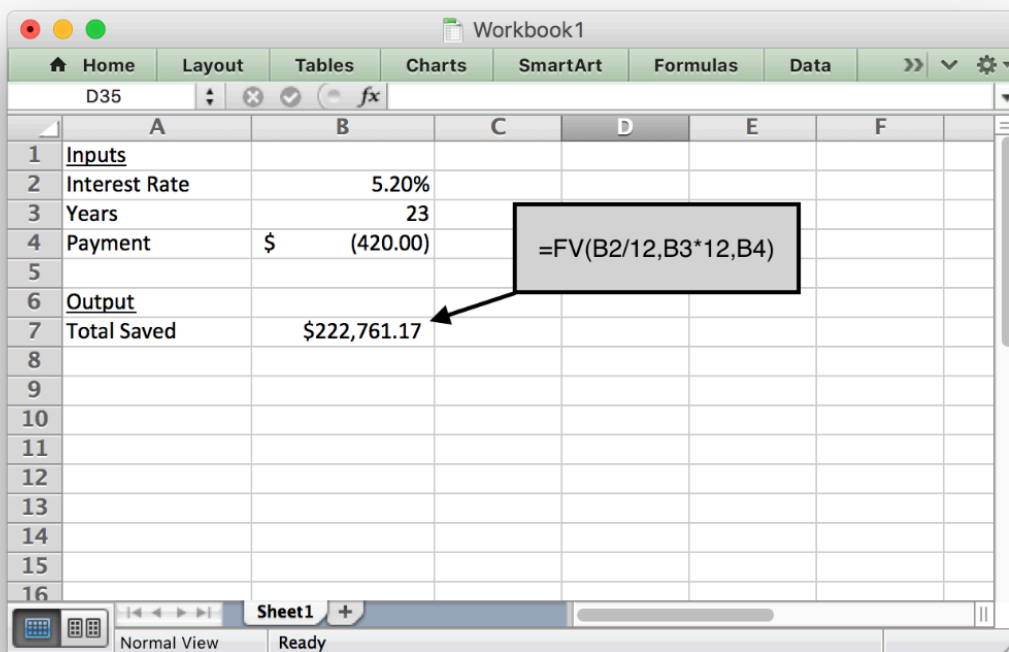
His employer contributes 4% of Doug's salary: $0.04 \times \$42,000 = \1680

Total annual contribution: $\$3360 + \$1680 = \$5040$

Total monthly contribution: $\frac{\$5040}{12} = \420

See the Excel input below to see how this information was used in Excel to compute the future value of Doug's retirement account.

Using the formula shown below shows us that Doug will have \$222,761.17 when he retires.



The screenshot shows an Excel spreadsheet titled 'Workbook1' with the following data:

	A	B	C	D	E	F
1	Inputs					
2	Interest Rate	5.20%				
3	Years	23				
4	Payment	\$ (420.00)				
5						
6	Output					
7	Total Saved	\$222,761.17				
8						
9						
10						
11						
12						
13						
14						
15						
16						

A callout box points to cell C7, containing the formula: `=FV(B2/12,B3*12,B4)`.

The Payment Function

When we do need to borrow money, it is important to make sure that the loan payment amount fits within your monthly budget. The payment function (PMT) in Excel helps determine what the monthly payment would be on a loan. Here are a few examples:

Example 5 – Student Loan

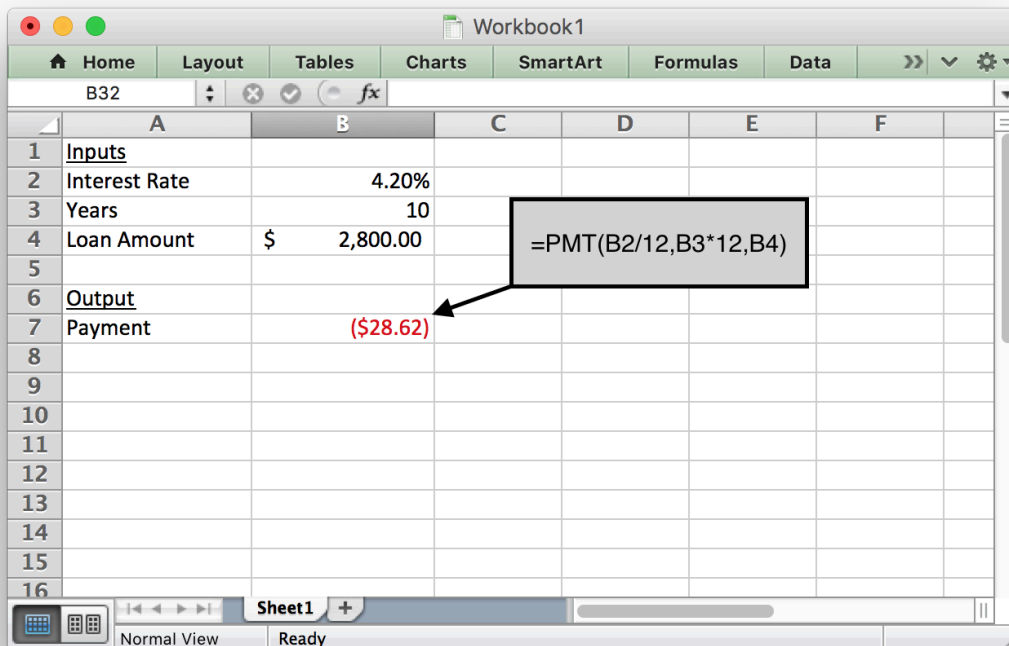
Emilee is an online student who needs to take out a student loan to finish her degree. She needs to borrow \$2800 and will pay it back after she graduates. She can pay it off over 10 years with an interest rate of 4.2%. What would her monthly payments be once she graduates?

Solution:

The payment function in Excel can help Emilee find her monthly payment. She needs to enter the interest rate, years, and loan amount. Because she wants to know a *monthly* payment, all inputs in the formula must be in months. We have to divide the annual interest rate by 12 to get a *monthly* interest rate. We also have to multiply the number of years by 12 to find the number of months.

Using the formula shown below tells us Emilee would pay \$28.62 per month to pay off her student loan.

Notice the loan amount is a positive number, because when Emilee gets the loan she will receive that money. But the monthly payment will be negative, because that is money she has to pay each month. Excel uses a negative sign, a red number, or a number in parentheses to indicate a negative value.



Example 6 – Small Business Example

Andrew is starting a construction company and needs to purchase tools and supplies to begin his business. His calculations show he needs \$45,000 to get his business started. He has \$9,000 saved to put toward his start-up expenses, so he only needs to borrow \$36,000. His bank will give him a 5-year small-business loan with an interest rate of 3.5%. What will his monthly payment be?

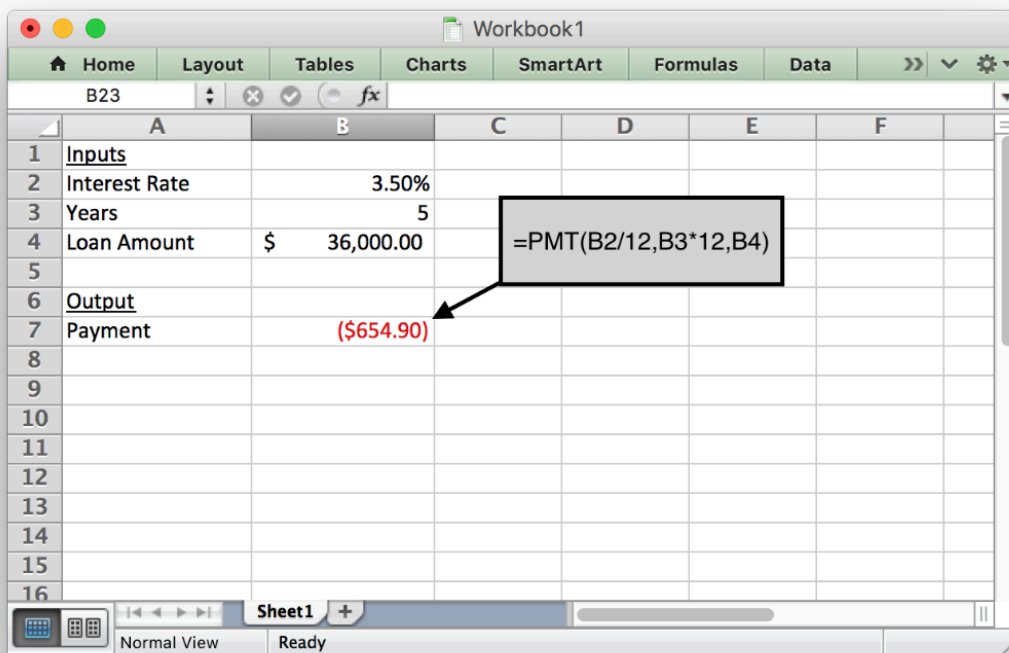
Solution:

The payment function in Excel would help Andrew answer this question. He needs to input the interest rate, the number of terms, and the total loan amount. Excel will then give the monthly payment as the output.

We entered the following formula into Excel to get the answer. Notice that because we want a *monthly* payment, we have to divide the annual interest rate by 12 to get a *monthly* interest rate. We also have to multiply the number of years by 12 to find the number of months.

Notice the loan amount is a positive number and the monthly payment is negative.

Andrew's monthly payment would be \$654.90.



Practice: Check Your Understanding

Problem 6

The three individuals shown below are borrowing money. Use Excel to find their monthly payments. Match each loan to the correct monthly payment amount.

Chloe plans to borrow \$1400. She will finance it for 5 years at 3% interest. Her monthly payment will be

Alyssa will borrow \$1350 and finance it for 4 years at 4.5% interest. Her monthly payment will be

Olu found a loan that offers 7% interest for 3 years. He is going to borrow \$1450. His monthly payment will be

Monthly payment amount choices:

- \$30.78
- \$25.16
- \$44.77

How Did I Do?

Chloe's monthly payment will be \$25.16.

Alyssa's monthly payment will be \$30.78.

Olu's monthly payment will be \$44.77.

Craig's Car Loan

In the opening video, we saw that Craig needed a loan to purchase a new car. We will use the Quantitative Reasoning Process to find which loan option Craig should choose. As we go through the process we will see the usefulness of the functions in Excel.

1 Understand the Problem

The real-world problem in this situation is that Craig needs to choose a loan for a new car. The loan is for \$5000 with an annual interest rate of 4%. Craig knows he will pay less interest if he finances the car for a shorter time period. But he can only afford to pay up to \$100 per month on a car payment. The bank offers 3, 4, 5, 6 or 7 year loans.

2 Identify Variables & Assumptions

The variables in this situation are:

- His monthly payment: P
- The length of the loan: n

His monthly payment depends on the length of the loan. Using the vocabulary of functions we would say that the monthly payment is a function of the length of the loan. The length of the loan is the input and the monthly payment is the output.

We are making the following assumptions:

- The cost of the car is \$5000.
- The APR is 4%.
- Craig wants to finance the car for the shortest time period possible.
- Craig can only pay \$100 per month for his car payment.

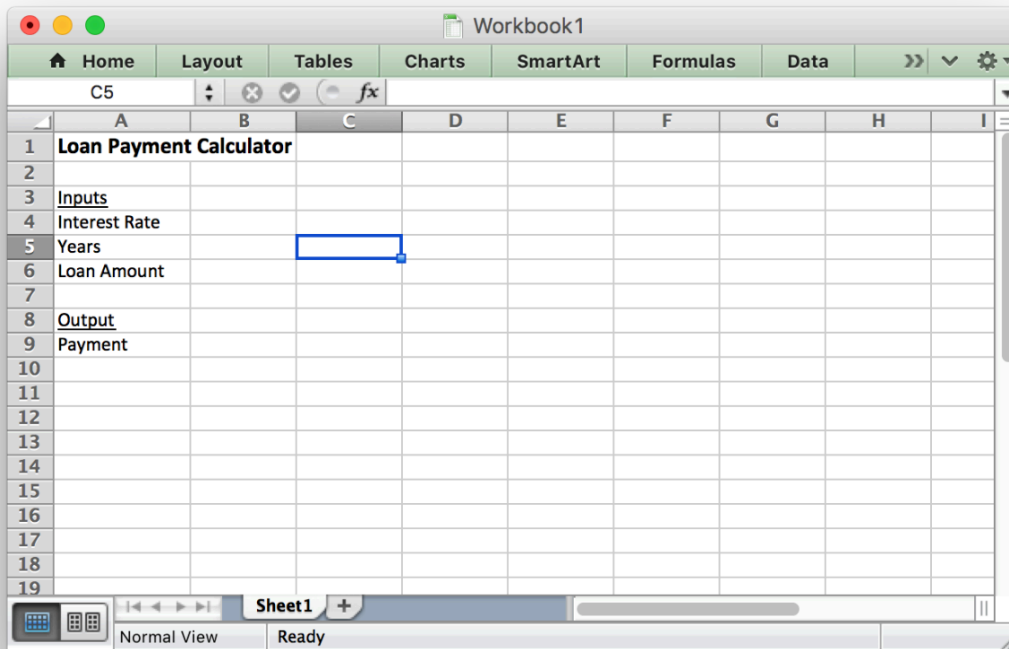
3 Apply Quantitative Tools

Follow the Excel calculations that help determine the length of the loan Craig should get.

Excel Instructions

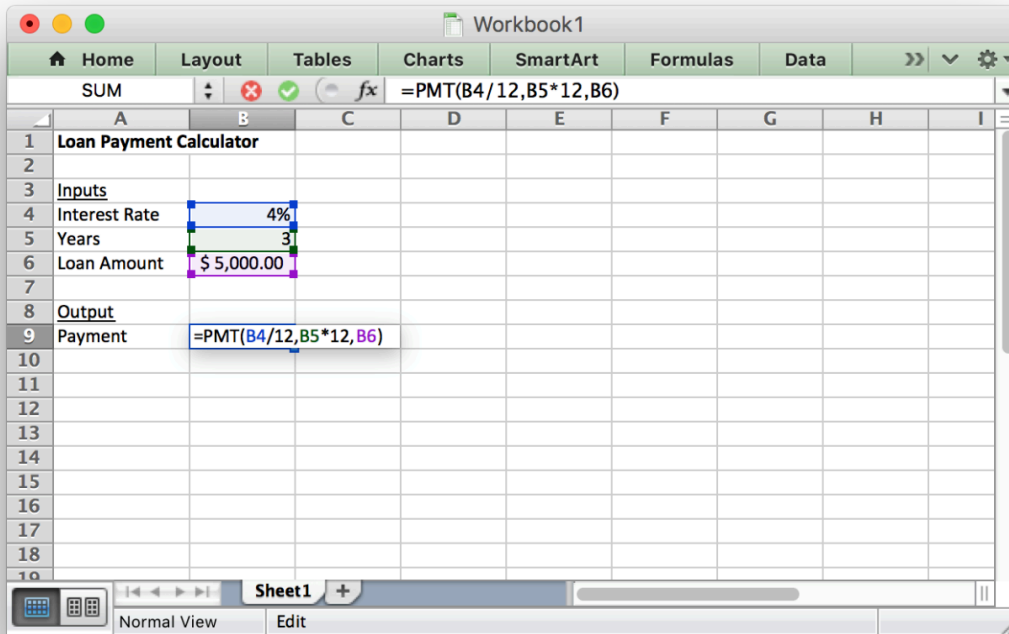
Step 1

We will use the payment function in Excel, so we know our inputs are the interest rate, the number of years, and the loan amount. Our output will be the monthly payment.



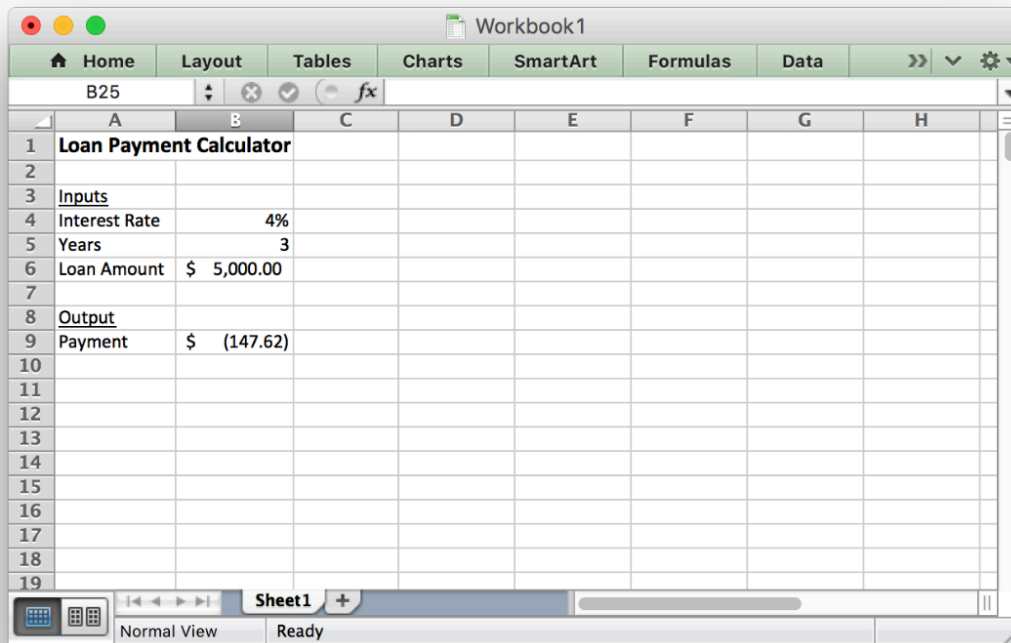
Step 2

The rate is 4% per year, we will start by using 3 years as the length of the loan, and the loan amount is \$5000. We have to be sure to convert all of our inputs to be in monthly units.



Step 3

This Excel calculation tells us that if Craig gets a 3-year loan his monthly payment would be \$147.62. This is more than the \$100 that Craig knows he can afford, so he should consider other loan lengths.



We will look at different inputs for this variable to determine which length of loan to choose.

Using the formulas we entered into Excel you should get the following values as you change the number of years for the loan:

Number of Years	Payment
3	\$147.62
4	\$112.90
5	\$92.08
6	\$78.23
7	\$68.34

4 Make an Informed Decision

Since Craig can only afford a \$100 loan payment per month, he should choose the 5 year loan with a car payment of \$92.08.

5 Evaluate Your Reasoning

When we reflect on the decision, we should consider whether we have made the best decision. Some people might argue that Craig should finance the car for 7 years because that makes the payment lower. However, we have to realize that means he has to pay interest for a longer period of time, so the entire loan will cost more money. We can find the total cost of the loan by multiplying the payment by 12 (to find out how much you pay per year) and then multiplying that by the number of years the loan is financed. Doing so for this problem tells us the total cost of the loan for each of the 5 options.

Number of Years	Payment	Total Cost of Loan
3	\$147.62	\$5314.32
4	\$112.90	\$5418.97
5	\$92.08	\$5524.96
6	\$78.23	\$5632.27
7	\$68.34	\$5740.90

This shows if Craig finances the car loan for 7 years he would pay \$5740.90 for the loan, but if he finances it for 5 years he will only pay \$5524.96. So Craig will save \$215.94 by selecting the shortest loan he can afford.

Excel - Absolute and Relative Cell References

One of the benefits of using Excel is that it allows you to use formulas. Additionally, you can copy and paste the formulas to help make it easier to create a useful spreadsheet.

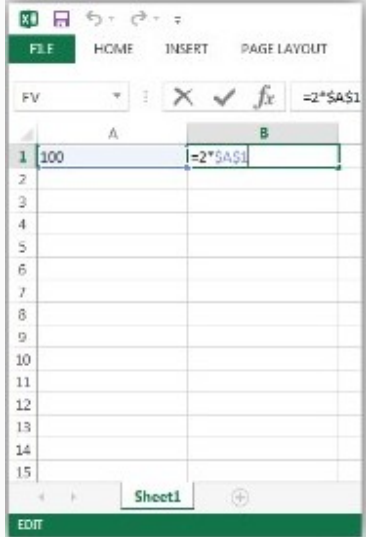
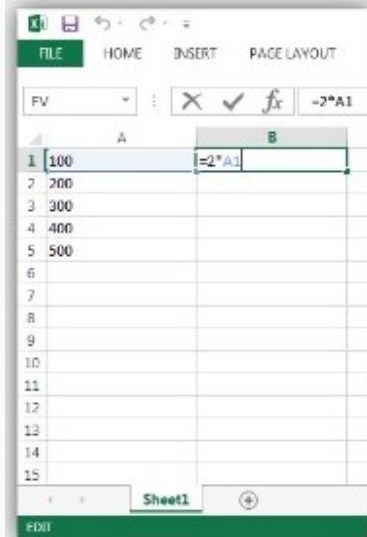
Cell References are used within Excel formulas to refer to information located in another cell in the spreadsheet. There are two types of cell references: *relative cell references* and *absolute cell references*.

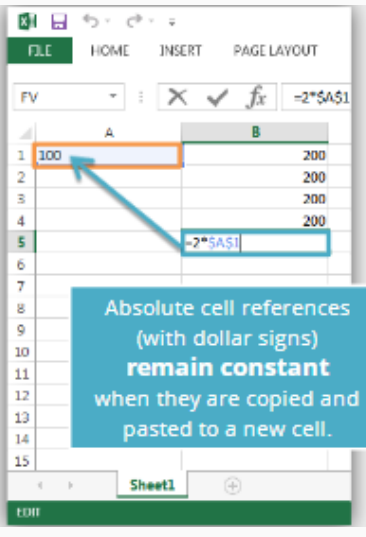
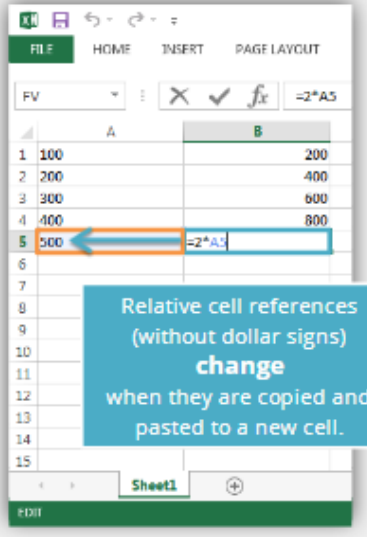
Relative cell references refer to a cell by giving its row and column (for example: A1). ***Absolute cell references*** use dollar signs when referring to a cell (for example: \$A\$1).

The difference between these two types of cell references is how they behave when they are copied and pasted to another cell. Relative cell references **change** when they are copied and pasted to a new cell. Absolute cell references **remain**

constant when they are copied and pasted. The following animation demonstrates how relative and absolute cell references work:

Example of How Absolute and Relative Cell References Work in Excel

Absolute References (with dollar signs)	Relative References (without dollar signs)
	

Absolute References (with dollar signs)	Relative References (without dollar signs)
 <p>Absolute cell references (with dollar signs) remain constant when they are copied and pasted to a new cell.</p>	 <p>Relative cell references (without dollar signs) change when they are copied and pasted to a new cell.</p>

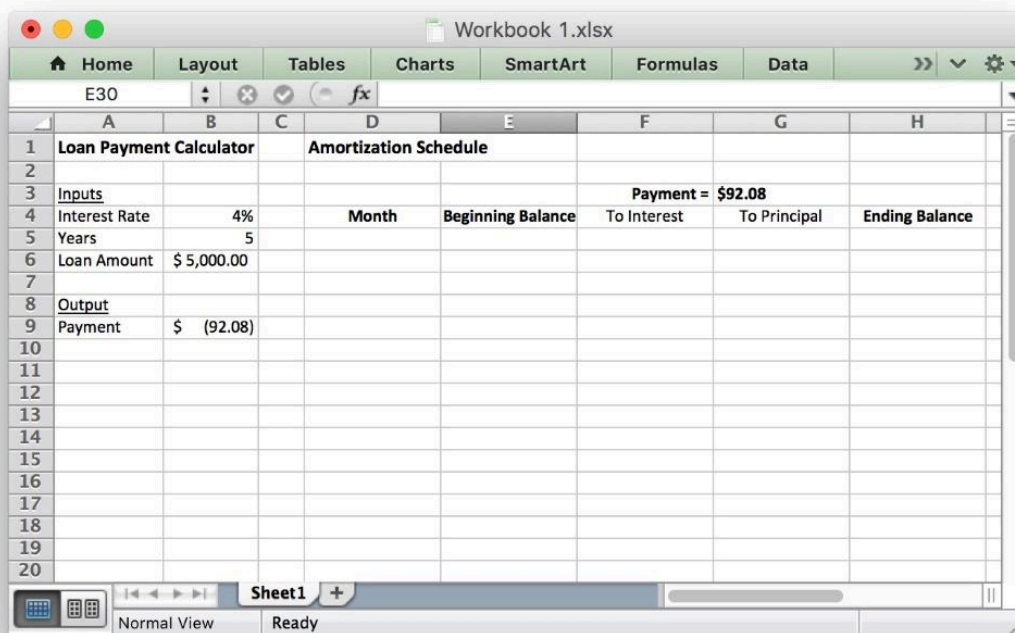
Excel Instructions - Amortization Tables

Now that we know how to use the Payment function in Excel to compute a monthly payment for a loan, we can learn how to create an amortization table. We will continue with the same example used in the previous problem to create an amortization schedule. Remember we are financing a \$5000 car for 5 years at 4% interest.

Follow these steps to create an amortization table for Craig's loan. The spreadsheet was intentionally left very plain in order to make it easier for you to reproduce.

Step 1

An Amortization Table breaks down your payment for each month and shows how much of the payment goes toward paying the loan principal and how much goes toward interest. We will add to the Excel spreadsheet we have been working on by creating headings:



Step 2

Now, we use formulas to fill in the Month 1 row. Pay attention to the dollar signs. They help the formulas copy correctly. Cell references with dollar signs are *absolute cell references* and cell references without dollar signs are *relative cell references*.

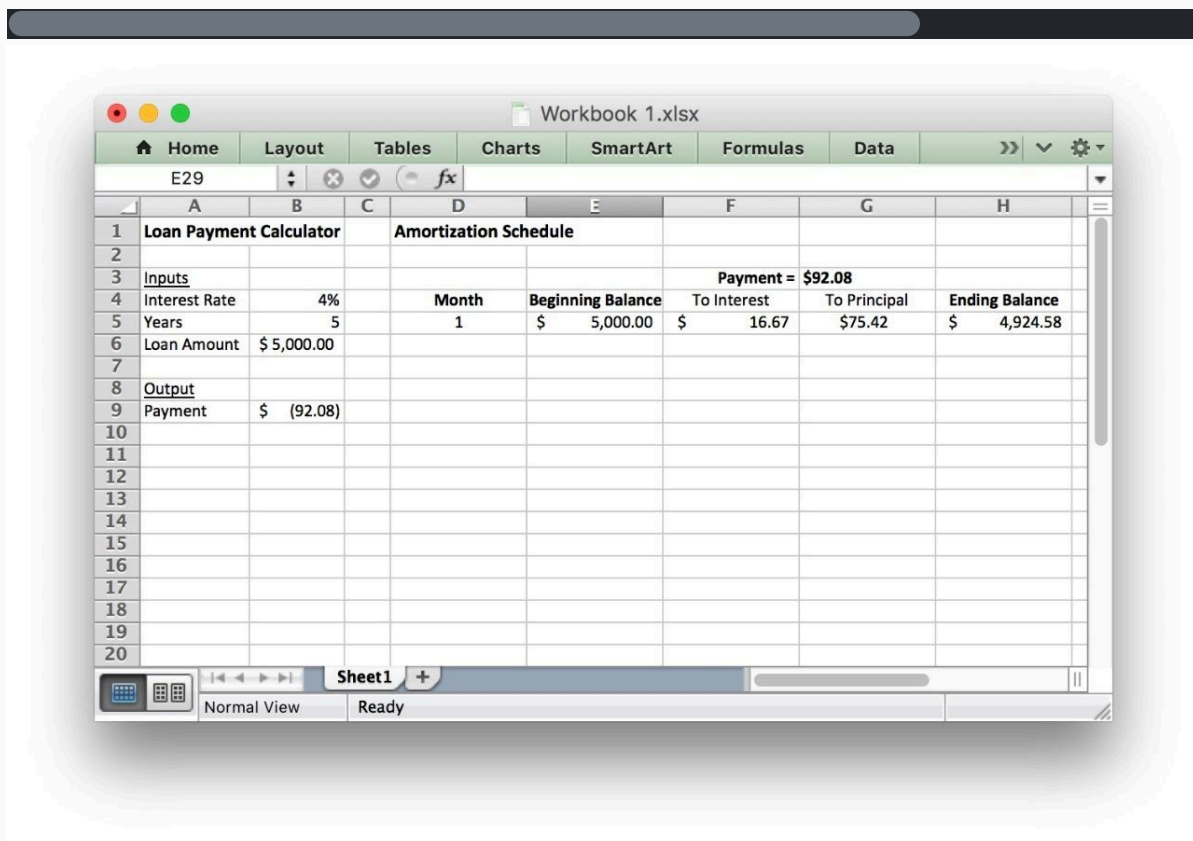
Here are the formulas you should type in for the first row:

Cell Number	Formula	Explanation
G3	=-B9	This produces the proper monthly payment. Since the value has a negative sign in front of B9 produces a positive value. Note

		directly into cell G3, the ending balance will be off by a per 1. This is because of the rounding done by typing 92.08 dir cell reference (which does not round the number).
D5	1	Type the number 1 to indicate Month 1.
E5	=B\$6	The Beginning Balance for Month 1 is the Loan Amount lis signs indicate an absolute cell reference.
F5	=B\$4/12*E5	Divide the interest rate from cell B4 by 12 to make it a mon by beginning balance in cell E5 to compute the interest acc
G5	=G\$3-F5	This subtracts the interest from the monthly payment to de payment goes to paying down the loan balance.
H5	=E5-G5	This subtracts the amount of principle paid from the beginr monthly ending balance.

Step 3

If you entered the formulas for the first row correctly, your spreadsheet should look like this:



Step 4

Now we will fill in the formulas for Month 2. After filling in this row, we will be able to copy these formulas to fill in the rest of the table.

Cell Number	Formula	Explanation
D6	=D5+1	This adds 1 to the month number from the previous cell to give us the new month number.
E6	=H5	The ending balance from the previous month becomes this month's beginning balance.
F6	Copy and paste the formula from cell F5	The formula should become =\$B\$4/12*E6
G6	Copy and paste the formula from cell G5	The formula should become =\$G\$3-F6
H6	Copy and paste the formula from cell H5	The formula should become =E6-G6

Note that absolute cell references (the ones with dollar signs) did not change when you copied and pasted them. But the relative cell references (the ones without dollar signs) changed to refer to the new row. This is the difference between absolute and relative cell references.

Step 5

If you entered the formulas for the second row correctly, your spreadsheet should look like this:

The screenshot shows an Excel spreadsheet with the following data:

Loan Payment Calculator							
Amortization Schedule							
Inputs				Payment = \$92.08			
Interest Rate	4%	Month	Beginning Balance	To Interest	To Principal	Ending Balance	
Years	5	1	\$ 5,000.00	\$ 16.67	\$75.42	\$	4,924.58
Loan Amount	\$ 5,000.00	2	\$ 4,924.58	\$ 16.42	\$75.67	\$	4,848.92
Output							
Payment	\$ (92.08)						

Step 6

Copy the entries in the Month 2 row and paste them in the remaining rows to fill in the rest of the table. The final row should be month 60 because it is a 5-year loan (5 years = 60 months). Only about 11 months are shown in the screenshot, but the table should continue until Month 60. If everything is done correctly, the Ending Balance in Month 60 should be \$0.00.

The screenshot shows an Excel spreadsheet with the following data:

Inputs		Month	Beginning Balance	To Interest	To Principal	Ending Balance
Interest Rate	4%	1	\$ 5,000.00	\$ 16.67	\$75.42	\$ 4,924.58
Years	5	2	\$ 4,924.58	\$ 16.42	\$75.67	\$ 4,848.92
Loan Amount	\$ 5,000.00	3	\$ 4,848.92	\$ 16.16	\$75.92	\$ 4,773.00
Output		4	\$ 4,773.00	\$ 15.91	\$76.17	\$ 4,696.82
Payment	\$ (92.08)	5	\$ 4,696.82	\$ 15.66	\$76.43	\$ 4,620.40
		6	\$ 4,620.40	\$ 15.40	\$76.68	\$ 4,543.72
		7	\$ 4,543.72	\$ 15.15	\$76.94	\$ 4,466.78
		8	\$ 4,466.78	\$ 14.89	\$77.19	\$ 4,389.59
		9	\$ 4,389.59	\$ 14.63	\$77.45	\$ 4,312.14
		10	\$ 4,312.14	\$ 14.37	\$77.71	\$ 4,234.43
		11	\$ 4,234.43	\$ 14.11	\$77.97	\$ 4,156.46

We can use the Amortization Table to find useful information. For example, we see that after 2 years (24 months), Craig will still owe \$3118.91 on the loan. We also see that in the first month he paid \$16.67 in interest, but in the 24th month he only paid \$10.67 in interest. This is because his account balance is lower and he is paying interest on a smaller amount of money.

We could also use the SUM function in Excel to add up the interest payments for all 60 months of the loan. This tells us Craig paid a total of \$524.96 in interest for the loan.

Because we used formulas to create the entire table, we could go back and change some of the inputs (interest rate, number of years, and original loan balance) and the entire table would update to reflect the new information.

Lesson Checklist

By the end of this lesson you should be able to do the following:

- Identify relevant variables used in a given model of a real-world situation.
- Identify input and output variables used in a given model of a real-world situation.
- Understand the definition of a function.
- Use function notation to define specific functions.
- Given a function and an input value(s), find the output value.
- Use the payment function in Excel.
- Use the future value function in Excel.
- Create an amortization schedule in Excel.

Optional Resources

The following video gives some good advice from the church on business loans:

- [How do I know if I should use a loan to grow my business?](#)

The following videos show the talk about mortgage debt by President Hinckley that was referenced by Henry J. Eyring in the video included in this lesson.

- [Link to the video \(Minutes 17:56-20:08 are recommended\)](#)

¹ ["Debt"](#), True to the Faith p. 48-49



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Access it online or download it at https://books.byui.edu/math_108x/lesson_7_functions_in_excel.

Lesson 8: Functions and Their Graphs

Opening Story

Lesson 8 - Opening Story

[L8 Opening Story Transcript](#)

Introduction

In this lesson we are focusing on the third step of the Quantitative Reasoning Process: *Apply Quantitative Tools*. Last week we learned about functions. This week we will expand our knowledge of functions to see how they help solve some important questions from everyday life when used as a tool in the Quantitative Reasoning Process.



In the opening story of this lesson, Sophia and Emmy used carbon-14 dating to solve the mystery about which statue was real and which was fake. Carbon-14 dating is a method that uses *exponential functions* to determine the age of an object that contains organic material.

Although Sophia and Emmy's story is a fictional bedtime story, similar uses of mathematical functions and the Quantitative Reasoning Process help solve many different types of problems encountered in every day life such as:

- How long will it take me to pay off this loan?
- How much money should I save each month if I want to buy a car?
- How much money should I save each month in order to retire at age 62?
- How many calories should I eat each day if I want to compete in a triathlon?
- Should I buy a car with better gas mileage?
- Should I accept a job offer that pays commission rather than a salary?
- Which health insurance plan is best for my situation?
- How much money do I need to start a community charity?

In fact, real life methods similar to those depicted in Sophia and Emmy's story are currently used to fight the poaching of elephants in Asia and Africa. The Poachers kill elephants for the ivory in their tusks. These poaching deaths have led to declining elephant populations over the past century. In an effort to protect elephants, international agreements

banned the trade of raw ivory from Asia and Africa in the 1970s and 1980s. However, 30,000 elephants are still killed every year by poachers who illegally take their tusks. Detecting poached ivory had previously been too expensive and time consuming for governments to enforce these trade laws.

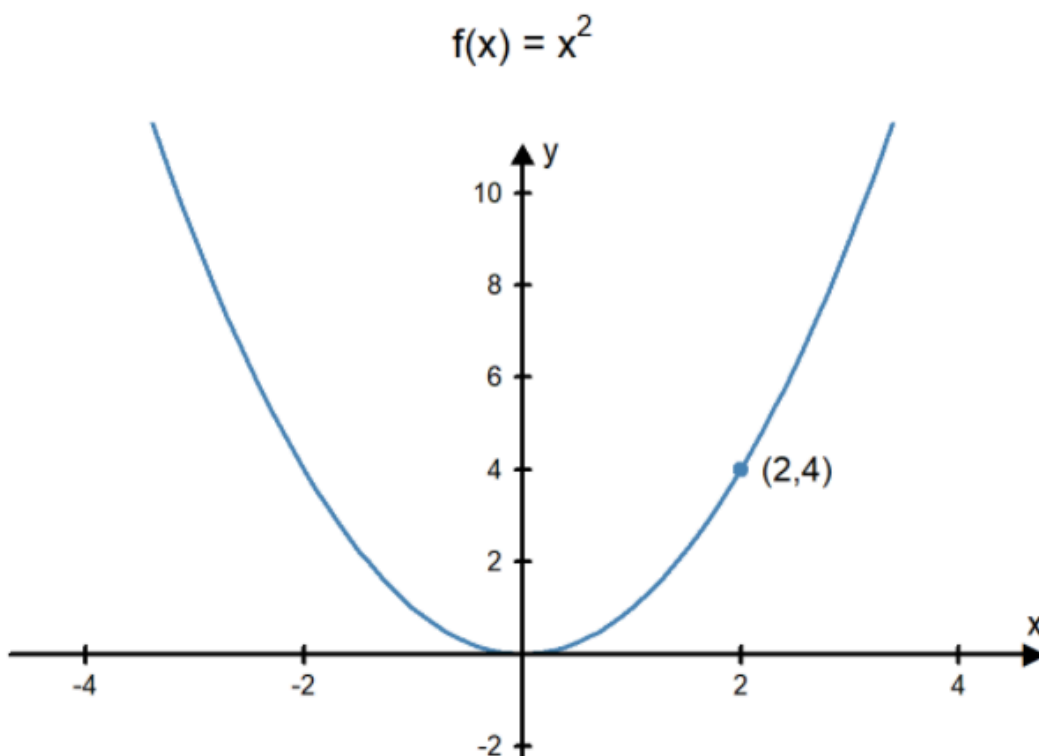
Recently researchers at the University of Utah developed methods to use carbon-14 dating to determine the age of ivory. The new method is less expensive than previous methods so it is more accessible to government agencies. The carbon-14 dating method is used to determine if the sample of ivory is old and comes from an elephant killed before the ivory trade ban, or if the ivory is recent and comes from a poached elephant. Governments can now identify illegally sourced ivory and fight poaching of elephants, all because of the wise use of mathematical functions and the Quantitative Reasoning Process. [1](#)

The Coordinate Plane

When we use functions it is helpful to create a graph of the function. Functions are graphed on the coordinate plane. The horizontal axis (also called the x -axis) represents the inputs of a function and the vertical axis (also called the y -axis) represents the outputs of the function.

Every point on the graph represents an input and output of the function. We represent each point as an ordered pair, (x, y) , where the first number is the input and the second number is the output. The point $(0, 0)$ is referred to as the *origin*.

When we graph a function, the graph is a visual representation of all the inputs and corresponding outputs of the function. For example, the following graph represents the function $f(x) = x^2$.



Every point on the curve represents an input and an output of the function. Notice that the point $(2, 4)$ is a point on the curve. That is because if you use 2 as an input in the function $f(x) = x^2$ then you get an output of 4. In other words, $f(2) = 4$. Some of the other ordered pairs related to this function are: $(-1, 1)$, $(3, 9)$, $(-2, 4)$, and $(0, 0)$. Notice that if you use the first number in each pair as an input of the function $f(x) = x^2$ then you get the second number as the output.

Three Types of Functions

Functions can be divided into *families* of functions. Each family represents a group of functions with similar properties. We will focus on three common families of functions: linear functions, quadratic functions, and exponential functions. There are many more families of functions, but these three function families are very helpful for solving everyday problems and making informed decisions.

Linear Functions

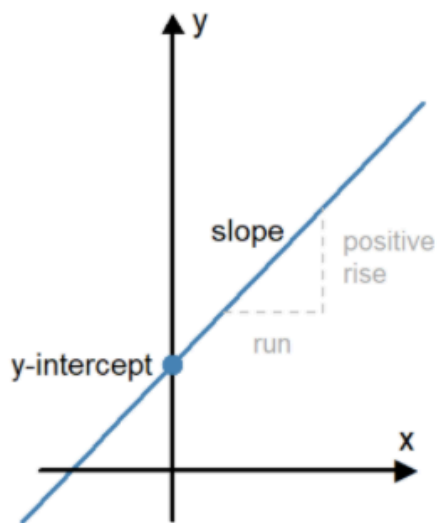
Linear functions help us describe or mathematically model real world situations where something is changing at a constant rate.

*A **linear function** is a function of the form $f(x) = mx + b$ where m and b are constants.*

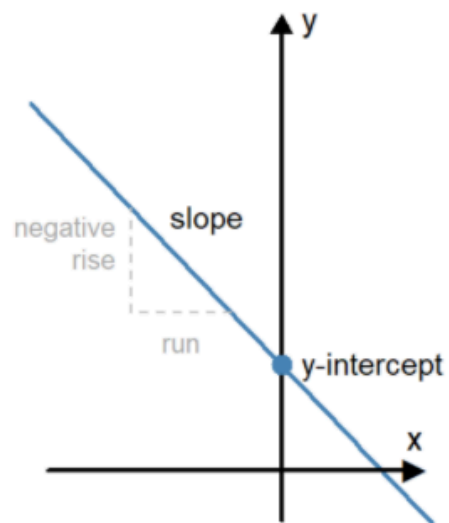
Two features that help us graph a linear function are the *slope* and the *y-intercept*. The equation $f(x) = mx + b$ is said to be in slope-intercept form.

- The y -intercept is the point on the graph where the line crosses the vertical axis (y -axis).
- The slope of a line is a number that tells us how steep the line is. If the slope is $\frac{2}{3}$, starting from any point on the line, we go up 2 units for every 3 units we go to the right. If the slope is -4 , we would start at any point on the line and then go down 4 units for each 1 unit we go to the right.

Positive Slope

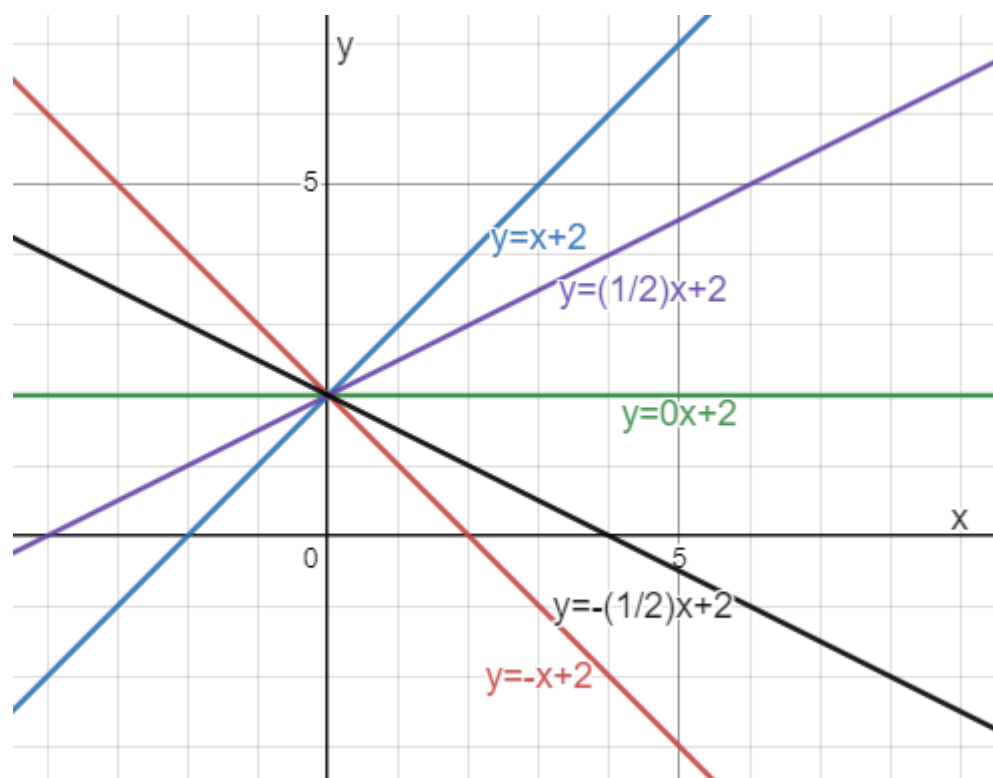


Negative Slope

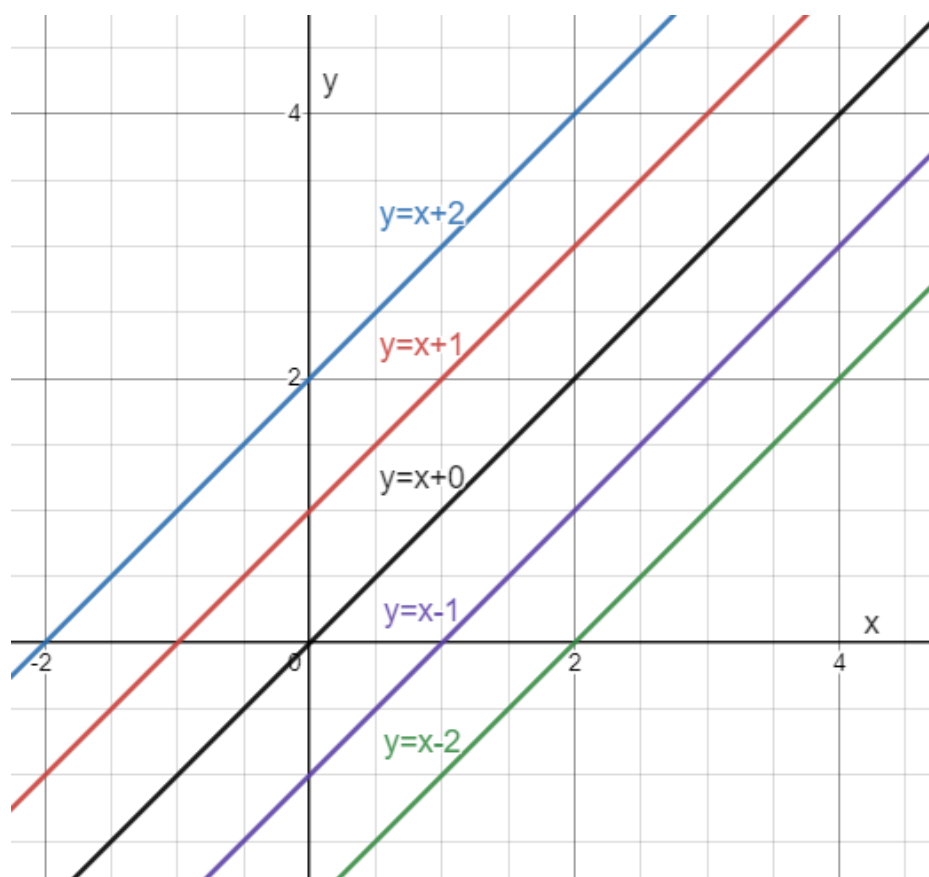


The graph of a **linear** function, $f(x) = mx + b$ is a straight line that crosses the y -axis at the point $(0, b)$ and has a slope of m .

The illustrations below show how changing the slope and the y -intercept changes the graph of the line.



Changing the slope of a line



Changing the y -intercept of a line

Example 1

Determine which of the following are linear functions:

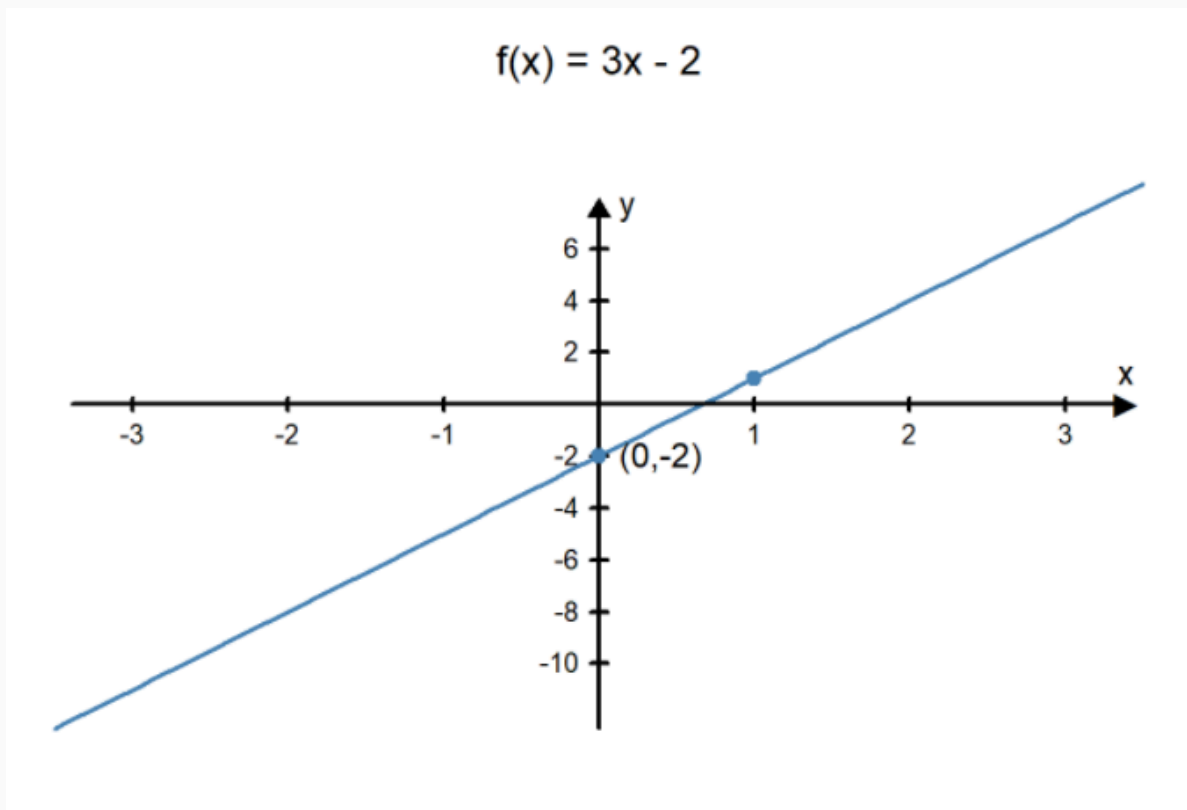
a. $f(x) = 3x - 2$

b. $3x + 4y = -7$

c. $y = 4x^2 - \sqrt{3}$

Solution:

a. The function $f(x) = 3x - 2$ is a linear function because its form matches the general form of a linear function: $f(x) = mx + b$. In this example, $m = 3$ and $b = -2$. We could graph this function by using the fact that it will cross the y -axis at the point $(0, -2)$ and has a slope of 3.



b. The function $3x + 4y = -7$ is also a linear function. It is not currently in the form $f(x) = mx + b$, but we could rewrite it in this form using some algebra rules. First, we solve the equation for y :

$$3x + 4y = -7$$

$$4y = -3x - 7$$

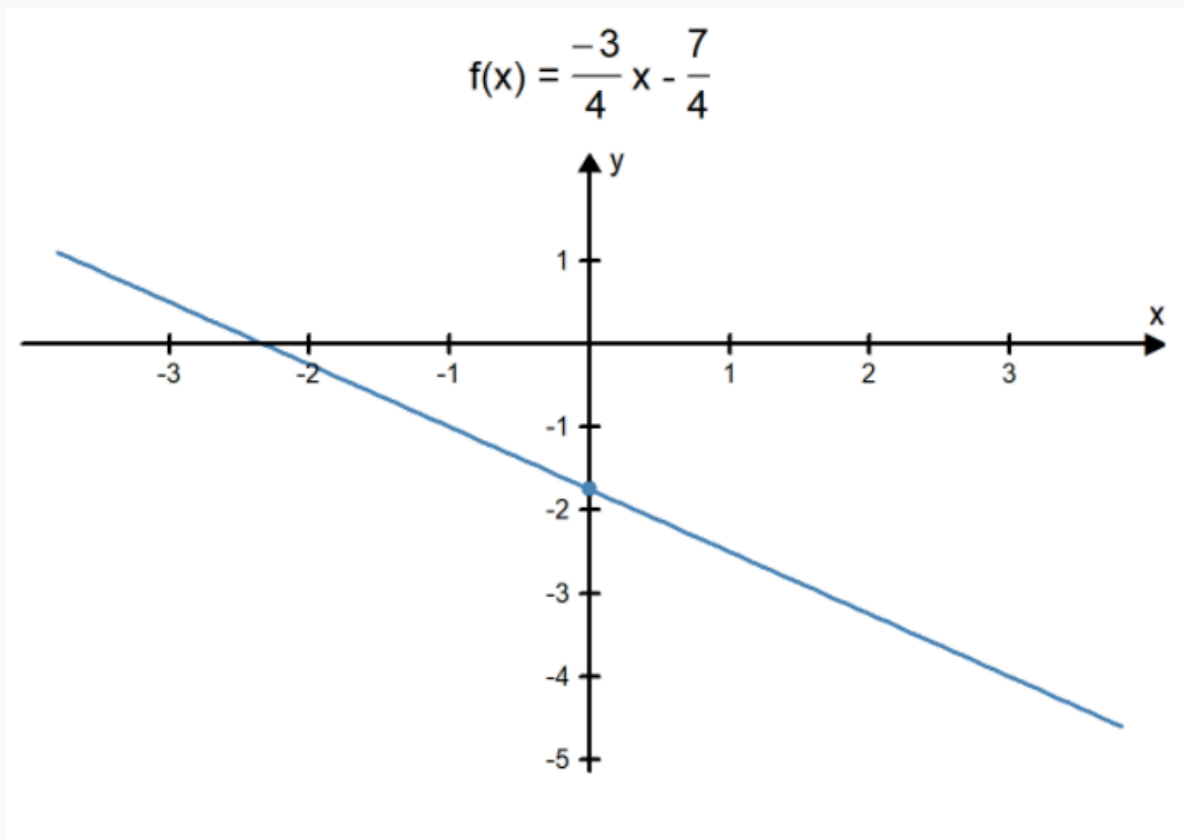
$$y = \frac{-3x - 7}{4}$$

$$y = \frac{-3}{4}x - \frac{7}{4}.$$

Recall from last week's lesson that $y = f(x)$, so we can change the y to an $f(x)$. This makes the equation:

$$f(x) = \frac{-3}{4}x - \frac{7}{4}.$$

Now this function is written in the slope-intercept form of a linear function. We could graph it using a y -intercept of $-\frac{7}{4}$ and a slope of $-\frac{3}{4}$.



c. The equation $y = 4x^2 - \sqrt{3}$ is not a linear function. Notice that in a linear function the x variable is not squared. Because equation c has an x^2 term, we know it is not a linear function.

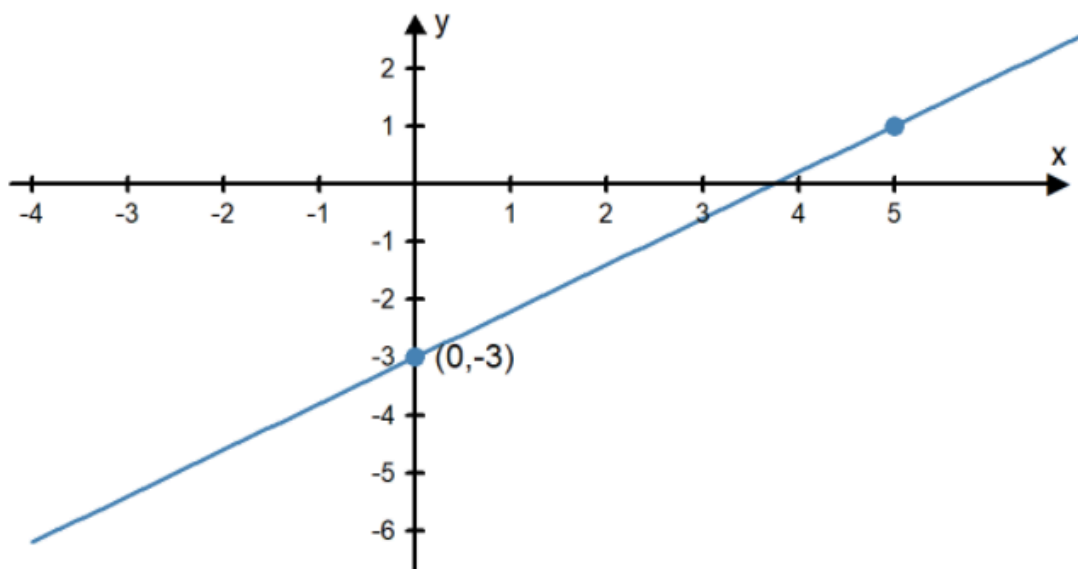
Example 2 - Graphing Linear Functions

Graph the linear function $f(x) = \frac{4}{5}x - 3$.

Solution:

The equation of this linear function is written in slope-intercept form, so we know the y -intercept is $(0, -3)$ and the slope is $\frac{4}{5}$. We can use this information to help us graph the function:

$$f(x) = \frac{4}{5}x - 3$$



Example 3 - Graphing Linear Functions

Graph the linear function $12x - 3y = 9$.

Solution:

First, we need to rewrite this equation in the slope-intercept form for a linear function. We start by solving for y .

$$12x - 3y = 9$$

$$-3y = -12x + 9$$

$$y = \frac{-12x + 9}{-3}$$

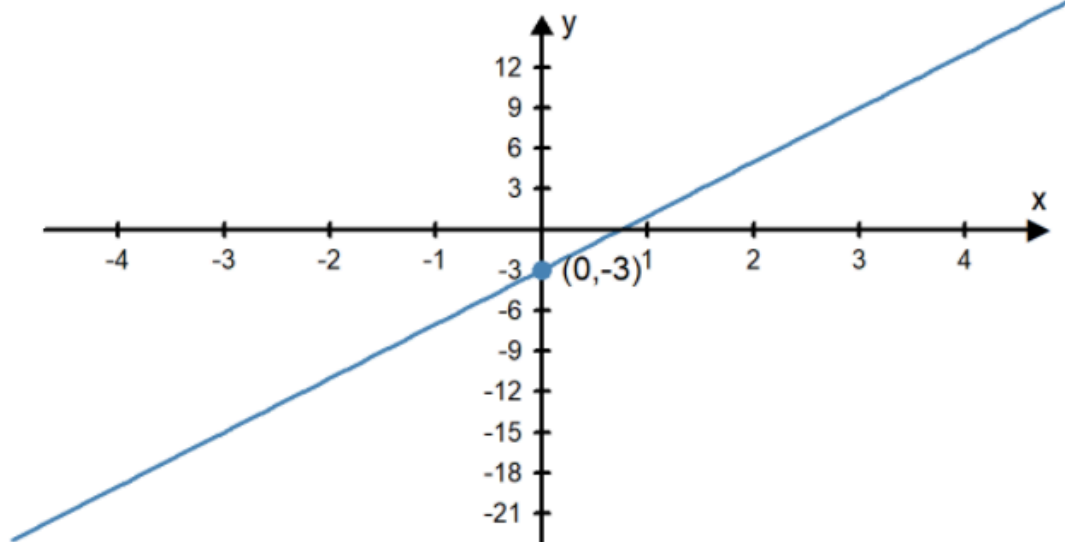
$$y = \frac{-12}{-3}x + \frac{9}{-3}$$

$$y = 4x - 3$$

$$f(x) = 4x - 3$$

We now know that we can graph this linear function by drawing a line that goes through the point $(0, -3)$ with a slope of $m = 4$.

$$f(x) = 4x - 3$$



A Linear Application - Calories per Day

A calorie is a unit of measurement that we use to measure energy. Sarah is a 22-year-old woman with an average weight and height but who does very little physical activity. Nevertheless, in a given day, she estimates that she spends about 1860 calories of energy. This means if Sarah eats more than 1860 calories in a day, her body will store the extra calories and she will gain weight. If she eats less than 1860 calories in a day, her body will need to use some of the stored calories to make up the difference and she will lose weight.

Sarah is trying to improve her health and realizes that by exercising, she can increase her caloric expenditure. Sarah decides to start walking each morning before she goes to work. She has determined that for her weight and the speed (15 minutes per mile), she will burn about 6 calories for each minute she walks. [2](#)

Her schedule varies every day so she can't always walk for the same amount of time. However, Sarah realizes that she can create a linear function that will give her the total amount of calories she can consume that is based on the number of minutes she walks. This will allow her to better monitor her caloric expenditure and meet her health goals.

The number of calories she spends on a day she doesn't go for a walk, 1860 calories, will be the y -intercept of the linear function. The slope of the function will be the number of calories burned per minute, or 6 calories per minute. This gives her the following linear function:

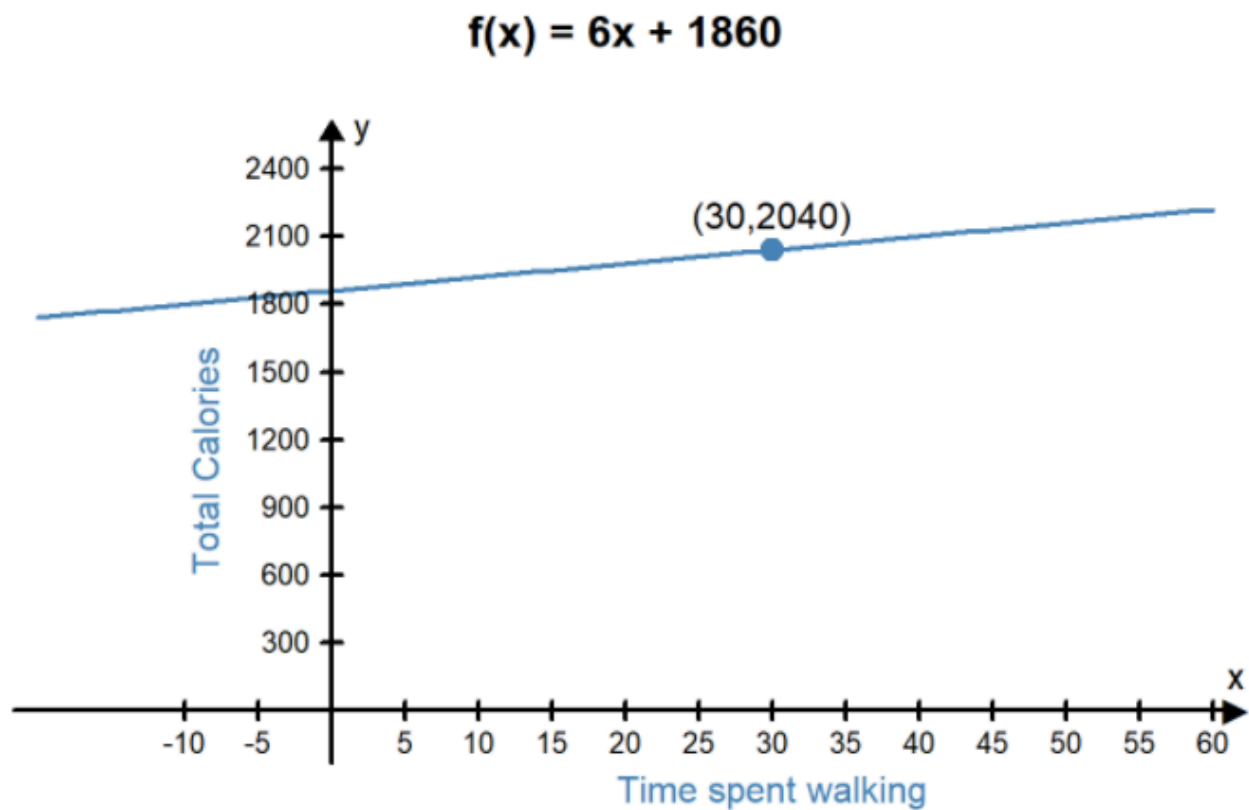
$$f(x) = 6x + 1860$$

Sarah makes the following chart to determine how many total calories she would need each day, depending on how long her walk is:

x = Length of Walk	$f(x)$ = Total calories
5 minutes	1890 calories

10 minutes	1920 calories
15 minutes	1950 calories
20 minutes	1980 calories
25 minutes	2010 calories
30 minutes	2040 calories
35 minutes	2070 calories
40 minutes	2100 calories
45 minutes	2130 calories
50 minutes	2160 calories
55 minutes	2190 calories
60 minutes	2220 calories

On a day when Sarah walked 30 minutes prior to work, she used the following graph to help her see how the walk effected the total number of calories she would burn that day. Based on this graph, she knows that her caloric expenditure is 2040 calories.



Quadratic Functions

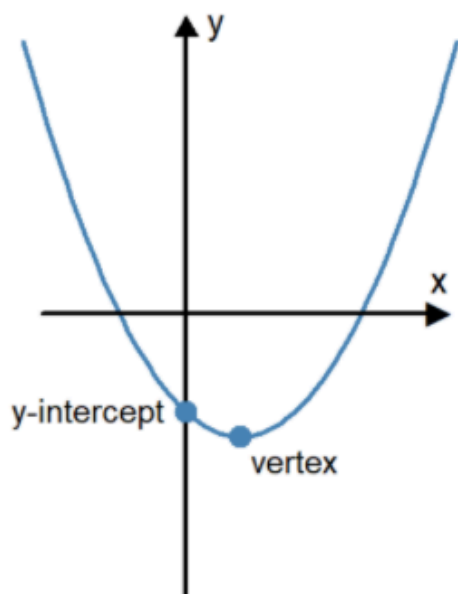
Another commonly used type of graph represents a quadratic function.

A **quadratic function** is a function in the form $f(x) = ax^2 + bx + c$ where a , b , and c are constants.

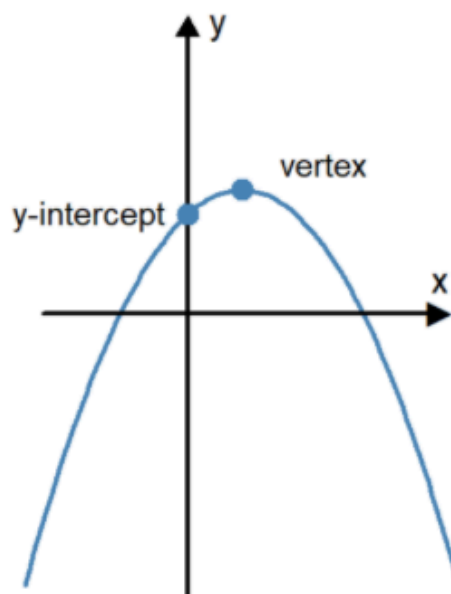
The graph of a quadratic function is always a u -shaped curve called a parabola. Three features that help us graph a quadratic function are the y -intercept, the *direction* the parabola opens, and the *vertex* of the parabola.

- The y -intercept is the point on the graph where the parabola crosses the vertical axis (y -axis).
- The direction of the parabola refers to whether the u -shape of the parabola opens up or opens down.
- The *vertex* of the parabola is either the highest or the lowest point on the parabola (depending on the direction of the parabola).

Parabola Opens Up



Parabola Opens Down



The graph of a **quadratic function** is a parabola. The vertex of the parabola has an x -coordinate of $x = \frac{-b}{2a}$. **You find the** y -coordinate of the vertex by finding $f(\frac{-b}{2a})$. The y -intercept is at $(0, c)$. If a is positive, the parabola opens up. If a is negative, the parabola opens down.

Example 4

Determine which of the following equations are quadratic functions.

a. $f(x) = -4^2 + 2x - 3$

b. $f(x) = \frac{-3}{x^2}$

c. $x^2 - 2y = 3x$

Solution:

Both equations a and c are quadratic functions.

Notice that equation a is already written in the form $f(x) = ax^2 + bx + c$. In this case $a = -4$, $b = 2$, and $c = -3$.

In equation c, the function is not written in standard form, but it can be rewritten as $f(x) = \frac{1}{2}x^2 - \frac{3}{2}x + 0$.

Equation b is not a quadratic function because the x^2 term is in the denominator of a fraction so it cannot be written in the form $f(x) = ax^2 + bx + c$.

Example 5

Sketch the graph of $f(x) = 2x^2 - 4x + 1$.

Solution:

First, we find the vertex of the parabola. We know it is at the point where $x = \frac{-b}{2a}$. So in this function, the vertex will be at

$$x = \frac{-b}{2a} = \frac{-(-4)}{2 \times 2} = \frac{4}{4} = 1$$

Since we know the x value of the vertex will be at $x = 1$, we can find $f(1)$ to find the y -value of the vertex.

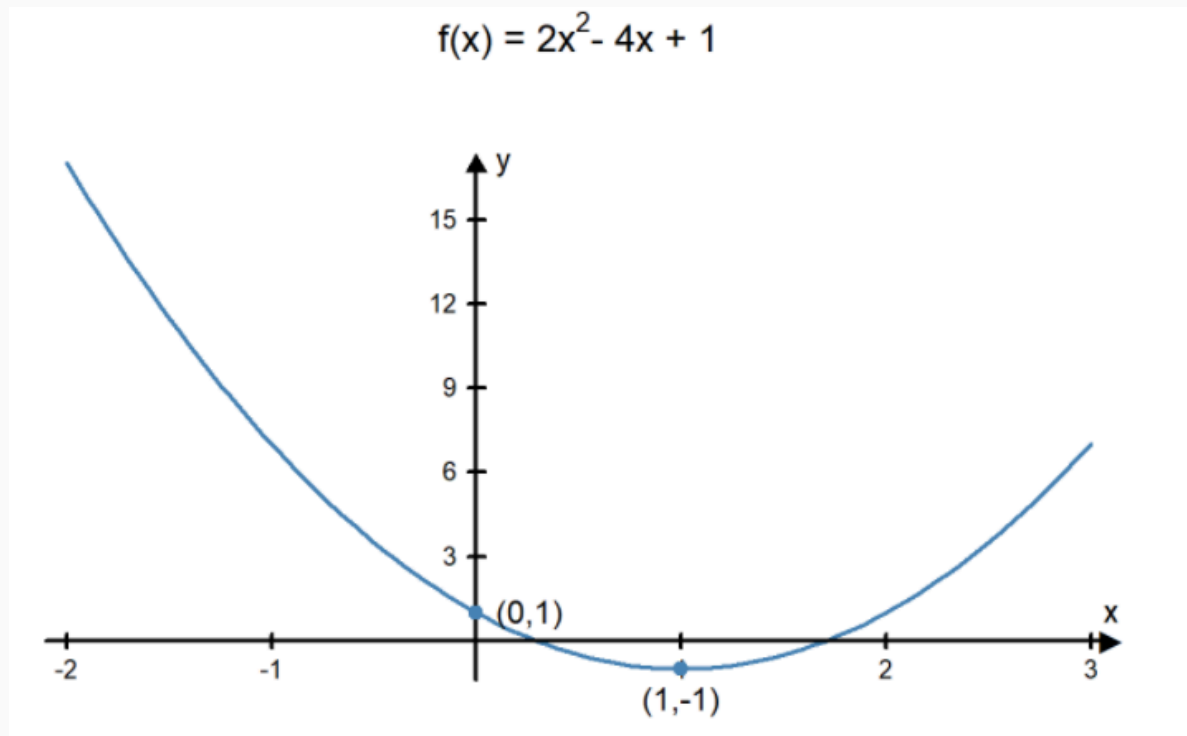
$$f(1) = 2(1^2) - 4(1) + 1 = 2 - 4 + 1 = -1$$

So we know the y -value of the vertex is $y = -1$. Therefore, the vertex of this parabola is $(1, -1)$.

Next, we find where the parabola crosses the y -axis. We know this happens at the point $(0, c)$, so in this example, the parabola will cross the y -axis at the point $(0, 1)$.

Finally, we determine whether the parabola opens up or down. In this parabola, $a = 2$. Because $a > 0$ we know the parabola opens up.

Putting all this information together gives us the following graph:



Practice: Graphing Quadratic Functions

Problem 1

Answer the following questions for the function $f(x) = 3x^2 - 18x + 16$, and then sketch the graph of the function.

- What is the x -coordinate of vertex?
- What is the y -coordinate of vertex?
- What is the y -intercept?
- Does the quadratic function open up or down?

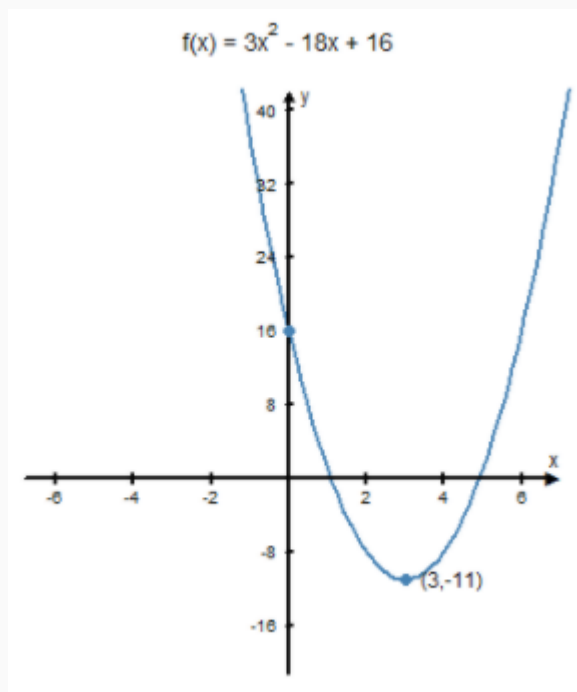
How Did I Do?

x -coordinate of vertex: 3

y -coordinate of vertex: -11

y -intercept: 16

Parabola opens up.



A Quadratic Application - Exploding Fireworks

One famous application of quadratic functions was discovered by the Italian mathematician and physicist Galileo Galilei.



In the 16th century, Galileo developed the following equation to show the path of a free-falling object:

$$f(x) = -16x^2 + v_0x + h$$

where x represents time, v_0 represents the initial velocity of the object, and h represents the initial height of the object (in feet). The input of the function represents how long the object has been in the air and the output of the function is the height of the object above the ground.

Andre is designing a fireworks show. He knows that he will fire a rocket from a location 2 feet above the ground and it will have an initial velocity of 203 ft/sec.

Andre needs to know how long to make the fuse of the rocket. For safety reasons, he needs to make sure the firework will be more than 500 feet above the ground when it explodes. He wants to have the firework explode when reaches its highest point.

Using Galileo's free-fall equation, Andre finds this equation for his rocket:

$$f(x) = -16x^2 + 203x + 2$$

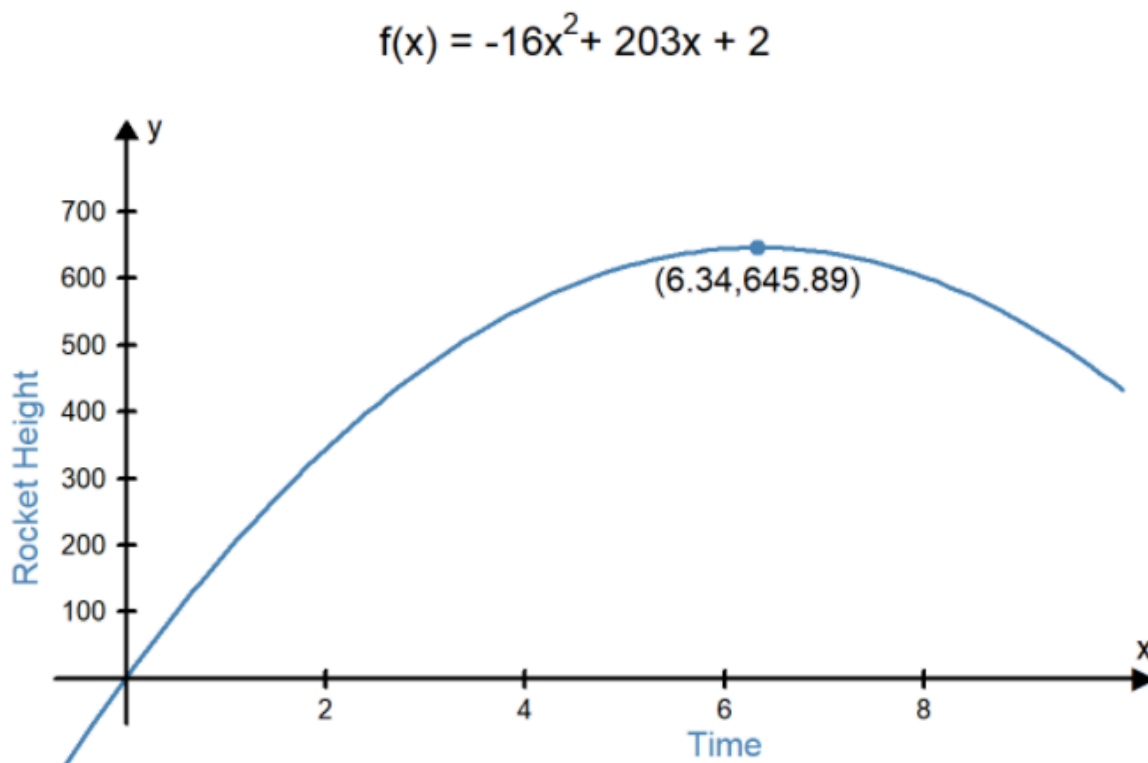
Because of the negative in the equation, he knows the parabola opens down, so the highest point will be at the vertex of the parabola. He finds the vertex of the parabola:

$$x = \frac{-b}{2a} = \frac{-203}{2 \times -16} = 6.34$$

$$f(6.34) = -16(6.34)^2 + 203(6.34) + 2 = 645.89$$

Since the vertex of the parabola is (6.34, 645.89), Andre knows the rocket will be 645.89 feet above the ground 6.34 seconds after he fires the rocket. So, he needs to use a fuse that will burn for 6.34 seconds.

Andre's graph verifies his solution:



Exponential Functions

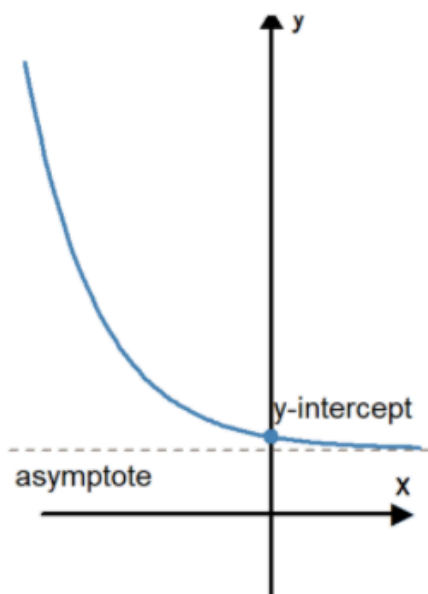
Exponential functions are used in many real-world applications such as computing compound interest in bank accounts and modeling the growth of populations.

*An **exponential function** is a function in the form $f(x) = a \cdot b^x + c$ where a , b , and c are constants and $b > 0$ and $b \neq 1$.*

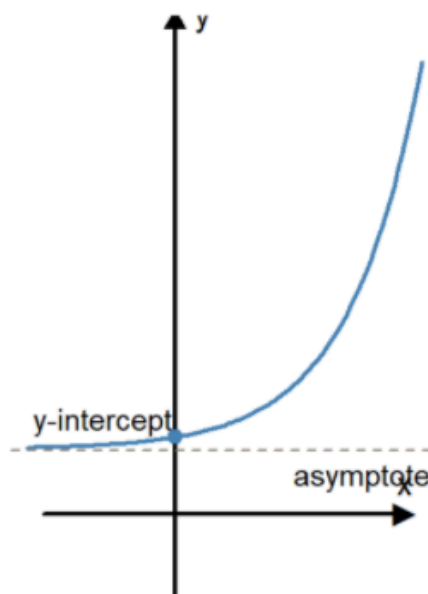
There are three features that help graph exponential functions:

- The y -intercept is the point where the graph crosses the vertical axis (y -axis).
- The horizontal asymptote is a line that the function gets closer and closer to, but never crosses.
- The graph can be either increasing or decreasing. An increasing graph goes up as you move along the graph from left to right. A decreasing graph goes down as you move along the graph from left to right.

Decreasing Exponential Graph



Increasing Exponential Graph



The graph of an **exponential** function has a y-intercept at $(0, a + c)$. Plotting another point on the graph helps determine the steepness of the graph. The graph has a horizontal asymptote at $y = c$. When $0 < b < 1$ the graph is decreasing when $b > 1$ it is increasing.

Example 6

Classify each of the following functions as being linear, quadratic, or exponential.

a. $f(x) = 3 \cdot 2^x + 4$

b. $f(x) = 4 - 3x$

c. $f(x) = 3x^2 - 4x + 2$

Solution:

a. This function is exponential. We know because the variable x is in the exponent.

b. This function is linear. We know because it is in the form $f(x) = mx + b$ where $m = -3$ and $b = 4$.

c. This function is quadratic. We know because it is in the form $f(x) = ax^2 + bx + c$ where $a = 3$, $b = -4$, and $c = 2$.

Example 7 - Graphing Exponential Functions

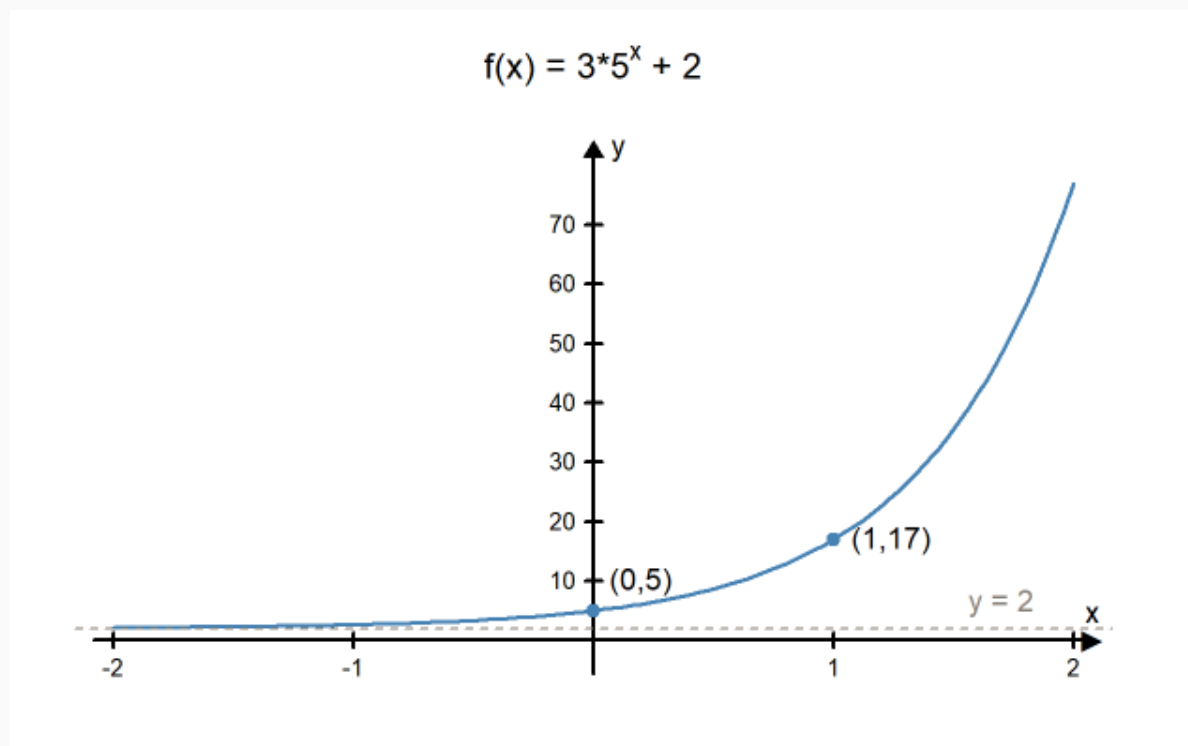
Graph the function $f(x) = 3 \cdot 5^x + 2$.

Solution:

This is an exponential function of the form $f(x) = a \cdot b^x + c$ where $a = 3$, $b = 5$, and $c = 2$.

- The y -intercept is at $(0, 3 + 2) = (0, 5)$.
- When $x = 1$, $f(1) = 3 \cdot 5 + 2 = 17$. The graph goes through the point $(1, 17)$.
- The horizontal asymptote is at $y = 2$.
- Since $b = 5$ is greater than 1, we know the function is increasing.

This information tells us the graph looks like this:



Example 8 - Graphing Exponential Functions

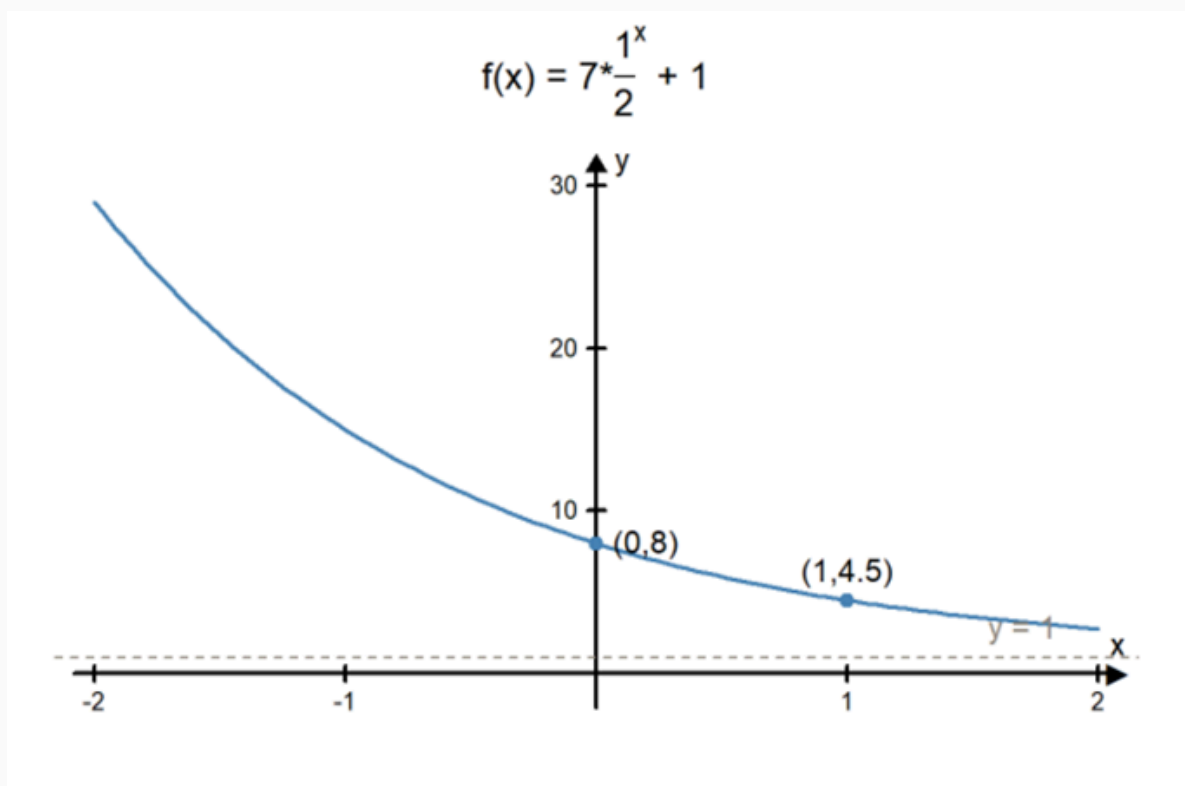
Graph the function $f(x) = 7\left(\frac{1}{2}\right)^x + 1$.

Solution:

This is an exponential function of the form $f(x) = a(b^x) + c$ where $a = 7$, $b = \frac{1}{2}$, and $c = 1$.

- The y -intercept is at $(0, a + c) = (0, 7 + 1) = (0, 8)$.
- When $x = 1$, $f(1) = 7\left(\frac{1}{2}\right) + 1 = \frac{9}{2} = 4.5$. The graph goes through the point $(1, 4.5)$.
- The horizontal asymptote is at $y = 1$.
- Since $b = \frac{1}{2}$ is between 0 and 1, we know the function is decreasing.

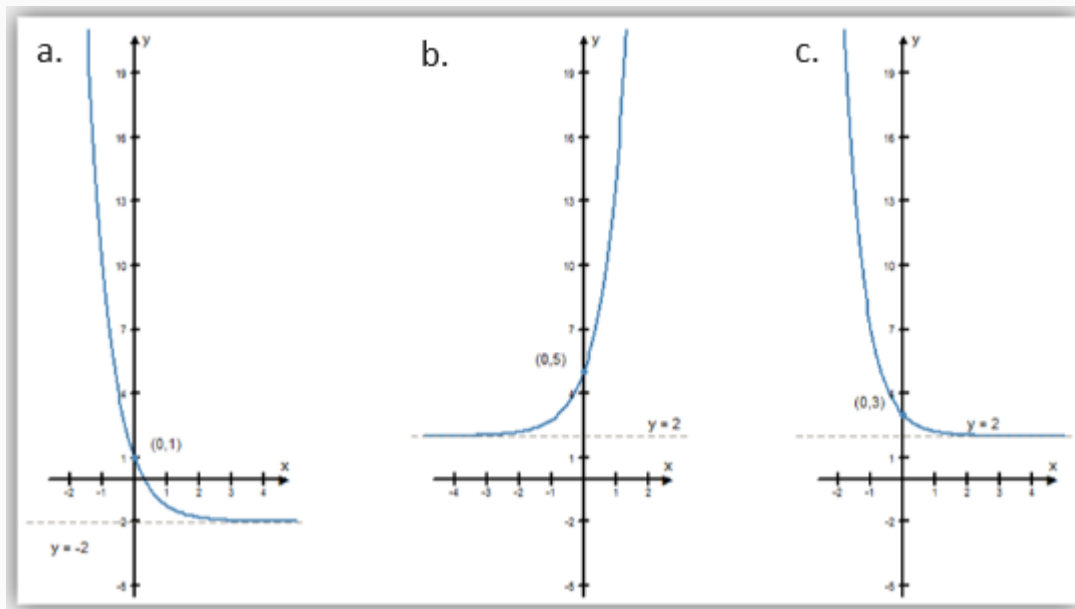
This information tells us the graph looks like this:



Practice: Graphing

Problem 2

Which is the correct graph of the exponential function $f(x) = 3\left(\frac{1}{4}\right)^x - 2$?



How Did I Do?

Graph a is correct.

Graph b is incorrect because $b = \frac{1}{4}$ in the equation. This means the graph should be decreasing instead of increasing.

Graph c is incorrect because $c = -2$ in the equation. This means the graph should have a horizontal asymptote at $y = -2$ instead of $y = 2$.

HIDDEN CONTENT

HIDDEN CONTENT

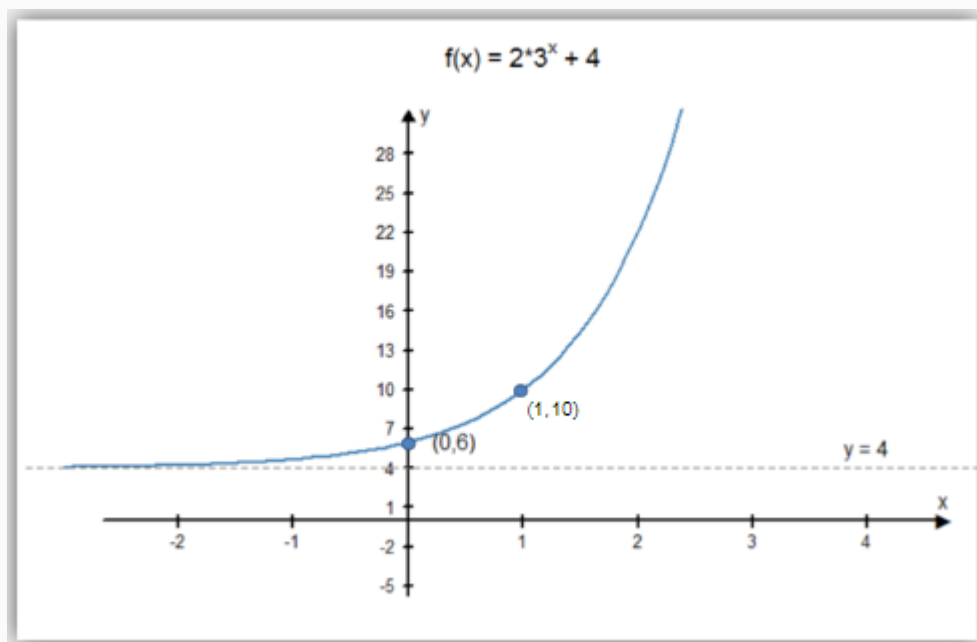
Problem 3

Answer the following questions for the function $f(x) = 2 \cdot 3^x + 4$, and then sketch the graph of the function.

- What is $f(0)$?
- What is $f(1)$?
- What is the horizontal asymptote?
- Is the graph increasing or decreasing?

How Did I Do?

- $f(0) = 6$
- $f(1) = 10$
- The horizontal asymptote is $y = 4$
- The graph is increasing



HIDDEN CONTENT

HIDDEN CONTENT

An Exponential Application - Carbon-14 Dating

In the introduction we learned that carbon-14 dating is used to determine the age of organic materials. This is a good example of an application involving exponential functions.

Carbon-14 dating works by comparing the ratio of carbon-14 to carbon-12 in the organic material. Carbon-12 is a stable form of carbon that does not decay over time. Carbon-14, on the other hand, is radioactive, and is therefore an unstable form of carbon. Living things have a consistent ratio of carbon-12 to carbon-14. However, when an organism dies, it

stops replenishing carbon-14 into its chemical makeup. Without new carbon-14 sources being introduced into the organism, the carbon-14 will decay over time into elements other than carbon. This will cause the carbon-12 to carbon-14 ratio to change with time. By looking at the current carbon-12 to carbon-14 ratio of the dead organism and comparing it to a carbon-12 to carbon-14 ratio in a living organism, scientists can use this difference to calculate how long an organism has been dead.

This is what is used to determine the age of ivory. While the tusk is still intact on a living elephant, the ratio of carbon-12 and carbon-14 will stay consistent. But when the tusk is removed from the elephant, the amount of carbon-14 begins to change.

The half-life of carbon-14 is about 5,730 years. This means that half of the carbon-14 decays away every 5,730 years. Knowing this, scientists can create the following exponential equation to describe the amount of carbon-14 that is present over time.

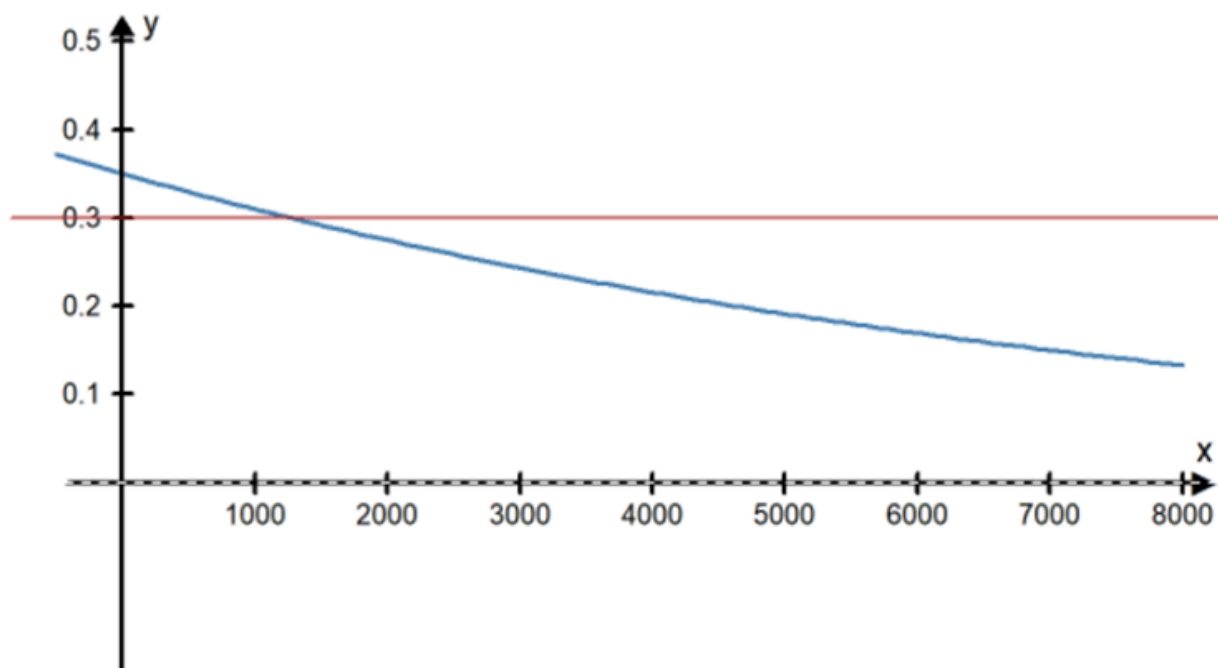
$$f(x) = A\left(\frac{1}{2}\right)^{x/5730}$$

In this equation A represents the amount of carbon-14 initially present in the living organism. The input for this function is time (in years) since the organism died and the output is the current amount of carbon-14 present in the organic material.

If a particular ivory tusk from an elephant had 0.35 grams of carbon-14 present when the elephant was living, the equation will become:

$$f(x) = 0.35\left(\frac{1}{2}\right)^{x/5730}$$

If researchers test a piece of ivory and find there are currently 0.30 grams of carbon-14 present, how old would this piece of ivory be? Looking at the graph, the red line shows that the y-value will be 0.3 after a little more than 1000 years. So this piece of ivory is over 1000 years old.



At the beginning of the lesson, we mentioned new techniques for carbon-14 dating developed by researchers at the University of Utah. Because carbon-14 has such a long half-life, it isn't very helpful at determining the age of organic materials that are less than about 50,000 years old. New methods developed by the University of Utah researchers are particularly helpful in fighting poaching because they found a way to make carbon-14 dating work for specimens that are only 30-50 years old.

Practice: Check Your Understanding

Problem 4

What type of function is this?

$$f(x) = -5x + 2$$

☐

Linear Function

☐

Quadratic Function

☐

Exponential Function

Problem 5

For the same function: $f(x) = -5x + 2$

- What is the slope?
- What is the y -intercept?

How Did I Do?

- The slope is -5
- The y -intercept is 2 .

Problem 6

What type of function is this?

$$f(x) = 2 \cdot 4^x + 7$$

- ☐ Linear Function
- ☐ Quadratic Function
- ☐ Exponential Function

Problem 7

For the same function: $f(x) = 2 \cdot 4^x + 7$

- What is the horizontal asymptote?
- What is the y -intercept?
- Is the graph increasing or decreasing?

How Did I Do?

- The horizontal asymptote is $y = 7$
- The y -intercept is 9
- The graph is increasing.

Problem 8

What type of function is this?

$$f(x) = 5 + 40x - 10x^2$$

- ☐ Linear Function
- ☐ Quadratic Function
- ☐ Exponential Function

Problem 9

For that same function: $f(x) = 5 + 40x - 10x^2$

- What is the x -coordinate of the vertex?
- What is the y -intercept?
- What is the direction (up or down)?

How Did I Do?

1. The x -coordinate of the vertex is 2
2. The y -intercept is 5
3. The quadratic opens down.

Social Media Marketing Example

Viktor is running a small business and wants to improve his marketing. He decides to use the Quantitative Reasoning Process to make some decisions about the type of marketing he wants to do.

1 Understand the Problem

Viktor's business is an online tractor sales business. Viktor does not have a store and runs the business out of his home. Most of his business is done through his website and over his phone. He believes his sales will increase if more people see his website. He decides to use Facebook, Twitter, and Instagram to market his business. He wants these social media tools to drive traffic to his website.

2 Identify Variables & Assumptions



Viktor finds a marketing company willing to run a social media marketing campaign for his business for a fixed monthly fee. The company offers three levels of service with different costs. As he thinks about his situation, Viktor identifies the following variables:

- The number of customers visiting his website per week
- The number of customers who purchase a tractor per week
- The percentage of paying customers who visited his website
- His average monthly sales
- The choice of marketing plan from the marketing company (Plan A, Plan B, or Plan C)
- The cost of hiring marketing company

He makes the following assumptions:

- The claims of the social media marketing company are accurate. They claim:
 - Plan A (highest expense) will increase the traffic to his website by 5% per month.
 - Plan B (medium expense) will increase the traffic to his website by 3.5% per month.
 - Plan C (lowest expense) will increase the traffic to his website by 2% per month.
- If more customers visit his website, he will sell more tractors.

3 Apply Quantitative Tools



Analytics for his website show that he currently gets around 75 unique visitors to his website per month. He hopes to double the number of visits per month to 150 unique visitors per month.

Because the marketing plans claim to increase traffic to his website by a fixed percentage every month, Douglas recognizes the number of visits can be described with an exponential function. He creates the following three exponential functions:

$$\text{Plan A: } f(x) = 75(1.05)^x$$

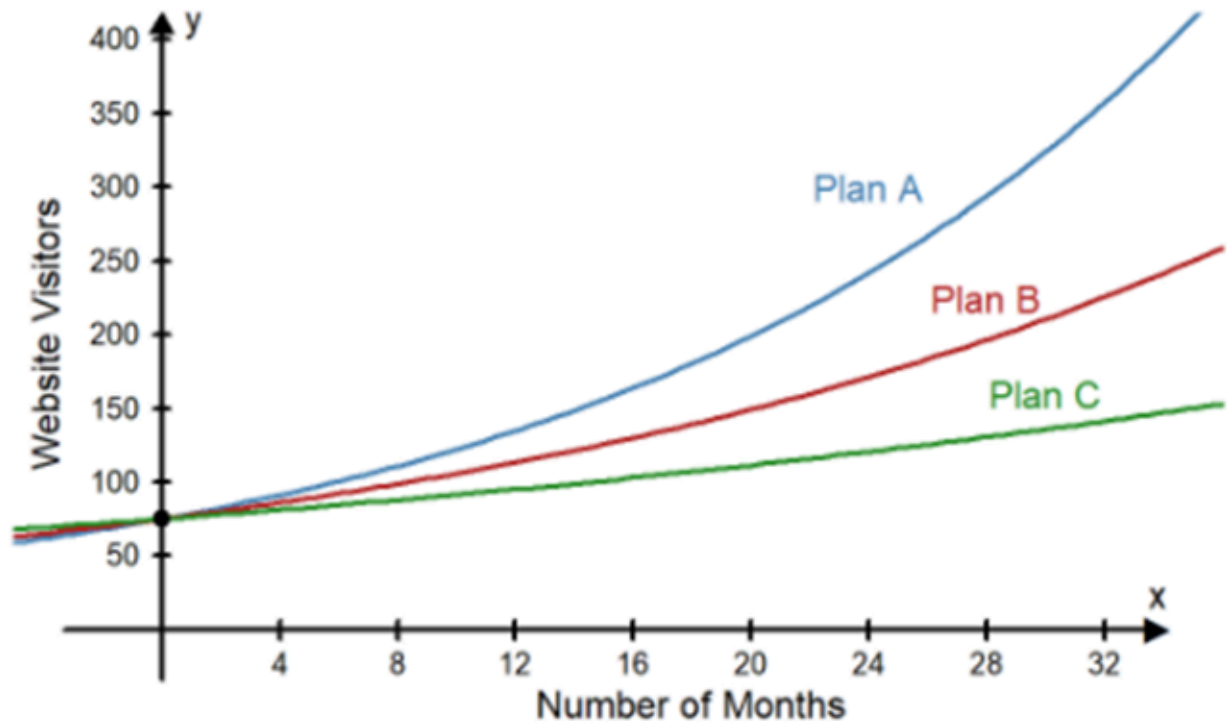
$$\text{Plan B: } f(x) = 75(1.035)^x$$

$$\text{Plan C: } f(x) = 75(1.02)^x$$

In these equations, the input x represents the number of months the social media marketing campaign has been running. The output $f(x)$ represents the number of visitors to the website that month.

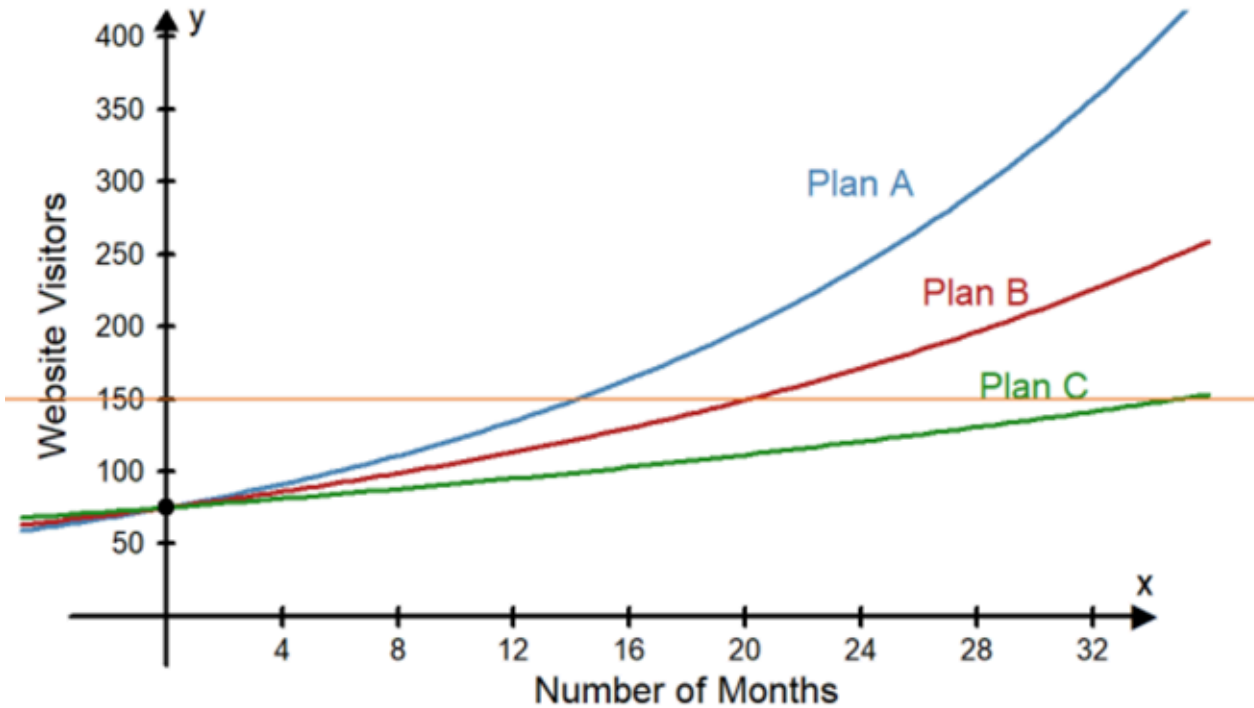
Viktor created the following graph with the three exponential functions on the same graph to compare the three plans.

Comparison of Three Social Media Marketing Plans



Because he is trying to get to 150 website visitors per month, he decides to draw a line on the graph at $y = 150$ so he can see how long it will take to reach 150 website visitors with each plan.

Comparison of Three Social Media Marketing Plans



Looking at the graph, he finds where the line intersects each curve. The x-value at the intersection point will represent the number of months required double the traffic to his website:

Plan	Approximate Number of Months
Plan A	about 14 months
Plan B	about 20 months
Plan C	about 35 months

He realizes that Plan A is the fastest, Plan B takes about 6 months longer than Plan A, and Plan C takes about 15 months longer than Plan B.

4

Make an Informed Decision

Using the results of his computations, Viktor compares the cost of each plan to the length of time required to double the traffic to his website (see chart above). He finds that Plan A is quite a bit more expensive than Plan B, but Plan B doesn't take that much longer to double the traffic to his website. Although Plan C is the cheapest, it takes much more time to get the result he wants.

Viktor selects Plan B.

5 Evaluate Your Reasoning



After using the marketing company for 6 months, Viktor decides to reevaluate his assumptions. He reviews his website traffic over the 6 months to see how things are going. According to the claims of the Social Media Marketing Company, he should have seen approximately a 3.5% increase of traffic to his website each month. He decides to compare the actual number of visits to his website to the predicted number of visits from the exponential function he used to make his decision. In the table below, the second column shows the predicted website visitors if the marketing plan worked as expected. The third column shows the actual number of visitors to the website.

Month	Predicted visitors	Actual visitors
1	$f(1) = 75(1.035)^1 = 77.63$	80
2	$f(1) = 75(1.035)^2 = 80.34$	81
3	$f(1) = 75(1.035)^3 = 83.15$	85
4	$f(1) = 75(1.035)^4 = 86.06$	79
5	$f(1) = 75(1.035)^5 = 89.07$	87
6	$f(1) = 75(1.035)^6 = 92.19$	93

He notices that there was a dip in Month 4 with fewer visitors than expected. He was not concerned because Month 4 was December, which is usually the slowest month for his business and he found the number of visitors was back up to the expected amount by Month 6. He determines the Social Media Marketing is working as expected.

Now that he knows more customers are visiting his website, he decides to examine the accuracy of his assumption that more website visitors will increase tractor sales. He compares his current sales to sales for the same months the previous year and finds on average his sales have increased by 4% since hiring the social media marketing company. Viktor is pleased with the results of his decision using the Quantitative Reasoning Process and plans to keep using the company.

Lesson Checklist

By the end of this lesson, you should be able to:

- Correctly apply the order of operations
- Correctly use the rules of exponents
- Use function notation to evaluate a function (linear, quadratic, exponential) at a point
- Distinguish among linear, exponential, and quadratic graphs
- Identify graphs of linear, quadratic, and exponential functions from their equations
- Interpret functional values or graphs in context of a given situation.

¹ [Age and legality of ivory revealed by Carbon-14 dating can fight poachers; July 1, 2013; Phys.org](#)

² <https://www.verywell.com/walking-calories-burned-per-minute-3887138>



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Lesson 9: Change Over Time

Opening Story

Lesson 9 - Opening Story

[L9 Opening Story Transcript](#)

Introduction

Most things change over time. Some things change quickly, like the weather or stock prices. Other things change slowly, like the depth of the Grand Canyon, or recovering from a significant injury. Change, whether fast or slow, is always happening around us. When we are making decisions using the Quantitative Reasoning Process, it is important to determine how quickly things will change. This is especially true of financial decisions.

When we consider saving money or taking out loans, the rate at which our savings or loan balance changes is an important factor. When we save money, we look at the rate at which interest builds up and is added to our balance. When we take out a loan, we want the amount of interest we pay each month to decrease as time passes. However, since interest on loans compounds, our loan balance can start to grow very rapidly. Loans can quickly get out of control if we do not manage our finances well.

Amadi's story from the opening video is a good example of a situation of change over time. Amadi wants to know how the amount of money he is saving for his home will change over time. He wants to make sure that he has enough money to purchase a home.

We will start by applying the Quantitative Reasoning Process to Amadi's example shown in the opening video.

Amadi's Savings

1 Understand the Problem

Amadi wants to save up enough money to purchase a home in 10 years. He already has \$12,000 saved, but he expects the home to cost \$50,000 and wants to know how much money he needs to save each month. He believes he can get an interest rate close to 1.5% APR on a savings account. He needs to determine how much money to save each month in order to reach his goal.

2 Identify Variables & Assumptions



The variables in this situation are:

- The amount Amadi adds to his savings account each month
- The total amount of money saved in 10 years

Amadi has to make the following assumptions:

- He will be able to use the \$12,000 he has already saved for his home
- He will be able to find or build a house for \$50,000
- His savings account will have an interest rate of 1.5%

The total amount of money Amadi will have saved in ten years is a function of the amount he saves each month.

3 Apply Quantitative Tools



The Excel payment function, PMT, from Lesson 7 will help Amadi determine how much money he needs to save each month. In Lesson 7 we used the PMT function to help us find loan payments. When computing payments on a savings account, the PMT function is used in a similar way.

Recall that the syntax for the PMT function is

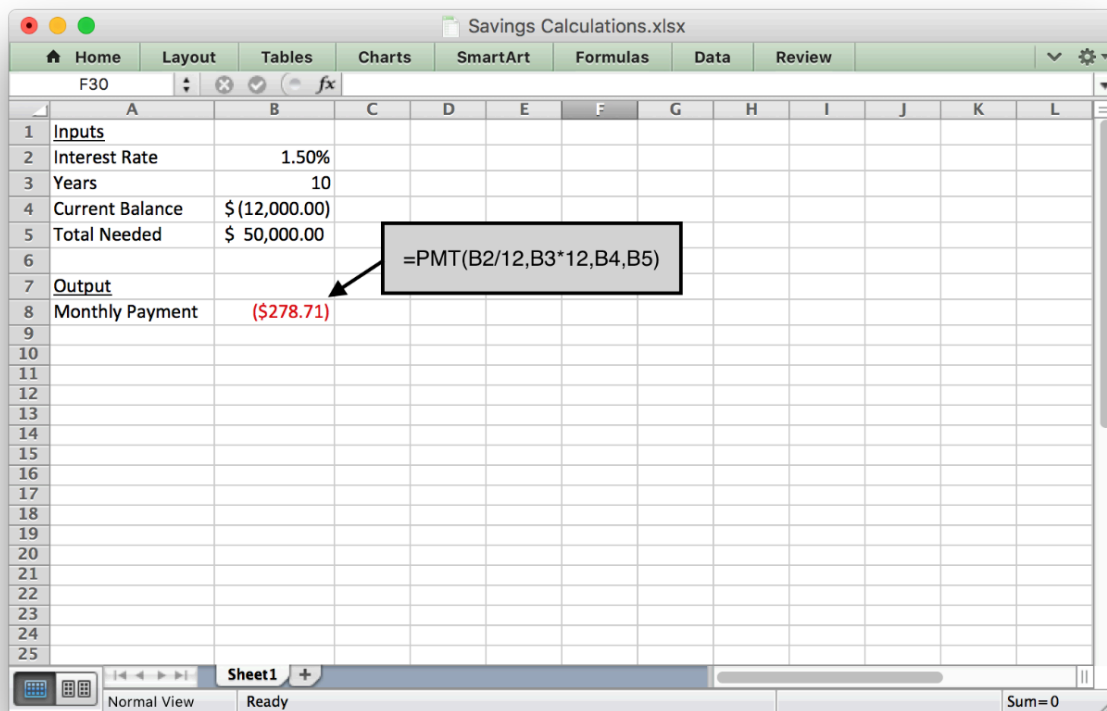
$$=PMT(\text{rate, number of periods, loan amount})$$

However, the function has a fourth optional input:

$$=PMT(\text{rate, number of periods, PV, [FV]})$$

In this case, PV stands for *present value* and FV stands for *future value*. [FV] is in square brackets to show it is optional. If we want to figure out how much to save each month, the current balance in our savings account is the present value (PV) and the total amount we need to save is the future value (FV).

Amadi used this command to set up his Excel file.



Notice that some numbers were positive and some were negative. Remember that a negative number is used for money “leaving your wallet” and a positive number is for money “entering your wallet”. Because Amadi is putting \$12,000 in his savings account now, the money is “leaving his wallet” and will be negative. His monthly payments are also negative. But the \$50,000 he will have saved at the end of 10 years is money he will get to take out of the bank and “put back in his wallet”, so it is entered in Excel as a positive number.

The Excel output from the PMT function shows Amadi he needs to save \$278.71 each month in order to have saved \$50,000 in 10 years.

4 Make an Informed Decision



After doing these calculations, Amadi realizes he needs to save at least \$278.71 each month. He carefully examines his monthly budget. He decides if he is careful with the money he earns, he can save \$300 each month. Saving a little extra every month will help, just in case he isn’t able to get the interest rate he wants or if he has an emergency and can’t deposit the full payment for a month or two.

5 Evaluate Your Reasoning



After saving for a year, Amadi takes some time to review and evaluate his plan. He finds that saving \$300 every month is difficult. It requires him to be extremely careful about the money he spends, but he is making it work. As he reevaluates his assumptions, he finds them to be fairly accurate. He is happy with the progress he has made.

Practice: Amadi's Savings

Problem 1

Use the future value (FV) function in excel to find how much money Amadi will have saved after one year, two years, and five years.

Account balance after 1 year =\$

Account balance after 2 years =\$

Account balance after 5 years =\$

How Did I Do?

Account balance after one year = \$15,806.10

Account balance after two years = \$19,699.68

Account balance after five years = \$31,614.08

Hint: Make sure you are entering in the correct information. Amadi started with \$12,000, then he adds \$300 per month in an account that earns 1.5% interest.

Change over Time

Notice that the amount of money in Amadi's account is changing over time. Each month the account balance changes as he earns interest and makes new deposits to the account. We often talk about how fast things change, or rates of change. Rates of change apply in many situations. For example, imagine you just bought a new car. Over time, the car will depreciate. The value of the car goes down as time passes. The cost of gas rises and falls. As the car ages, systems wear out and the cost of maintaining the car typically goes up.

In this lesson we will learn some mathematical principles that can help us quantify change. Predicting and understanding how things change helps us effectively use the Quantitative Reasoning Process to make good decisions in situations where change occurs over time.

Constant and Variable Slopes

Graphs of functions give us a visual way to see how fast a quantity is changing. When we look at graphs of functions, the rate of change is represented by the slope of the line or the curve. Some functions have constant slopes and some functions have variable slopes. This distinction is important to consider when we study change over time.

Constant Slope

Let's look at an example of a function with a constant slope. Karina is working at her first job after graduating from college. When she started working, she was paid an annual salary of \$32,000 per year. Every year, her employer gives her a raise of \$1,600 per year. Her annual salary can be represented as a function of the time she has worked at the store.

Let y represent her annual salary and let x represent the number of years she has worked for the company. Then, we could say:

$$y = 1600x + 32000$$

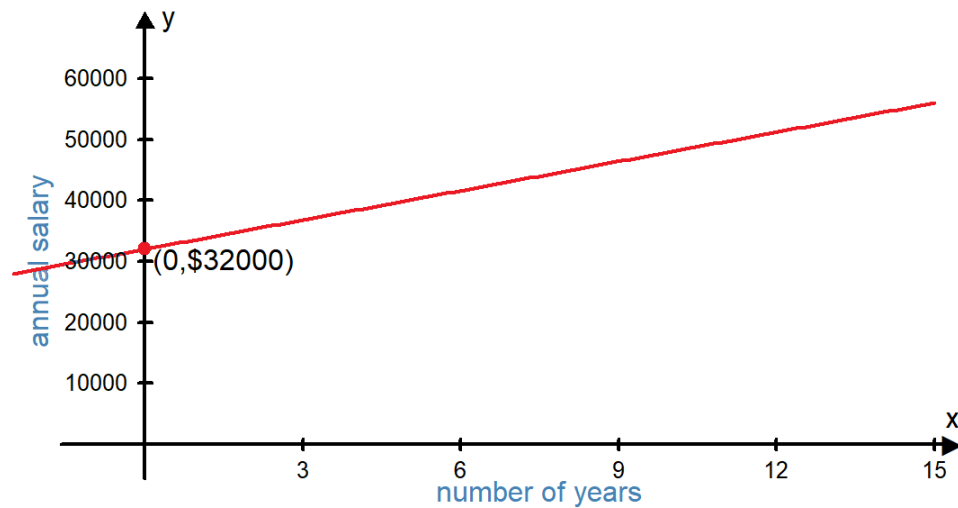
Let's look at some inputs and outputs of this function:

x = number of years	y = annual salary
0	$y = 1600(0) + 32000 = \$32,000$
1	$y = 1600(1) + 32000 = \$33,600$
2	$y = 1600(2) + 32000 = \$35,200$
3	$y = 1600(3) + 32000 = \$36,800$
4	$y = 1600(4) + 32000 = \$38,400$
5	$y = 1600(5) + 32000 = \$40,000$

Our chart shows that the function $y = 1600x + 32000$ accurately represents Karina's situation. In this chart we see that for each additional year Karina worked for the company, her salary increased by \$1600 per year.

This is a linear function with a slope of $m = 1600$ and a y -intercept of 32000. When we look at the graph of the function, the slope of the line represents the rate of change.

Karina's Salary



Slope is the change in y divided by the change in x . In this equation, we can see that the slope is 1600.

$$\text{slope} \frac{\text{change in } y}{\text{change in } x} = \frac{\text{change in annual salary}}{\text{change in years worked}} = \frac{1600}{1}$$

When the number of years worked changes by 1 year, Karina's annual salary changes by \$1600. Notice that this slope is consistent. Every year, Karina's salary increases by the same amount. It always goes up by \$1600 per year. We see this on the graph as a straight line. The slope is always the same. From any point on the graph, we can find another point if we go up by 1600 and move right 1.

The slope of a function is the *rate of change*. The slope tells us how fast the output value is changing given a particular change in the input. If the graph is a straight line, the rate of change (and the slope) is constant between any two points.

*Linear functions represent situations with a **constant** rate of change.*

Variable Slope

Now let's look at a similar example where the slope is not constant. Adriana is also working at her first job after graduation. Like Karina, when she started her job she made \$32,000 per year. However, instead of getting a \$1600 raise each year, she gets a 5% raise each year. The function that will describe Adriana's salary each year is:

$$y = 32000(1.05)^x$$

where x represents the number of years Adriana has worked at the store and y represents her annual salary.

Let's look at a table showing us Adriana's salary over the five years she works at the store.

x = number of years	y = annual salary
0	$y = 32000(1.05)^0 = \$32,000$
1	$y = 32000(1.05)^1 = \$33,600$
2	$y = 32000(1.05)^2 = \$35,280$
4	$y = 32000(1.05)^4 = \$38,896.20$
5	$y = 32000(1.05)^5 = \$40,841.01$

Notice that the changes in Adriana's salary are different each year. The first year her salary goes up from \$32,000 to \$33,600, so she gets a \$1600 raise. The second year her salary goes up from \$33,600 to \$35,280. This time she got a \$1680 raise. Let's create a column showing the amount of her raise in the table.

x = number of years	y = hourly wage	Raise Amount
0	$y = 32000(1.05)^0 = \$32,000$	
1	$y = 32000(1.05)^1 = \$33,600$	\$1600
2	$y = 32000(1.05)^2 = \$35,280$	\$1680
3	$y = 32000(1.05)^3 = \$37,044$	\$1764
4	$y = 32000(1.05)^4 = \$38,896.20$	\$1852.50
5	$y = 32000(1.05)^5 = \$40,841.01$	\$1944.81

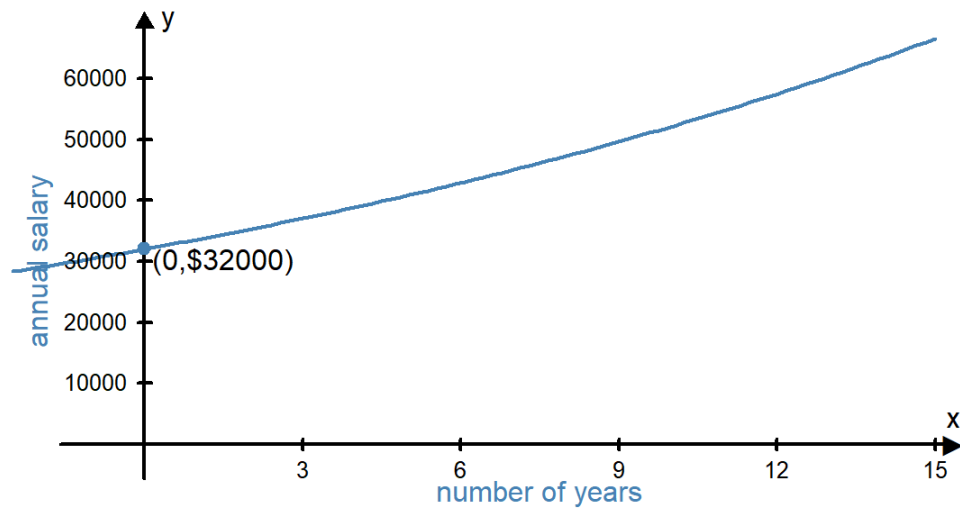
Notice that the raise amount is not constant. This function has a *variable slope* or a *variable rate of change*.

Recall that if a function has a constant slope, it is a linear function. If a function has a variable slope, it will be a *non-linear* function.

*Non-linear functions represent situations with **variable** rates of change.*

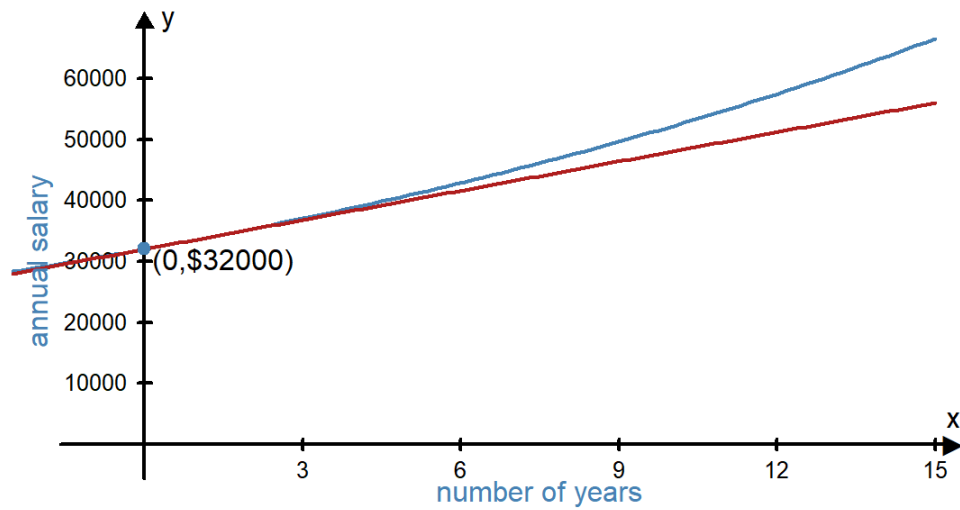
There are many different types of non-linear functions. The non-linear function in this example is exponential.

Adriana's Salary



If we graph both Karina and Adriana's salary functions on the same graph, it makes it easier to compare them and see that the graph of Adriana's salary function is not a line. Notice that at first, their salaries are similar, but after a few years Adriana's salary starts to pull away from Katrina's salary. Notice Katrina's salary continues to grow at a constant rate, but Adriana's salary grows by increasingly larger amounts each year. This is the difference between linear and non-linear rates of change.

Adriana and Katrina's Salaries

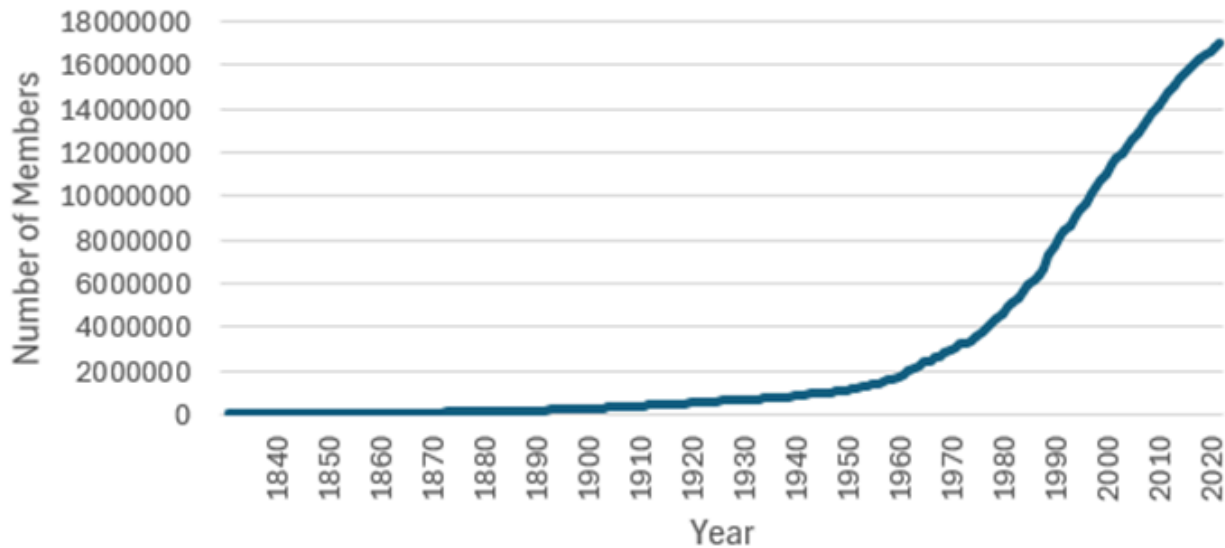


Church Membership Example

Let's look at another example of change over time. Since the organization of the Church of Jesus Christ of Latter-day Saints in 1830 with six members, the membership has grown significantly. Has the change been constant or variable? One way to determine the type of change is to look at a graph of church membership to see if the pattern is linear or non-linear.

In Lesson 5 we learned that when we want to create a chart showing a trend over time, we use a time series plot. Using Excel, we created the following time series plot for church membership.

Membership in the Church of Jesus Christ of Latter-day Saints



This graph is clearly not a line. Because the graph is non-linear, we know the slope is not constant and that membership of the church has a variable rate of change. Notice that in the 1800s in the early history of the church, the number of new members added per year was small especially when compared to the number of members added to the church in recent years. We can see this in the graph. Let's look at some specific numbers.

- In 1875 there were 107,167 members of the church. By 1876 membership had grown to 111,111. This is an increase of 3,944 members that year.
- In 1975 there were 3,572,202 members of the church. By 1976 membership had grown to 3,742,749. This is an increase of 170,547 new members that year. ¹

These numbers verify what we see in the graph. The slope of the curve tells us the number of new members per year. The slope is not constant, so we know the number of new members per year is variable. We describe the change in church membership over time as non-linear.

Understanding Interest

As part of your monthly budget, you should plan to save some money every month for an emergency or a large purchase. You could physically keep the money (for example: in an envelope, a home safe, or under your mattress) or you could put it in a savings account in your bank. The advantage of storing your money in a savings account is that you will then earn interest on the money you save. Banks pay compound interest.

Compound interest is earned both on the original amount **and** on any accumulated interest.

When you earn compound interest, you are earning interest on your interest. This means your account balance can grow very quickly. Putting money under your mattress every month leads to a *constant rate of change*. It only grows according to the amount you put in. Putting money in an account that pays compound interest leads to a *variable rate of change*. Not only does your account increase by the amount you put in, it also increases as you earn interest.

Compound Interest Equations - Savings

There are three mathematical equations we need in order to compute the account balances of savings accounts that pay compound interest. These mathematical equations are used by Excel's future value command, FV. We will be learning these equations to help us better understand what Excel is doing when it does calculations. You don't need to memorize these equations, but if you are given one of these equations you should be able to enter the inputs and find the corresponding output.

In these equations there are six variables. We will use the same variable names that are used in Excel:

- FV = Final Balance (future value)
- PMT = Monthly Payment
- PV = Principal Balance (present value)
- r = Annual interest Rate (written as a decimal)
- n = number of compoundings per year
- Y = number of years

Here are the three equations:

Situation	Equation
Deposit PV dollars in an account and leave it for Y years	$FV = PV\left(1 + \frac{r}{n}\right)^{(nY)}$
Start with no money in the account, but add PMT dollars per month for Y years	$FV = PMT \frac{\left(1 + \frac{r}{n}\right)^{(nY)} - 1}{\left(\frac{r}{n}\right)}$
Start with a balance of PV dollars and add PMT dollars per month for Y years	$FV = PV\left(1 + \frac{r}{n}\right)^{(nY)} + PMT \frac{\left(1 + \frac{r}{n}\right)^{(nY)} - 1}{\left(\frac{r}{n}\right)}$

As you can see, the equations get complicated. But it makes it easier to see what is going on if you notice the third equation is just the sum of the first two.

Excel is useful because it allows us to do these calculations without using the equations.

Example 1

Carlos wants to start a savings account. He has \$2000 he can deposit now and plans to add \$50 each month. The account will earn 2.0% interest and is compounded monthly. Carlos wants to know how much money he will have saved in 10 years.

Let's do the problem using the equation and using Excel. Notice that we get the same amount either way.

Solution - Using the Equation:

Using the equation from above, Carlos wants to find the value of FV . He knows the values of the other variables:

- $PMT = \$50$
- $PV = \$2000$
- $r = 0.02$
- $n = 12$
- $Y = 10$

Putting all these values into the equation gives us:

$$\begin{aligned} A &= PV \left(1 + \frac{r}{n} \right)^{(nY)} + PMT \frac{\left(\left(1 + \frac{r}{n} \right)^{(nY)} - 1 \right)}{\left(\frac{r}{n} \right)} \\ &= 2000 \left(1 + \frac{0.02}{12} \right)^{(12 \times 10)} + 50 \frac{\left(\left(1 + \frac{0.02}{12} \right)^{(12 \times 10)} - 1 \right)}{\left(\frac{0.02}{12} \right)} \\ &= 2442.40 + 6635.98 \\ &= 9078.38 \end{aligned}$$

At the end of 10 years, Carlos will have \$9078.38 in his account.

Solution - Using Excel:

Instead of using the equation above, Carlos could use Excel to find the answer. He is looking for the future value of his account, so he will use the FV command in Excel.

The syntax for the FV command is:

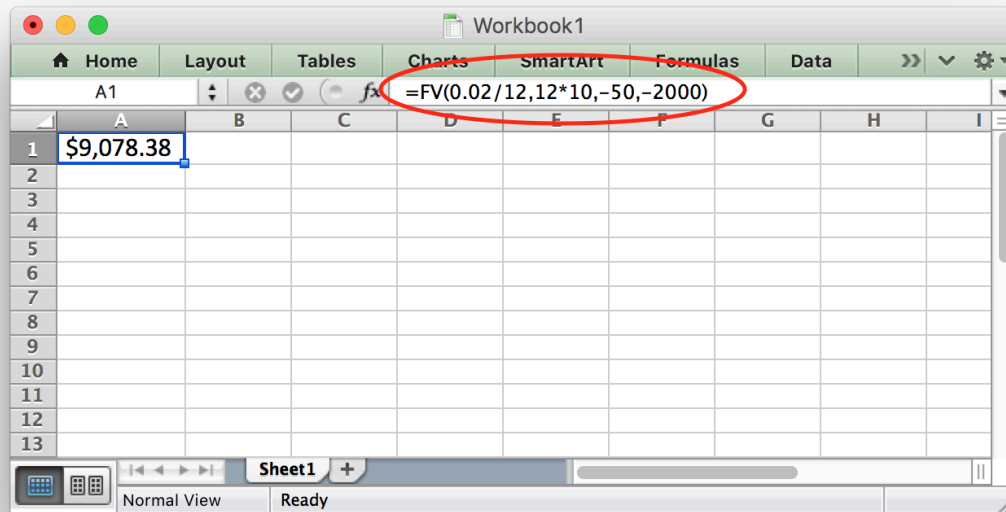
$$= FV(rate, nper, PMT, [PV])$$

The rate Excel uses is a monthly rate, so Carlos will need to divide his annual interest rate by 12 to find the monthly rate. Excel also asks for nper. This refers to the total number of periods, or the number of payments. He will make 12 payments for year for 10 years. So he needs to multiply 10*12 to find the value of nper.

Carlos also has to remember to include negatives where appropriate. In Excel, PMT and PV would be negative in a savings situation because that is money you are taking out of your wallet and putting in the bank right now. The future value of his account will be positive because it will be money he can take out of the bank and put back into his wallet.

Carlos types the following command into Excel to find his answer:

$$= FV(0.02/12, 10*12, -50, -2000)$$



At the end of 10 years, Carlos will have \$9078.38 in his account.

Practice: Finding Future Value

Problem 1

Maria wants to start a savings account. She has \$5,000 she can deposit now and plans to add \$50 each month. The account will earn 2% interest and is compounded monthly. Maria wants to know how much money she will have saved in 10 years.

Replace each variable with the appropriate value in the formula below, and then type the equation in a calculator to find Maria's account balance after 10 years.

$$FV = PV\left(1 + \frac{r}{n}\right)^{(nY)} + PMT \frac{\left(\left(1 + \frac{r}{n}\right)^{(nY)} - 1\right)}{\left(\frac{r}{n}\right)}$$

How Did I Do?

We know that

- PV = beginning account balance = \$5000
- r = interest rate, entered as a decimal = 0.02
- n = how many times per year payments are made = 12
- Y = number of years = 10
- PMT = monthly payment = 50

So we get the equation

$$FV = 5000\left(1 + \frac{0.02}{12}\right)^{(12 \times 10)} + 50 \frac{\left(\left(1 + \frac{0.02}{12}\right)^{(12 \times 10)} - 1\right)}{\left(\frac{0.02}{12}\right)}$$

Typing this equation into a calculator gives a future value of \$12,741.98

Problem 2

Rahul wants to start a savings account. He has \$6,600 he can deposit now and plans to add \$175 each month. The account will earn 1.7% interest and is compounded monthly. Rahul wants to know how much money he will have saved in 6 years.

The syntax for the FV command in Excel is:

$$= FV(rate, nper, PMT, [PV])$$

Use the FV command in Excel to find Rahul's account balance in 6 years.

How Did I Do?

The excel command = $FV(0.017/12, 6*12, -175, -6600)$ gives a future value of \$20,563.35

Non-interest vs Interest Savings Plans

How much of a difference does compound interest make? In order to see this, we can compare two different retirement savings situations. In the first situation, imagine that you save money each month for retirement by putting it in an envelope hidden under your mattress. In the second situation, imagine that you invest the money in a retirement account that earns an average of 7.5% interest annually.

In both situations, we will assume you just graduated from college and are beginning to save for retirement. You currently have \$1200 and will save an additional \$270 each month. You plan to retire in 35 years.

The Non-interest Plan

Currently, you have \$1200 saved in an envelope hidden under your mattress. Every month you plan to add \$270 to the envelope. This adds up to an additional \$3240 per year. Because the account balance changes by the same amount every year, there is a constant rate of change. We can use a linear function to describe the amount of money you have saved. (The compound interest equations don't apply because you are not earning any interest.)

$$f(x) = 3240x + 1200$$

In this equation the input, x , is the number of years you have been saving. The output, $f(x)$, is the amount of money you have saved. We can find how much money you will have saved in 35 years by using an input of $x = 35$.

$$f(35) = 3240(35) + 1200 = \$114,600$$

The Interest Savings Plan

Currently, you have \$1200 saved in a retirement account that earns an average annual interest rate of 7.5% compounded monthly. That means every month the current balance in your account is used to calculate the interest, which they add to your balance, so the next month you start earning interest on the interest you earned last month. Every month you plan to add \$270 to the account. Because the account balance changes by a different amount every month as the balance continues to grow, the rate of change is variable.

We can use the compound interest equation to create an exponential function:

$$FV = PV\left(1 + \frac{r}{n}\right)^{(nY)} + PMT \frac{\left(\left(1 + \frac{r}{n}\right)^{(nY)} - 1\right)}{\left(\frac{r}{n}\right)}$$

$$FV = 1200\left(1 + \frac{0.075}{12}\right)^{(12x)} + 270 \frac{\left(\left(1 + \frac{0.075}{12}\right)^{(12x)} - 1\right)}{\left(\frac{0.075}{12}\right)}$$

This equation looks complicated, but if you do some arithmetic to simplify the equation, you find it is a rather simple exponential function.

Click here to see the algebra for this problem

$$FV = PV\left(1 + \frac{r}{n}\right)^{(nY)} + PMT \frac{\left(\left(1 + \frac{r}{n}\right)^{(nY)} - 1\right)}{\left(\frac{r}{n}\right)}$$

$$FV = 1200\left(1 + \frac{0.075}{12}\right)^{(12x)} + 270 \frac{\left(\left(1 + \frac{0.075}{12}\right)^{(12x)} - 1\right)}{\left(\frac{0.075}{12}\right)}$$

$$FV = 1200(1 + 0.00625)^{(12x)} + 270 \frac{\left((1 + 0.00625)^{(12x)} - 1\right)}{(0.00625)}$$

$$= 1200(1.00625)^{(12x)} + 270 \frac{\left((1.00625)^{(12x)} - 1\right)}{(0.00625)}$$

$$= 1200(1.00625)^{(12x)} + \frac{270}{0.00625} ((1.00625)^{12x} - 1)$$

$$= 1200(1.00625)^{(12x)} + 43200((1.00625)^{12x} - 1)$$

$$= 1200(1.00625)^{(12x)} + 43200(1.00625)^{12x} - 43200$$

$$= 44400(1.00625)^{(12x)} - 43200$$

$$= 44400(1.00625^{12})^x - 43200$$

$$= 44400(1.0776)^x - 43200$$

Simplifying the equation yielded:

$$f(x) = 44400(1.0776)^x - 43200$$

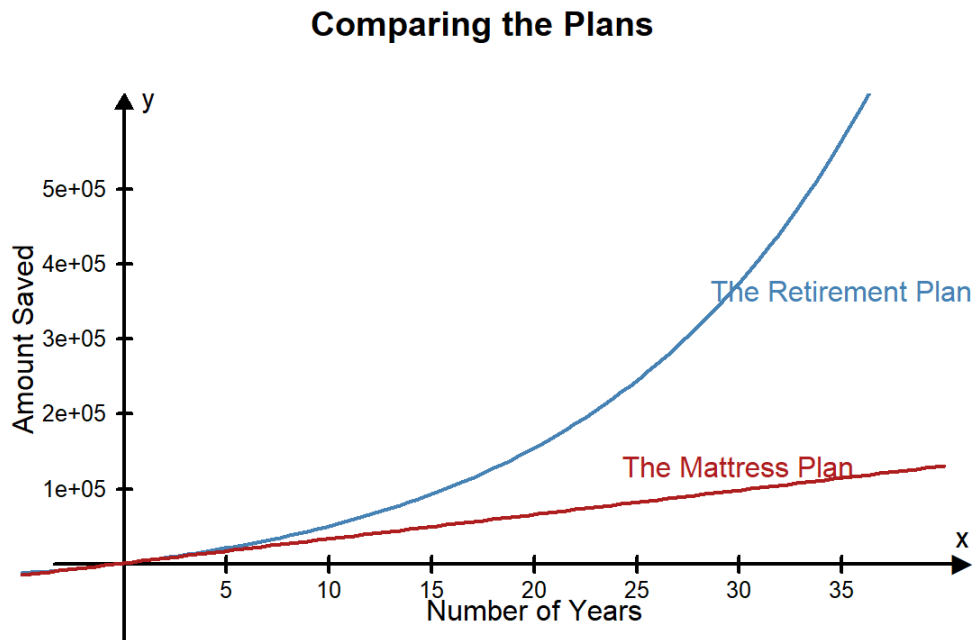
In this equation, just like in the mattress plan, the input, x , is the number of years you have been saving. The output, $f(x)$ is the amount of money you have saved. We can find how much money you will have saved in 35 years by computing $f(35)$.

$$f(35) = 44400(1.0776)^x - 43200 = \$564,093.13$$

Comparing the Non-interest and Interest Savings Plans

Comparing the balance of the Mattress Plan and the Retirement Plan after 35 years shows the difference compound interest can make. In both situations, you started with the \$1200 and saved \$270 every month for 35 years. When you saved the money under your mattress you ended with \$114,600 and when you saved the money in an account earning compound interest you ended with \$564,093.13. The difference in the account balances is due entirely to earning compound interest.

The following graph shows the graphs of the two savings plans over time:



This graph clearly shows the advantage of earning compound interest.

Earning Interest vs Paying Interest

As we saw when we compared saving money in a retirement account vs saving money under your mattress, it is to your advantage to earn compound interest when you are saving money. However, when you borrow money you are charged compound interest. So when you are paying back a loan compound interest works against you instead of for you. As quoted in an Ensign article, Albert Einstein supposedly said "Compound interest is the eighth wonder of the world. He who understands it, earns it...He who doesn't, pays it." [2](#)

President Gordon B. Hinckley taught that we cannot obtain self-reliance "when there is serious debt hanging over a household. One has neither independence nor freedom from bondage when he is obligated to others." He went on to issue this challenge: "Look to the condition of your finances. I urge you to be modest in your expenditures; discipline

yourselves in your purchases to avoid debt to the extent possible. Pay off debt as quickly as you can, and free yourselves from bondage” [3](#)

One reason debt becomes a source of temporal and spiritual bondage is because it accrues compound interest.

Paying Off Debt

1 Understand the Problem

Danielle has been using credit cards recklessly and has accumulated \$12,271 in credit card debt. The debt is divided onto four different cards with different account balances and different APRs. The following table summarizes Danielle’s debt.

Credit Card	Account Balance	APR
Card 1	\$3121	12.99%
Card 2	\$7012	18.99%
Card 3	\$1211	22.99%
Card 4	\$927	24.99%

Danielle is creating a plan to become debt-free. She wants to know how much money she should pay each month to pay off all her credit card debt in 5 years. She also wants to know the total amount of money she will pay to the credit card companies in interest.

2 Identify Variables & Assumptions

Danielle identifies the following variables:

- How much she needs to pay per month on each account.
- The amount of interest she will pay on each account by the time all four cards are paid off.

She makes the following assumptions:

- She will pay off all her credit card debt in 5 years.
- She will not make any new charges on any of the cards.

3 Apply Quantitative Tools

Danielle uses the PMT function in Excel to determine what her four monthly payments need to be in order to pay each card off in 5 years.

	A	B	C	D	E	F	G	H	I	J	K
1	Card #1			Card #2			Card #3			Card #4	
2	Inputs			Inputs			Inputs			Inputs	
3	Interest Rate (r)	12.99%		Interest Rate (r)	18.99%		Interest Rate (r)	22.99%		Interest Rate (r)	24.99%
4	Years (Y)	5		Years (Y)	5		Years (Y)	5		Years (Y)	5
5	Current Balance (PV)	\$3,121.00		Current Balance (PV)	\$7,012.00		Current Balance (PV)	\$1,211.00		Current Balance (PV)	\$ 927.00
6											
7	Output			Output			Output			Output	
8	Monthly Payment (PMT)	(\$71.00)		Monthly Payment (PMT)	(\$181.86)		Monthly Payment (PMT)	(\$34.13)		Monthly Payment (PMT)	(\$27.20)
9											
10											
11											
12											
13											
14											
15											
16											
17											

Now Danielle wants to figure out how much money she will pay in interest in the 5 years it takes her to pay off her cards.

For each credit card she finds the total amount she will have to pay back by multiplying her monthly payment by 12 to get an annual total and then multiplying by 5 to get the Total Payments.

Then she finds the interest paid on each card by taking the Total Payments and subtracting the Current Balance.

Credit Card	Current Balance (PV)	Monthly Payment	Total Payments	Interest Paid
Card #1	\$3121	\$71.00	\$4260	\$1139.00
Card #2	\$7012	\$181.86	\$10,911.60	\$3899.60
Card #3	\$1211	\$34.13	\$2047.80	\$836.80
Card #4	\$927	\$27.20	\$1632.00	\$705.00
Total	\$12,271	\$314.19	\$18,851.40	\$6580.40

4 Make an Informed Decision



From her calculations, Danielle realizes that although she only owes \$12,271 on all the credit cards combined, she will repay \$18,851.40 which includes paying \$6,580.40 in interest. Danielle will need to pay a total of \$314.19 in credit card payments each month to pay off all of the balances in 5 years.

5 Evaluate Your Reasoning



As Danielle reflects on her calculations, she decides that she needs a budget to make sure she can make the monthly payments. She also realizes that if she doesn't stick to her budget she will probably continue to use her credit cards. For

her plan to work, she will have to hold to her assumption that she will not use her credit cards anymore.

After 5 years of careful budgeting and hard work, Danielle is able to pay off her credit cards. She has also realized the role compound interest had in her financial situation. She had to pay back much more money than she borrowed because her credit card balances accrued compound interest. Now that her credit card debt is paid off, she is able to revise her budget to save money and earn compound interest instead of paying compound interest.

Practice : Credit Card Debt

Problem 3

Li Min has accumulated \$4,752 in debt on her credit card. The card has an APR of 21.99%. Li Min is creating a plan to become debt-free. She wants to know how much money she should pay each month to pay off her credit card in 3 years. Use the payment (PMT) function in excel to find the monthly payment.

Monthly payment = \$

How Did I Do?

Monthly payment = \$181.46

Problem 4

From the previous problem, Li Min now knows she needs to pay \$181.46 each month to pay off her balance of \$4,752 in credit card debt in 3 years.

Now she wants to know the total amount of money she will pay to the credit card company to pay off the card and the total amount of interest she will pay. Find these amounts.

Total payments = \$

Total interest = \$

How Did I Do?

Total payments = \$6532.56

Total interest = \$1780.56

Hint: To find Li Min's total payments, multiply her monthly payments by the number of months it takes her to pay off her credit card. To find Li Min's total interest, subtract her current balance of \$4,752 from her total payments.

Lesson Checklist

By the end of this lesson, you should be able to:

- define the term rate of change as the slope of a curve at a given point
- understand the difference between constant and variable rates of change
- define compound interest
- understand the connection between rate of change and accrued savings (or debt).

Optional Resource

The following video talks about the advantages of saving to purchase a car rather than getting a car loan.

[How to Drive a Free Car For Life](#)

¹ [Wikipedia](#)

² https://www.lds.org/ensign/2016/07/young-adults/an-eternal-perspective-on-personal-and-family-finance?lang=eng#footnote9-13187_000_015

³ <https://www.lds.org/general-conference/1998/10/to-the-boys-and-to-the-men?lang=eng>



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Unit 3

Lesson 10: Making Predictions From Data
Lesson 11: Solving Systems of Equations
Lesson 12: Probability and Confidence Intervals
Lesson 13: Applying the Quantitative Reasoning Process



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Lesson 10: Making Predictions From Data

Opening Story

Lesson 10 - Opening Story

[\(L10 Opening Story Transcript\)](#)

Introduction

As Sophia and Emmy traveled through Yellowstone National Park (USA) they encountered situations where *data analysis* was used to make a decision or a prediction.

Data analysis is the method of collecting, graphing, and analyzing data in order to understand patterns in the data.

In other words, data analysis is used to uncover the truth about a current scenario, a future event, or even what happened in the past.

Elder Bednar stated, “I long have been impressed with the simple and clear definition of truth set forth in the Book of Mormon: The Spirit speaketh the truth and lieth not. Wherefore, it speaketh of things as they really are, and of things as they really will be; wherefore, these things are manifested unto us plainly, for the salvation of our souls.”¹

While we live in a time where God has revealed many truths, there are still many things that God has not yet revealed. As individuals, our futures are still full of uncertainty. This lesson focuses on analyzing data to make prediction, which gives us quantitative tools to better understand things “as they really are” and predict things “as they really will be.”²

Input and Output Variables

One of the most powerful ways to discover how things “really are” is to use known information to help predict, or explain the unknown. This is done using input and output variables, as we have discussed in earlier lessons.

Mathematicians and statisticians use a variety of different names to refer to input and output variables. We will continue to refer to these variables as inputs and outputs. But it can be helpful to know their other names.

Input Variable	Output Variable
x -variable	y -variable
independent variable	dependent variable

explanatory variable	response variable
predictor variable	response variable

We have seen several examples of input and output variables in previous lessons. In Lesson 8 we read about Sarah, a woman who was trying to improve her health. She used the function $f(x) = 6x + 1860$ where the input variable, x , represented the number of minutes she walked each day and the output variable, $f(x)$ or y , represented her total caloric expenditure for the day. The total number of calories burned changed in *response* to how long she walked, and how long she walked *explained* or *predicted* how many calories she would burn. Said differently, she was free (or *independent*) to choose how long she walked, while the total number of calories burned was *dependent* on her choice of how long she walked. This is just an example of how the alternative names of input and output variables could be used. As stated previously, this lesson will continue to use only the terms “input” and “output” for variables.

When it is possible two variables might be related, we can use data analysis to explore relationships between the input variable and the output variable. We can create a scatter plot with the input variable on the horizontal axis (x -axis) and the output variable on the vertical axis (y -axis). Looking at the scatter plot helps us see if a pattern or trend exists that can be described using a function.

Trendlines

When data shows a pattern or trend, we can use the data to make predictions. The predictions may be about some future event, an estimate of what happened in the past, or even a statement about where things are currently. We often use trendlines to describe a trend or correlation in data.

*A **trendline** is the graph of a function that describes the pattern or trend in data represented by input and output variables.*

Although the term *trendline* includes the word “line”, trendlines are not always lines, they can also be curves. We will be using linear trendlines, quadratic trendlines, and exponential trendlines.

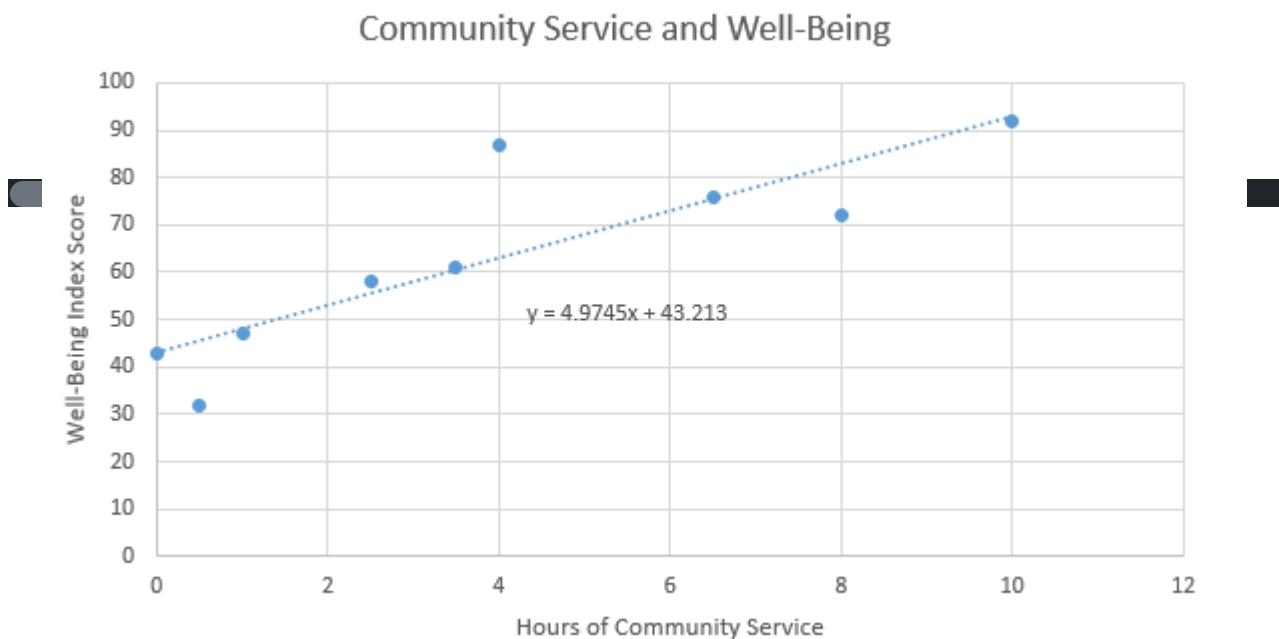
Community Service Example

A study conducted in 2014 showed that “Americans who actively work to better their communities have higher overall well-being than those who do not.”³ Although the actual data of the study is confidential and cannot be shared here, the following data is recreated to reflect the results of the actual study. This fabricated data shows the score earned by 10 fictitious individuals on a well-being index along with the number of hours of community service they have completed in the past month.

Well-Being Index Score (Scale of 0-100)	Hours of Community Service Completed
32	0.5
43	0
47	1
58	2.5

61	3.5
72	8
76	6.5
87	4
92	10

To study the trend shown by this data we use Excel to create a scatter plot. We decide if the trend is linear, quadratic, or exponential and then add the appropriate trendline and equation to the graph.



Looking at the graph, we can see that all the points don't lie directly on the line. However, the line does show the general trend of the data. In this case, Excel tells us the general relationship between the Well-Being Index Score and the Hours of Community Service can be described with the function

$$y = 4.9745x + 43.213$$

where the input, x , represents the number of hours of community service completed in the past month and the output, y , represents the score on the well-being index.

We can use this function to predict the well-being score for individuals, based on the number of hours of community service they have completed in the previous month.

For individuals who have completed 10 hours of community service in the past month,

$$y = 4.9745(10) + 43.213 = 92.958.$$

For individuals who have completed 5 hours of community service in the past month,

$$y = 4.9745(5) + 43.213 = 68.0855.$$

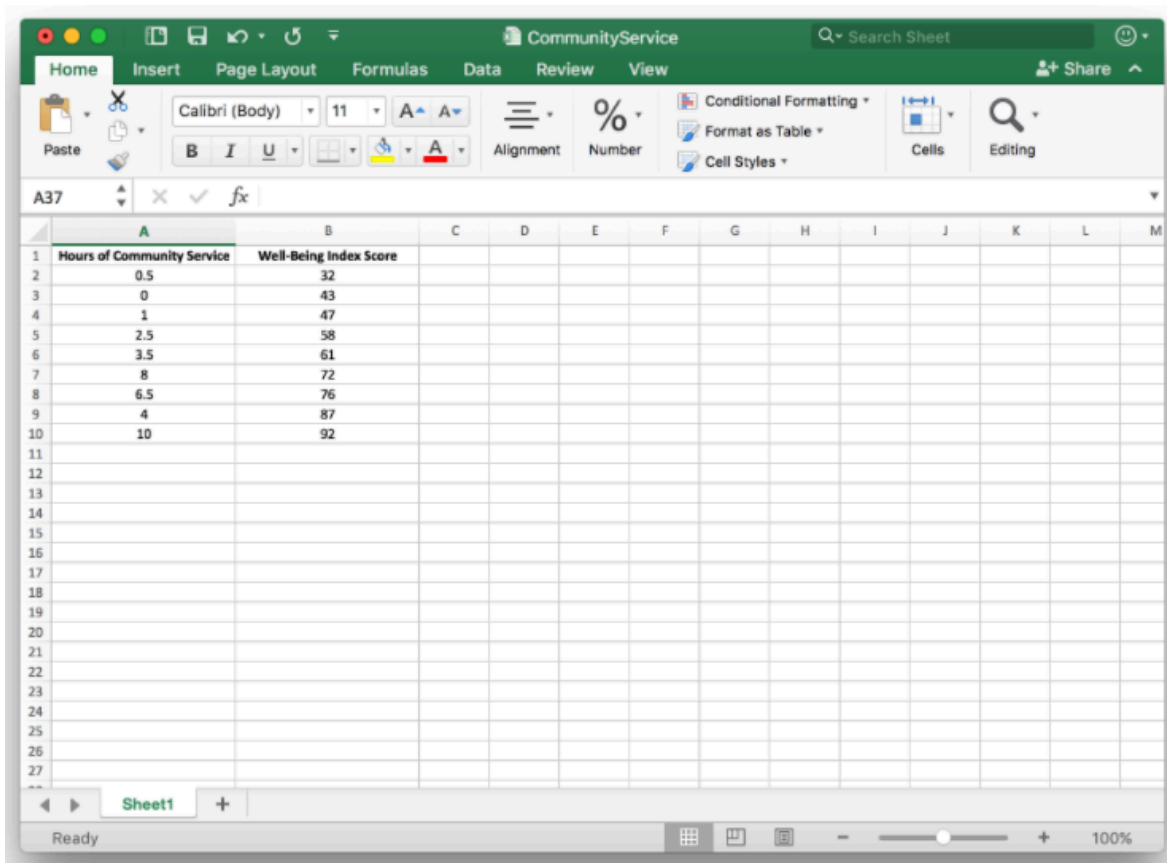
The prediction is that on average, individuals who complete 10 hours of community service will have a higher well-being score than individuals who complete 5 hours of community service.

Excel Instructions – Trendlines

The five steps below show how to create the community service scatter plot and trendline.

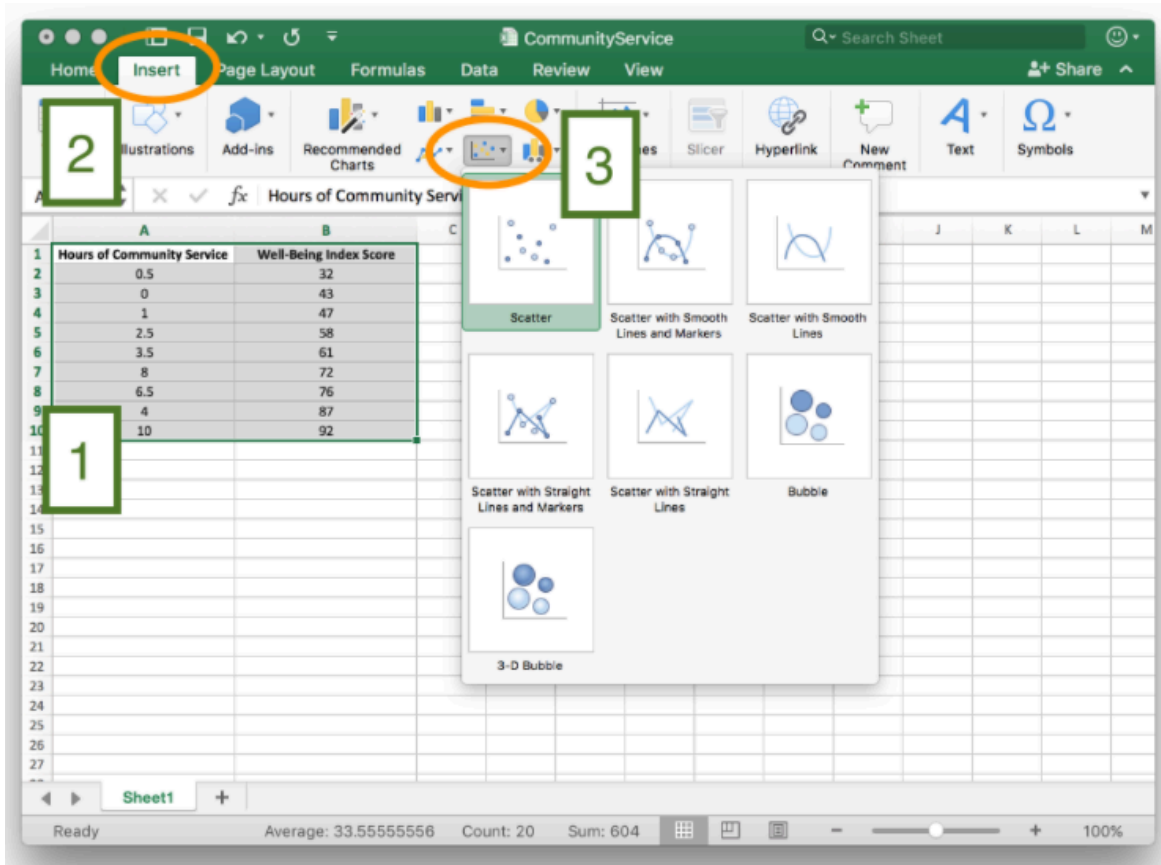
Step 1

Create an Excel file that lists the input variable in the first column and the output variable in the second column.



Step 2

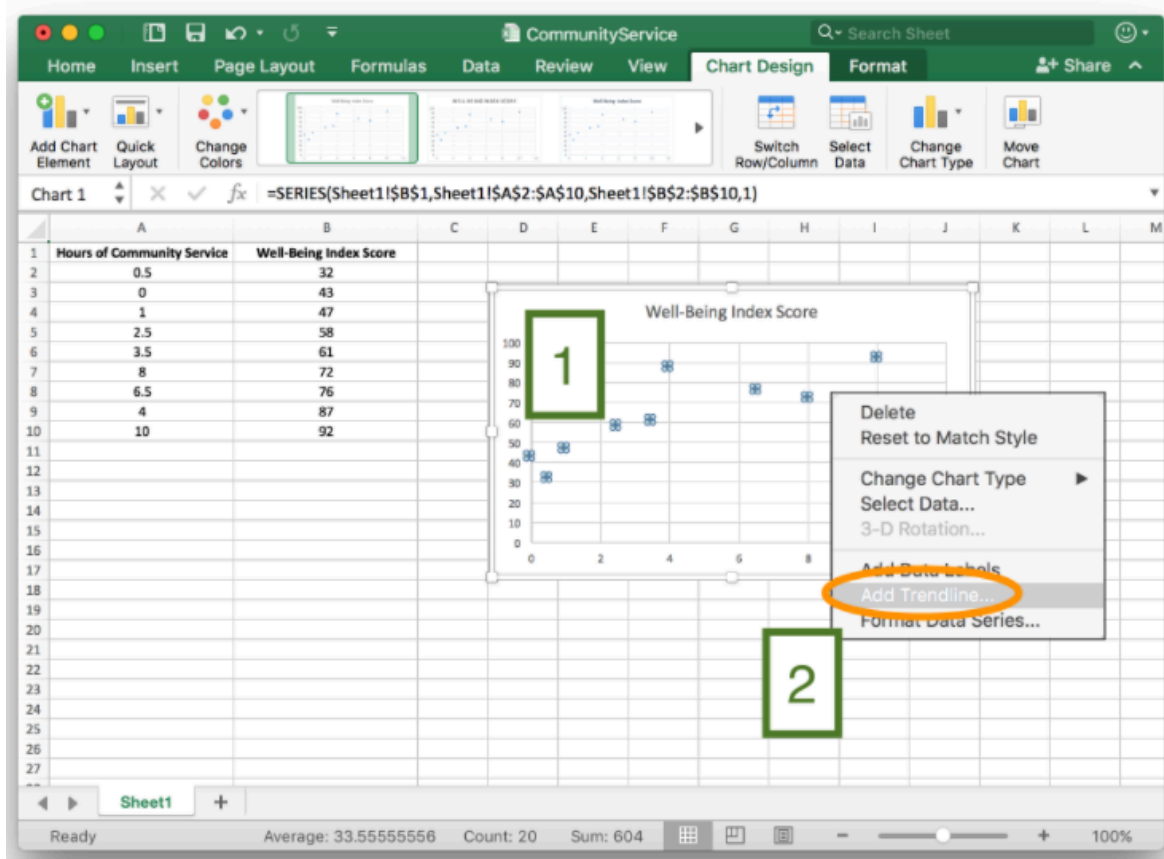
1. Highlight the data you want to graph, then
2. Select "Insert".
3. Select the "Scatter plot" icon and the "Scatter option".



Step 3

Once the scatter plot has been created,

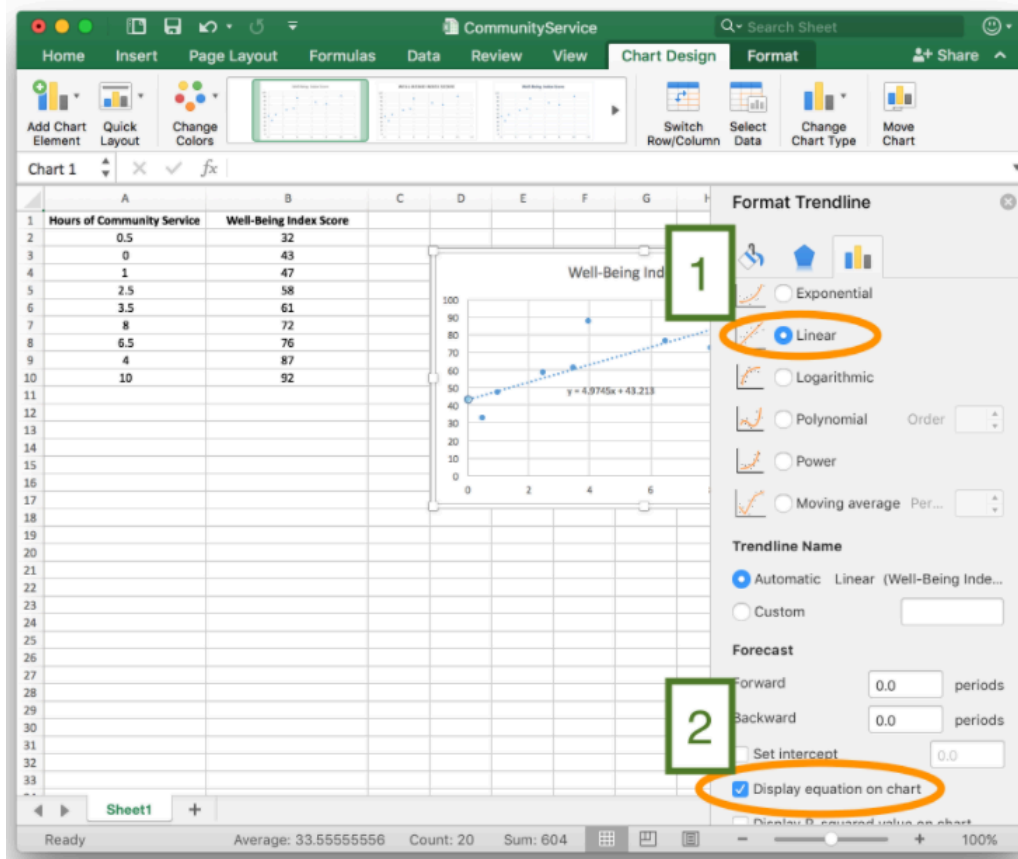
1. Right-click on one of the points.
2. From the right-click menu, select "Add Trendline"



Step 4

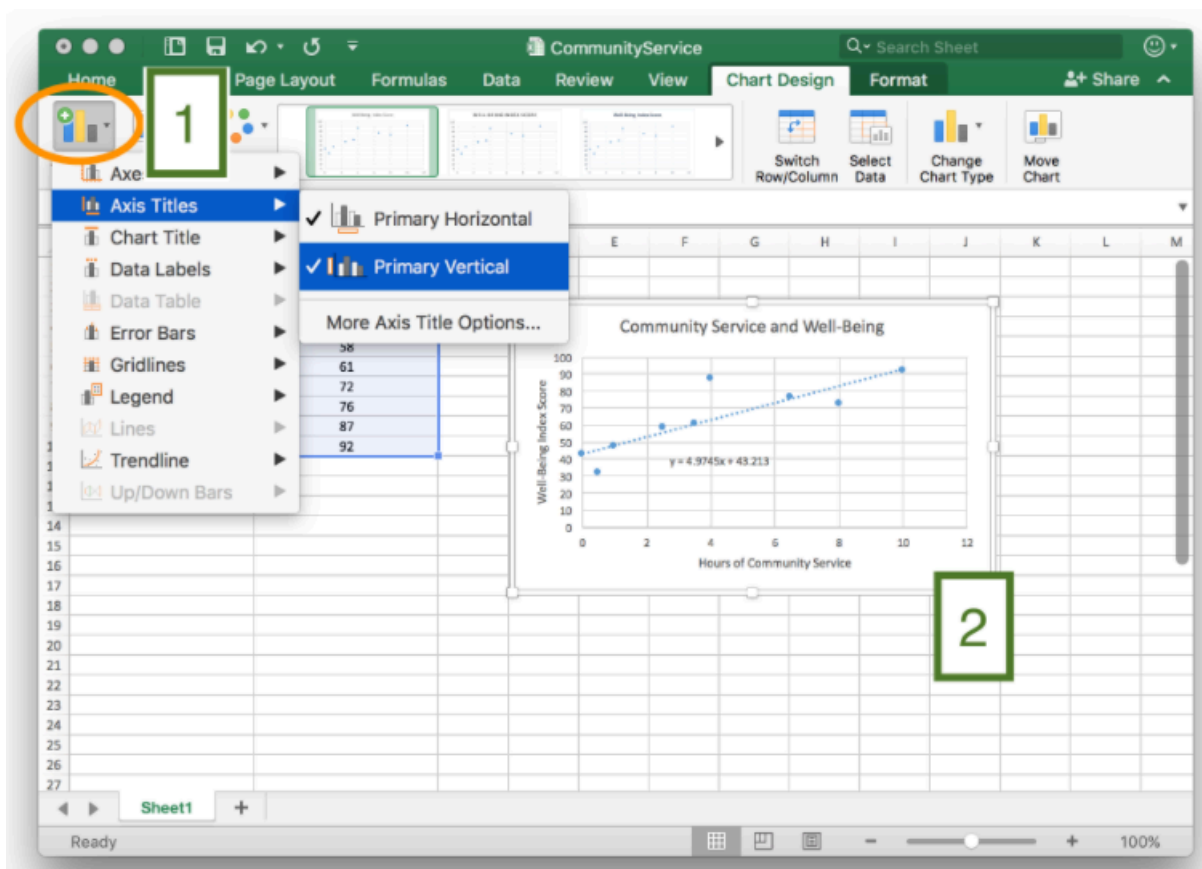
From the "Format Trendline" menu,

1. Because the scatter plot for these data show a linear trend, select "Linear". (Select "Polynomial" to create a quadratic trendline or "Exponential" to create an exponential trendline.)
2. Check the box to "Display Equation on Chart".



Step 5

Add chart elements [1], such as axis titles and chart titles to create a finished graph [2].



Practice: Creating a Trendline

Problem 1

The following data gives the height and shoe size for each student in a high school class. The height is measured in inches and the shoe sizes are in men's US sizes.

Please enter this data into Excel. These follow the "Excel Instruction - Trendlines" given above to create a linear trendline that describes these data. Find the equation of the trendline.

Height	Shoe Size
71	9.5
70	10
70	11
62	6.5
71.5	11.5
61	4.5
71	12
61	4.5
67	10
67	10
66	11
66	8
64	5.5
65	9

How Did I Do?

$$y = 0.6057x - 31.555$$

Correlation vs Causation

In Lesson 3 we learned about flaws in logical reasoning. If we are not careful, the use of trendlines can lead to a very common flaw in logical reasoning.

When a scatter plot shows a trend or relationship between two variables we say that the variables are *correlated*. However, this does not necessarily mean that changes in the input variable cause a change in the output variable.

A correlation between two variables does not imply there is a causal relationship.

A causal relationship means that changes in one variable cause a change in the other variable. Often, when variables are correlated there is an alternative explanation, other than causation, for the relationship. Here are a few examples.

Lice: A Sign of Good Health

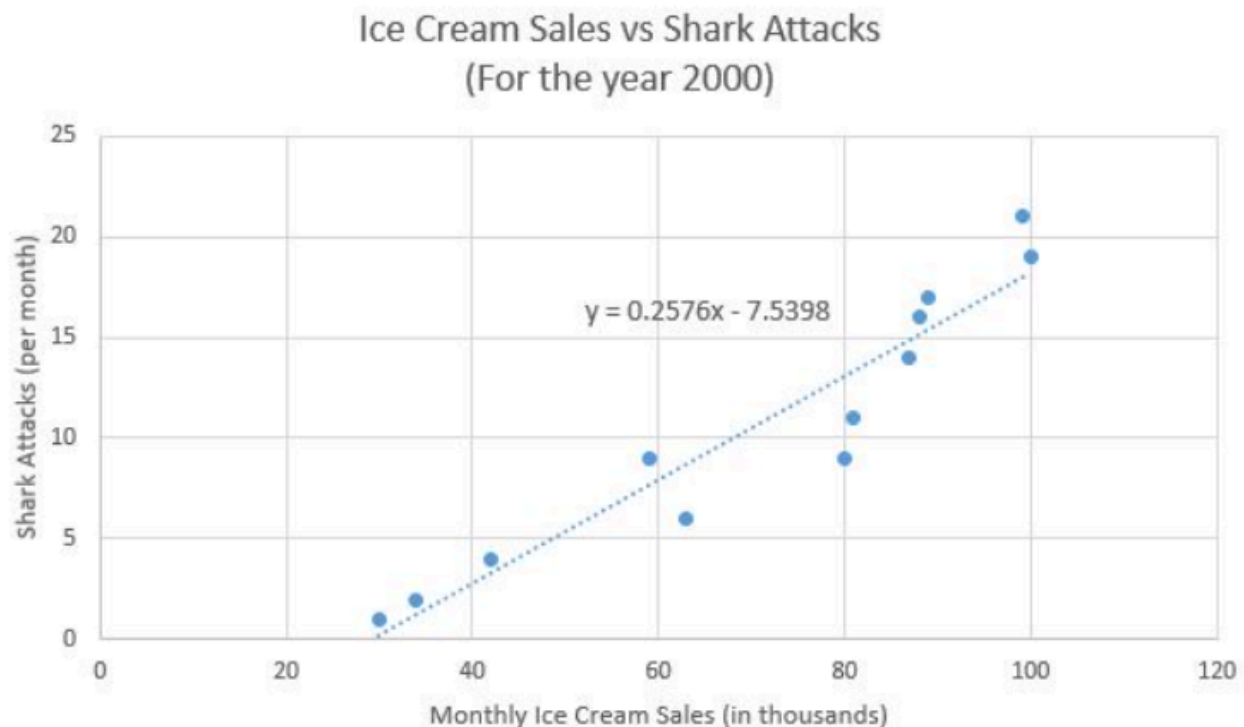
Correlation/Causation

[Correlation/Causation Transcript](#)

As explained by President Eyring, it is easy to assume a casual relationship when none exists.

Eating Ice Cream Causes Shark Attacks!

Here is another classic example. The following scatter plot and trendline show the relationship between monthly ice cream sales and the number of shark attacks per month. ⁴



This scatter plot clearly shows a linear trend where increased monthly ice cream sales correspond with a high number of shark attacks. However, what we are seeing in the graph is a *correlation* between ice cream sales and shark attacks. We cannot conclude that eating ice cream *causes* shark attacks.

The correlation seen in these data has a more rational explanation. Because of warmer weather, people are more likely to eat ice cream in the summer than they are in the winter. So ice cream sales are higher during summer months. Similarly, warmer weather during the summer means that people are more likely to go to the ocean and go swimming. Due to there being more people in the water, as well as because of the migratory patterns of sharks, shark attacks are more common during the summer than in the winter. So the relationship between ice cream sales and shark attacks is due to the fact that both are more common during summer months when the temperature is warmer. This *alternative explanation* explains the trend we see in the data.

In the case of sharks and ice cream, it is pretty obvious that we should look for an alternative explanation. However, no matter what the data represents, we should always remember that seeing a correlation or trend in the data does not tell us that there is a causal relationship where changes in one variable cause a change in the other variable. Here is an example where it is more tempting to say a causal relationship exists. We first saw this example in Lesson 3.

Soda Leads to Heart Attacks

A research study published in 2013 examined the effect that drinking soda (or other sugary drinks) has on cardiovascular health. The data examined in this study showed a trend where men who drank more soda had higher risk of having a heart attack. However, as stated in a CBSNews article reporting on the study, “other doctors caution that just because there may be a link between sugary drinks and heart attacks does not mean sugar is causing them.”⁵ The article went on to point out that men who drink sugary drinks often consume them along with burgers, fries, and other unhealthy food choices. Having a generally unhealthy diet and other poor health habits may be the actual cause of increased heart attack risk.

Community Service and Well-Being

We should apply the rule that correlation does not imply causation to the community service example from the previous section. This example shows a trend where people who do more community service tend to have higher scores on a well-being index. However, we cannot conclude that it is the community service that causes them to have better well-being. We just know there is a correlation and that an individual who does more community service is more likely to have a higher score on the well-being index. It might be the community service that causes a feeling of well-being, but there might also be an alternative explanation. The information we have been given is not enough to tell.

Linear, Quadratic, or Exponential?

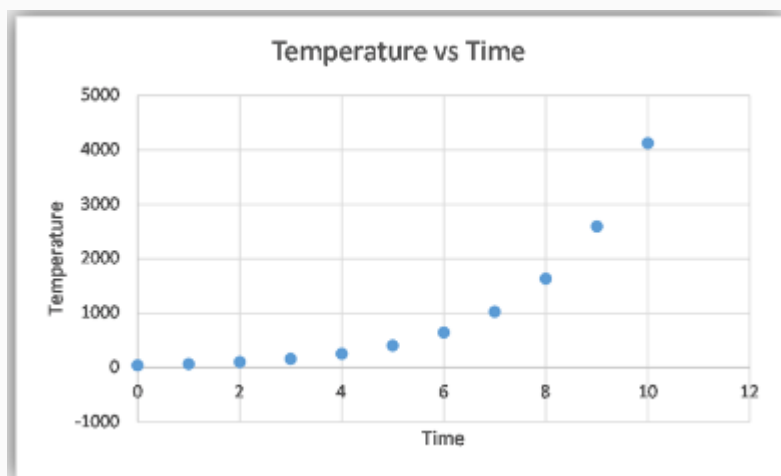
Excel can create linear, quadratic, and exponential trendlines. To decide which of these three types of trendlines would apply in a particular situation we look at the shape of the scatter plot. If the general shape of the points are close to being a line, we use a linear trendline. If the general shape of the graph resembles a u-shaped parabola, we use a quadratic trendline. If the general shape is non-linear and increasing (or decreasing), we use an exponential trendline.

Linear trendlines are used with data that is linear and has a constant rate of change. Quadratic and exponential trendlines are used with data that is non-linear and has a variable rate of change.

Practice: Linear, Quadratic, or Exponential?

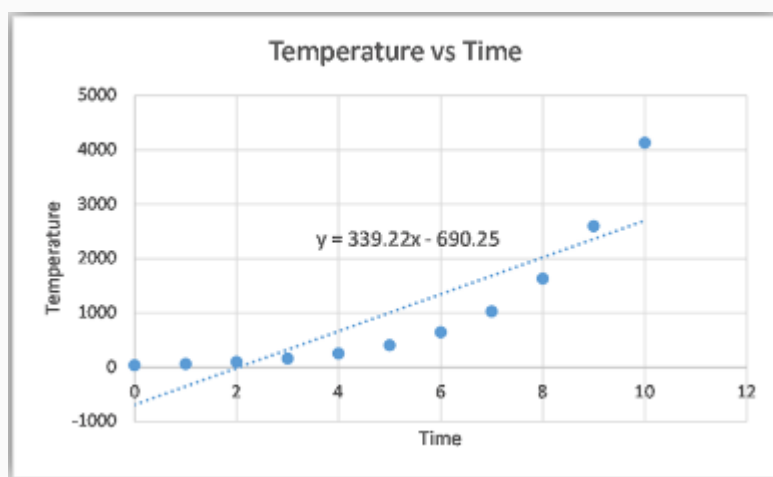
Problem 2

Consider the following scatterplot.



Click on the buttons below to view the linear, quadratic, and exponential trendlines. After comparing the three trendlines, select the one that best fits the data.

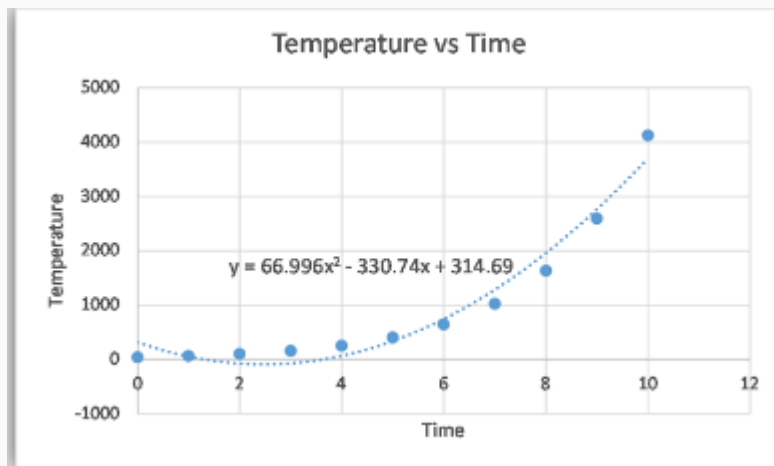
Linear



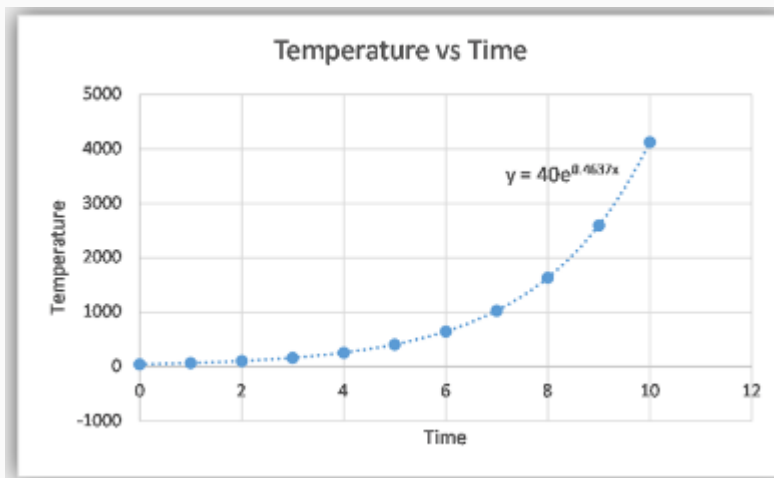
That is not correct. There is a better type of trendline for this data.

That is not correct. There is a better type of trendline for this data.

Quadratic



Exponential



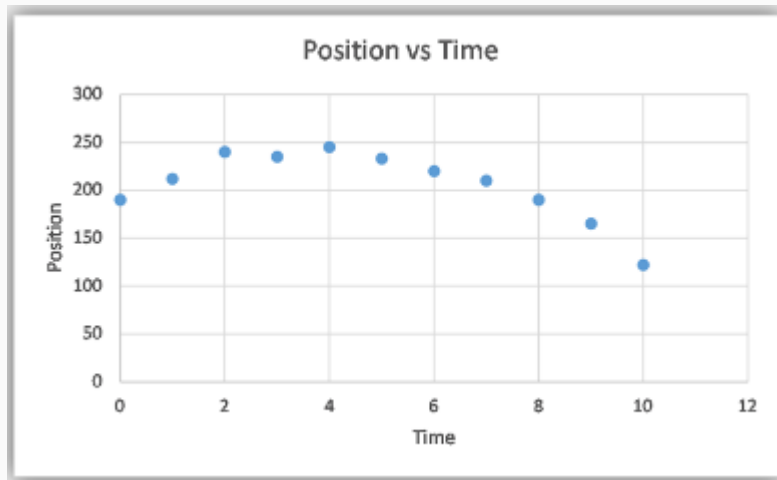
The **exponential** trendline best describes this data. The overall trend of the data is increasing and represents change over time.

Which type of trendline best fits this data?

- ☐ Linear
- ☐ Quadratic
- ☐ Exponential

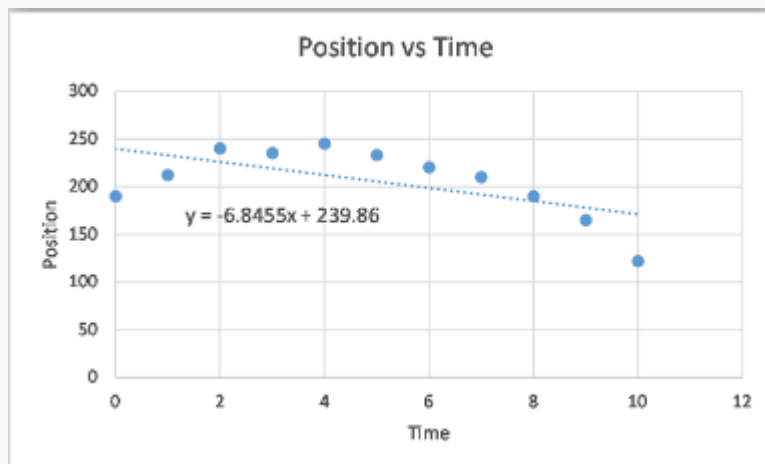
Problem 3

Consider the following scatter plot.

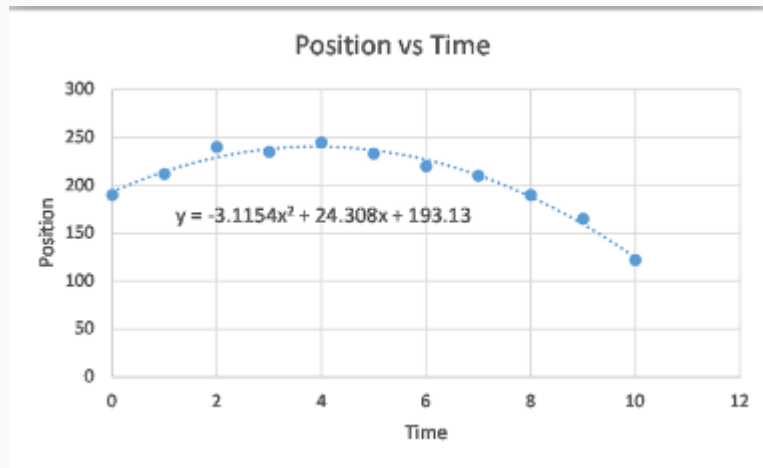


Click on the buttons below to view the linear, quadratic, and exponential trendlines. After comparing the three trendlines, select the one that best fits this data.

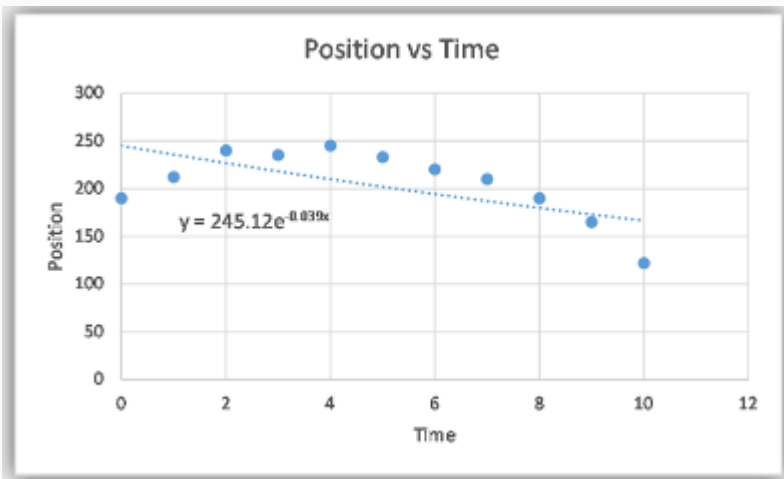
Linear



Quadratic



Exponential



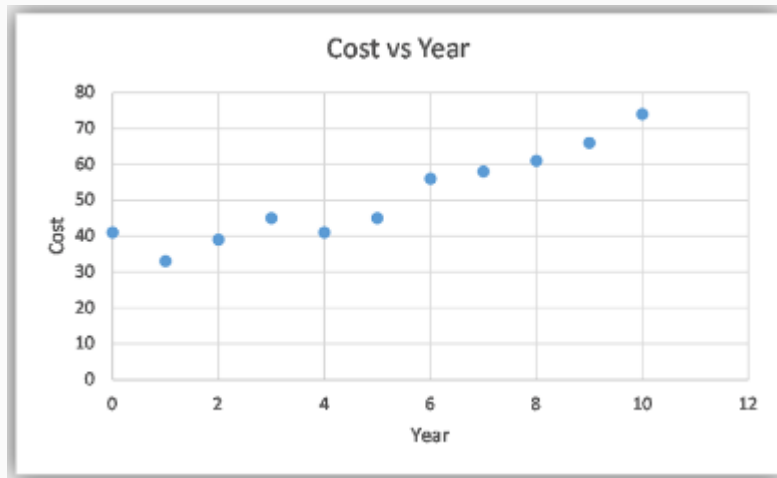
A **quadratic** trendline best describes this data. The overall trend of the data is a u-shaped parabola.

Which type of trendline best fits this data?

- ☐ Linear
- ☐ Quadratic
- ☐ Exponential

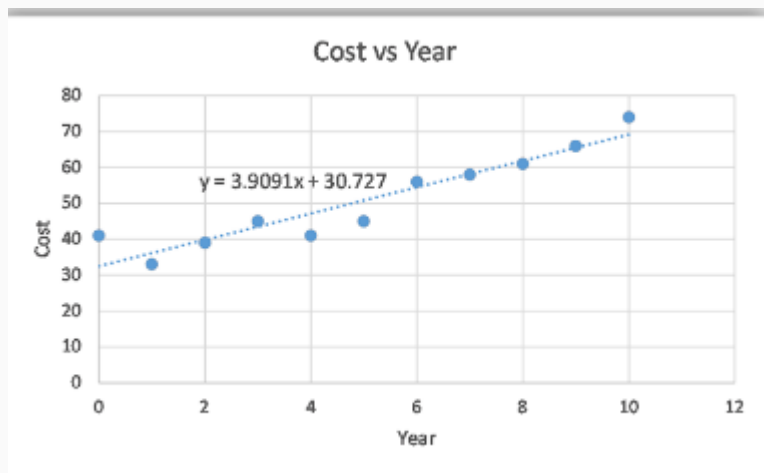
Problem 4

Consider the following scatter plot.

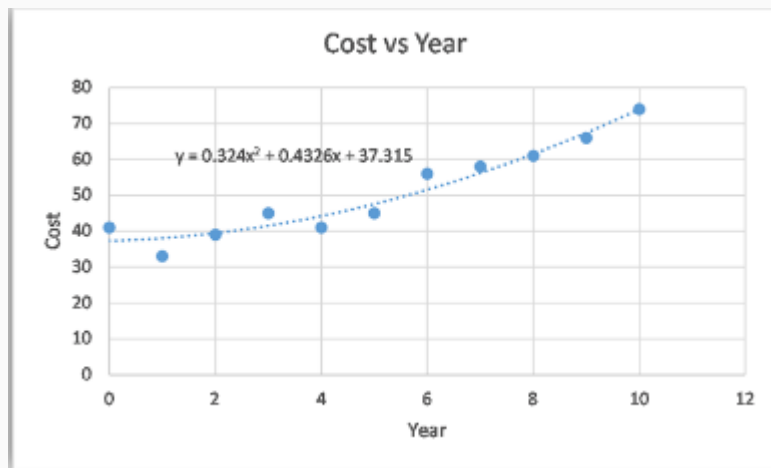


Click on the buttons below to view the linear, quadratic, and exponential trendlines. After comparing the three trendlines, select the one that best fits this data.

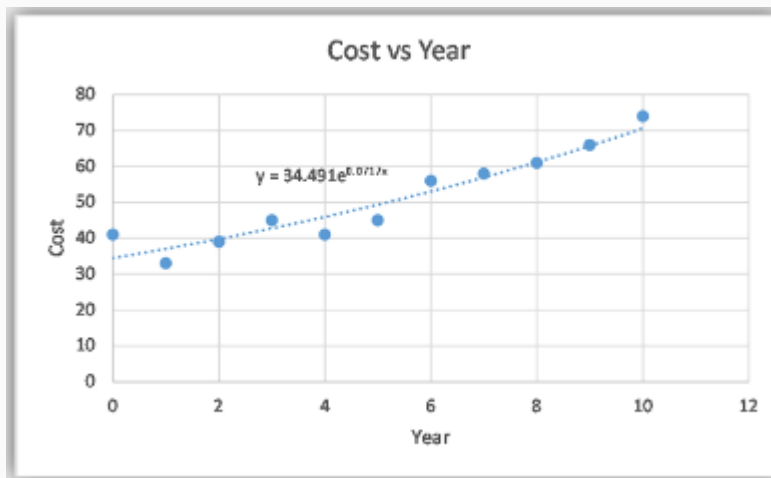
Linear



Quadratic

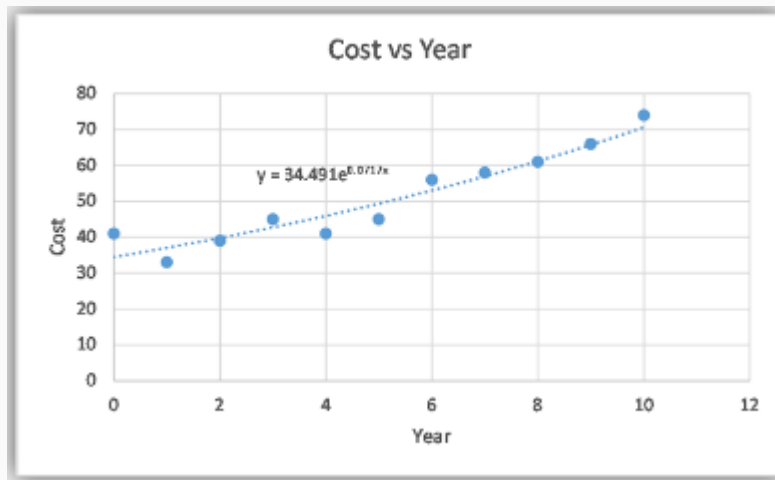


Exponential



Which type of trendline best fits this data?

- ☐ Linear
- ☐ Quadratic
- ☐ Exponential



A linear trendline best describes this data. The linear model is simpler than the quadratic or exponential models and describes the data fairly well. The quadratic or exponential models only fit the data slightly better than the line does. We typically use the simplest model possible, so we only use the quadratic and exponential models when they fit the data much better than the line does.

BUTTON TEXT

HIDDEN CONTENT

BUTTON TEXT

HIDDEN CONTENT

NOTE

The quadratic and exponential models only fit the data slightly better than the line does. We typically use the smallest model possible, so we only use the quadratic and exponential models when they fit the data much better than the line does.

Writing Exponential Functions

Because of the rules of exponents, exponential functions can be written a variety of different ways. For example,

$$\begin{aligned}
 f(x) &= 2^{-x} \\
 &= 2^{-1 \times x} \\
 &= (2^{-1})^x
 \end{aligned}$$

$$\left(\frac{1}{2}\right)^x$$

Therefore, even though the equations $f(x) = 2^{-x}$ and $f(x) = \left(\frac{1}{2}\right)^x$ look different from one another, they represent the same function and have the same graph.

Excel uses the exponential number e when giving the equation for an exponential function. The exponential number e is a constant that is approximately equal to 2.7182. Like π , the exponential number e is a non-terminating, non-repeating decimal. Because e is a number that shows up naturally in a variety of applications, it was given a special name and symbol.

We can use the rules of exponents to show that $f(x) = e^{1.098612x}$ and $f(x) = 3^x$ are different equations that represent the same function.

$$\begin{aligned} f(x) &= e^{1.098612x} \\ &= (e^{1.098612})^x \\ &\approx (2.7182^{1.098612})^x \\ &\approx 3^x \end{aligned}$$

Predictions from Trendlines

Creating a scatter plot and trendline gives us a function to describe the relationship between two variables. We can use this function to make predictions. The following example demonstrates how we could use predictions from trendlines as part of the Quantitative Reasoning Process.

1 Understand the Problem

Olivia has a 10 year old Honda Civic. It is completely paid for so Olivia has been saving money each month for the purchase of her next car. She currently has \$10,500 saved to purchase a new car.

The new car Olivia wants to buy will cost \$24,000, including all taxes and fees. Olivia needs to determine whether she will get enough money from the sale of her old Honda Civic to purchase the new car.

2 Identify Variables & Assumptions

As Olivia considers the variables and assumptions in her situation, she identifies the following key variables:

- The total cost of the new car.
- The condition of the Honda Civic she is selling.
- The amount of money she will get from selling her Honda Civic.

She realizes she is making the following assumptions:

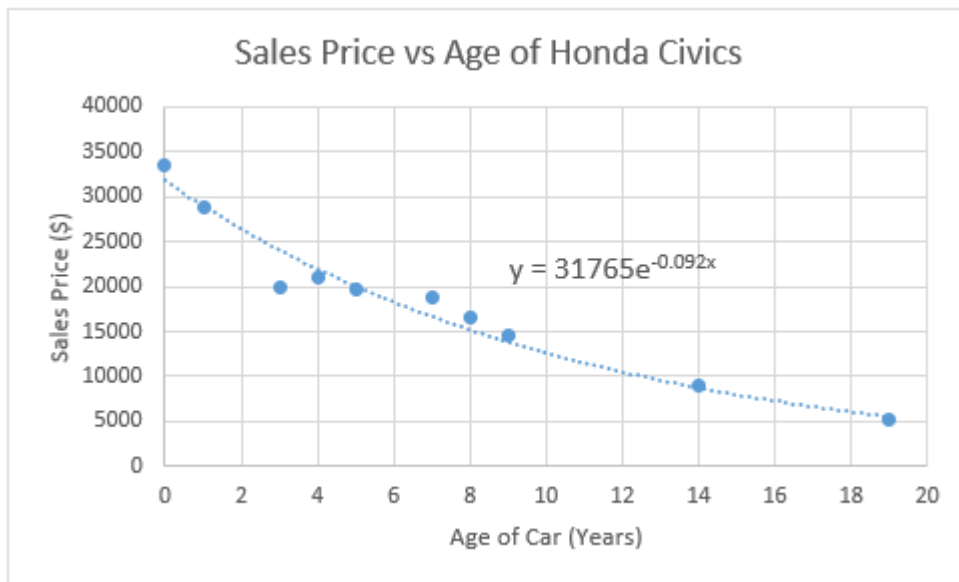
- She will pay cash for her new car and not borrow any money.
- She can use all the money from the sale of the Honda Civic for the purchase of her new car.
- She can buy a new car for \$24,000.
- She has \$10,500 to use to purchase her new car.
- If she can sell her Honda Civic for at least \$13,500, she can purchase the new car.

3 Apply Quantitative Tools

Olivia knows she needs to be able to sell her 10 year old Honda Civic for at least \$13,500. She finds several Honda Civics for sale, but none of them are the same year as hers, so she isn't sure how much she will earn from the sale of her old car. Olivia creates a spreadsheet listing the age and sale price of Honda Civics in her area with similar features and condition as her Honda Civic.

Age of Car	Sale Price
0	\$33,500.00
1	\$28,900.00
3	\$19,990.00
4	\$20,950.00
5	\$19,600.00
7	\$18,900.00
8	\$16,590.00
9	\$14,490.00
14	\$8990.00
19	\$5100.00

The scatter plot shows how the value of the Honda Civic changes over time. After looking at the scatter plot, Olivia realizes the data is non-linear so she decides to use an exponential trendline to describe the relationship between the age and sale price of Honda Civics.



From the scatter plot shown above, we see that the function relating age and sale price of Honda Civics is

$$y = 31765e^{-0.092x}$$

where x represents the age of the car and y represents the sale price.

Olivia uses this equation to predict the amount of money she will get from selling her 10-year old car:

$$y = 31765e^{-0.092(10)} = \$12,658.96$$

Rounding up, Olivia concludes that she will be able to sell her car for around \$12,660. However, she realizes that this prediction gives an average price and that there will be variation in the selling price of Honda Civics. So she might be able to sell her car for a little more or a little less than this predicted price.

4 Make an Informed Decision



Because Olivia has \$10,500 saved, if she sells her car for \$12,660 she would have a total of \$23,160. However, Olivia knows she needs at least \$24,000 for the new car she wants to purchase. She will likely be about \$840 short of the amount she needs to purchase the new car. She decides to wait to sell her car until she has saved more money.

5 Evaluate Your Reasoning

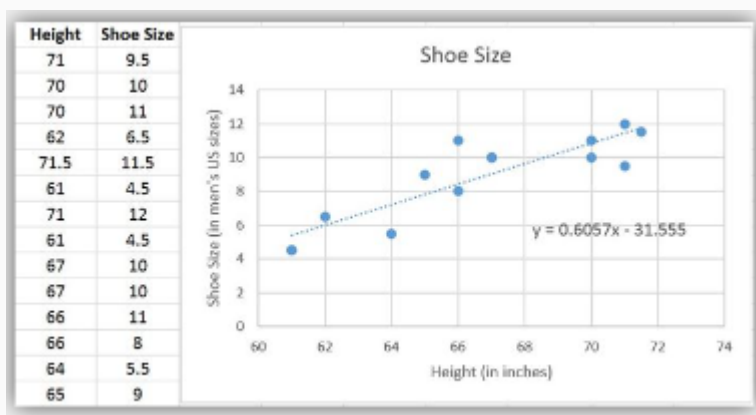


As Olivia reflects on her decision, she is disappointed. Because she was excited to purchase the new car, she looks again at the new car options. She finds a less expensive car that she likes and that will meet her needs. Since the alternative car is only \$22,000 it fits within her current budgeted amount. She revises her decision and goes ahead and puts her Honda Civic up for sale. She lists the sale price as \$13,500 so she still has room to negotiate with potential buyers about the price.

Practice: Making a Prediction

Problem 5

Earlier in this lesson, you found the following trendline:



Use the trendline to predict the shoe size of a student who is 68 inches tall.

How Did I Do?

To predict the shoe size of a student who is 68 inches tall, we enter 68 as the input of the trendline function:

$$y = 0.6057x - 31.555$$

$$y = 0.6057(68) - 31.555$$

$$y = 41.1876 - 31.555$$

$$y = 9.6326$$

The student would need a shoe size of about 9.5 or 10.

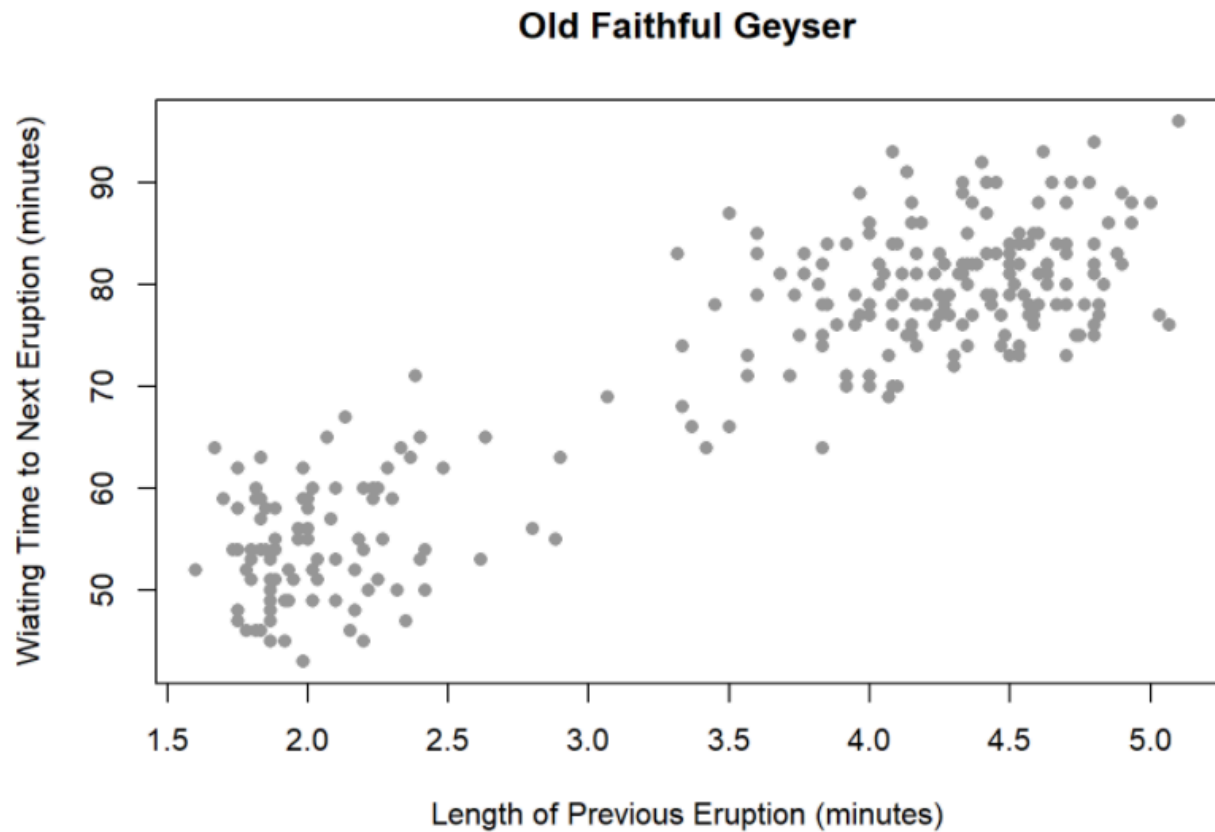
Example: Old Faithful

In the opening video, we saw that Sophia and Emmy predicted the time of the next eruption of Old Faithful Geyser. Old Faithful is one of the most famous features of Yellowstone National Park. The park's website boasts that Old Faithful is "the most famous geyser in the world". [6](#)

Old Faithful is named for its predictable eruptions. Visitors to the park come from all over the world to see the eruption of Old Faithful. [7](#) The geyser is so predictable that the park actually posts signs letting visitors know how many minutes until the next eruption. Most other geysers in Yellowstone and elsewhere erupt less frequently and their eruptions are much more difficult to predict.

The scatter plot below shows how the park uses the length of the previous eruption to determine how long it will be until the next eruption. The scatter plot shows the general pattern found in actual data recorded by researchers for past

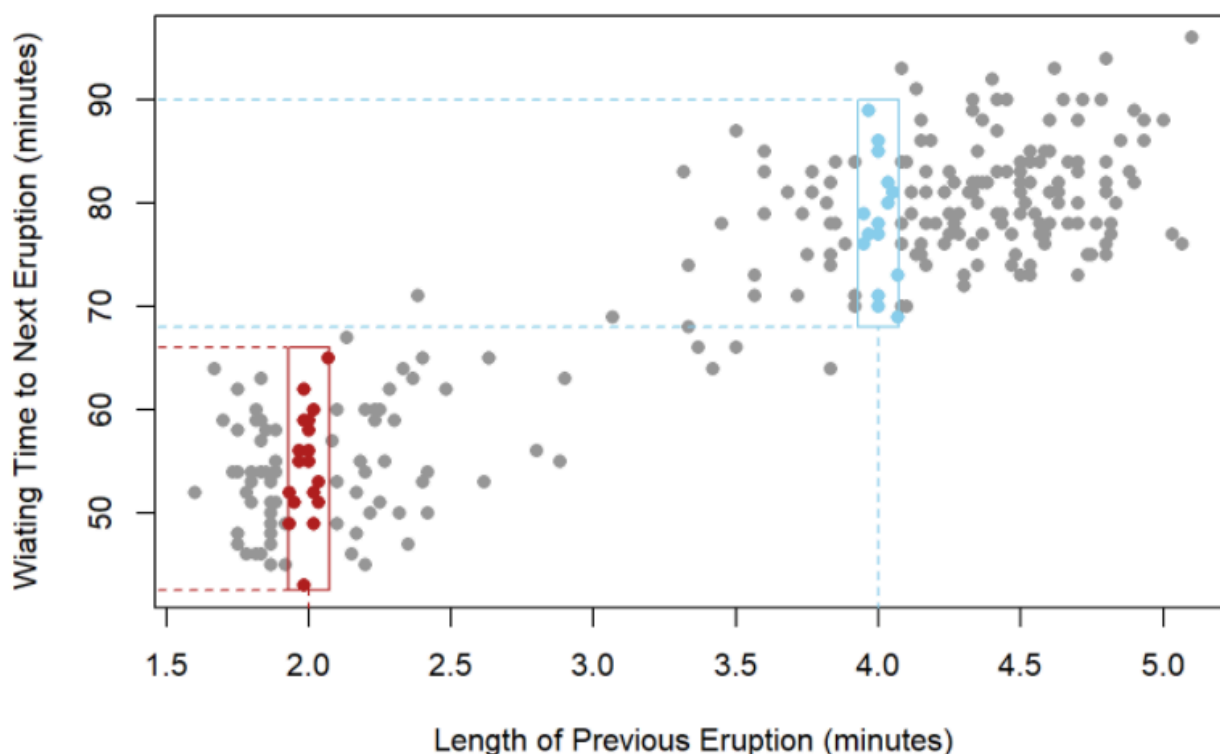
eruptions.



Notice that if the previous eruption of Old Faithful lasted for 4.0 minutes, then we would predict the time until the next eruption to be anywhere from 68 to 90 minutes. See the blue points in the plot below.


The red points in the plot predict that if the previous eruption lasted 2.0 minutes, we would expect a waiting time between 42 and 66 minutes until the next eruption. We can see that the longer the previous eruption lasted, the longer we would expect to wait until the next eruption.

Old Faithful Geyser



If we could fit a mathematical model to the data, then we could create an equation that would allow us to predict the average waiting time to the next eruption based on the length of the current eruption.

Note that the general trend in the data is linear. This suggests a line would be the most appropriate mathematical model for these data.

 A scatter plot graph titled Old Faithful Geyser. The horizontal axis is labeled Length of Previous Eruption (minutes). The vertical axis is labeled Waiting Time to Next Eruption (minutes). A linear trendline is drawn on the graph. Equation of the Line: $Y = 33.47 + 10.73X$.

The mathematical model we get from this data is given by the equation

$$y = 33.47 + 10.73x$$

where the input variable, x , represents the length of the most recent eruption and the output variable, y , represents the waiting time to the next eruption. If the most recent eruption lasted 4.0 minutes, then the model would estimate that the next eruption will occur in

$$y = 33.47 + 10.73(4.0) = 76.39 \text{ minutes.}$$

Park rangers can add 76.39 minutes to the time at which the previous eruption ended, and then post a sign ⁸ near Old Faithful stating when the next eruption would occur. However, note that as shown in the plot above, the actual waiting time could be anywhere from 68 to 90 minutes. So it would be wise to let visitors know that the eruption time could be a little earlier or a little later than the predicted 76.39 minutes from the end of the previous eruption.

Practice: Old Faithful

Problem 6



On a visit to Yellowstone National Park you time the eruption of the Old Faithful Geyser. The geyser erupted for 3.7 minutes and the eruption ended at 3:14 PM.

Use the model we found above to give a prediction for the time at time at which the geyser will erupt next.

How Did I Do?

The next eruption will be about 73 minutes after 3:14 PM, or 4:27 PM.

[View Live Webcam of Old Faithful Geyser](#)

Example: Snow Machines in Yellowstone

In the opening video, Sophia and Emmy encountered a park ranger who enforced the rule that their snow machines had the best available technology. Yellowstone National Park officials have been given the task of conserving natural resources in the park, while at the same time allowing for use of the park by the public. Conservation and public use can be at odds with one another. The use of snow machines in Yellowstone is an example of where there has been a lot of controversy.



In order to protect the environment in Yellowstone National Park, all snow machines are required to have the best available technology (BAT).⁹ This is because older snow machines have less efficient engines that emit more hydrocarbons and carbon monoxide, which are harmful to the environment. However, restricting the types of snow machines significantly restricts winter access to Yellowstone.

The officials at the National Park Service used data analysis to help make decisions related to the conservation and use of Yellowstone National Park, such as the use of snow machines. The following example outlines the process used by Yellowstone officials to determine the winter use policy in Yellowstone.

1 Understand the Problem

By the late 1990's the number of snow machines entering Yellowstone during the winter had increased to the point there was a noticeable impact on air quality. In 1999, an environmental group petitioned the National Park Service and asked them to ban recreational snowmobiling in Yellowstone (and all other National Parks). In 2000, in response to noticeable impacts on air quality and the request by an environmental group, the National Park Service decided to phase out the use of snow machines in Yellowstone National Park. In 2001, the National Park Service was sued over their proposed ban of snow machines. Because the National Park Service has a legal mandate to allow for public use of park resources, a federal court overturned Yellowstone's snow machine ban. Yellowstone officials then decided to carry out several studies in order to determine the best way to balance environmental conservation and public use of the park.¹⁰

2 Identify Variables & Assumptions

Some of the key variables identified by researchers were:

- The types of vehicles entering the park
- The number of vehicles entering the park
- Hydrocarbon Emissions
- Carbon Monoxide Emissions
- Types of snow machine engine (2-stroke engines, 4-stroke engines, BAT engines)
- The researchers made several important assumptions:

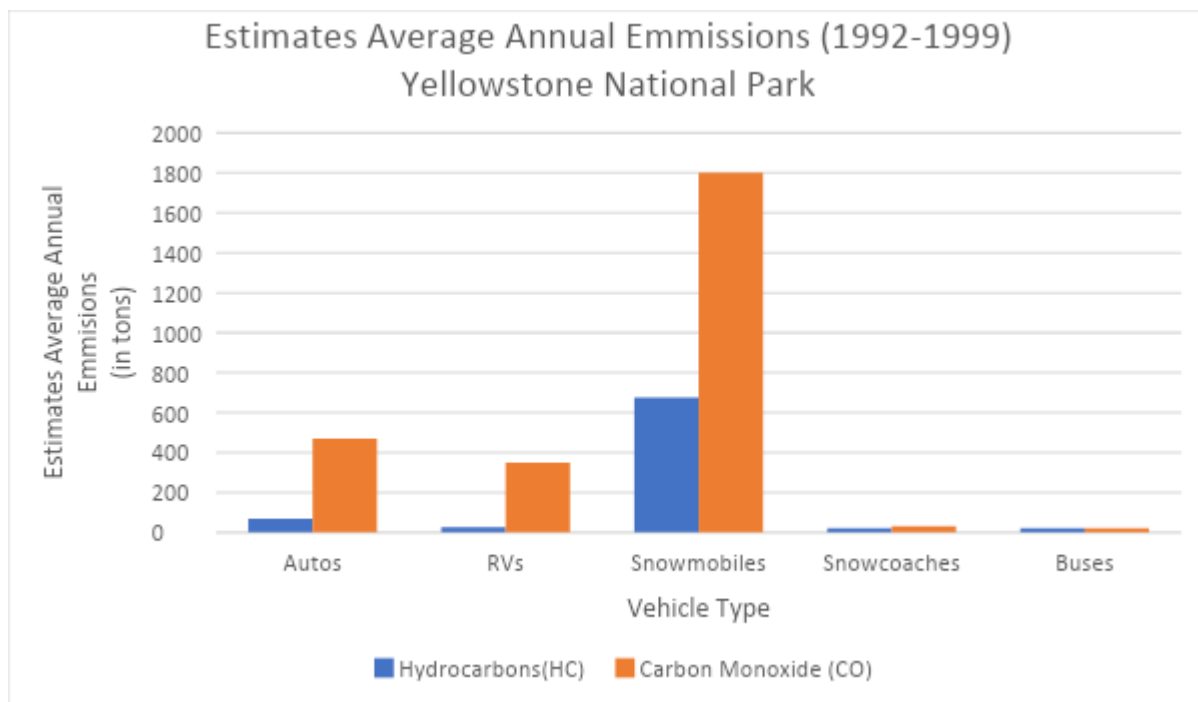
Hydrocarbon and carbon monoxide are harmful to the environment

- Decreasing the number of vehicles entering the park will decrease hydrocarbon and carbon monoxide emissions
- BAT snow machines have lower hydrocarbon and carbon monoxide emissions than non-BAT snow machines
- Traveling with a guide will reduce harmful emissions

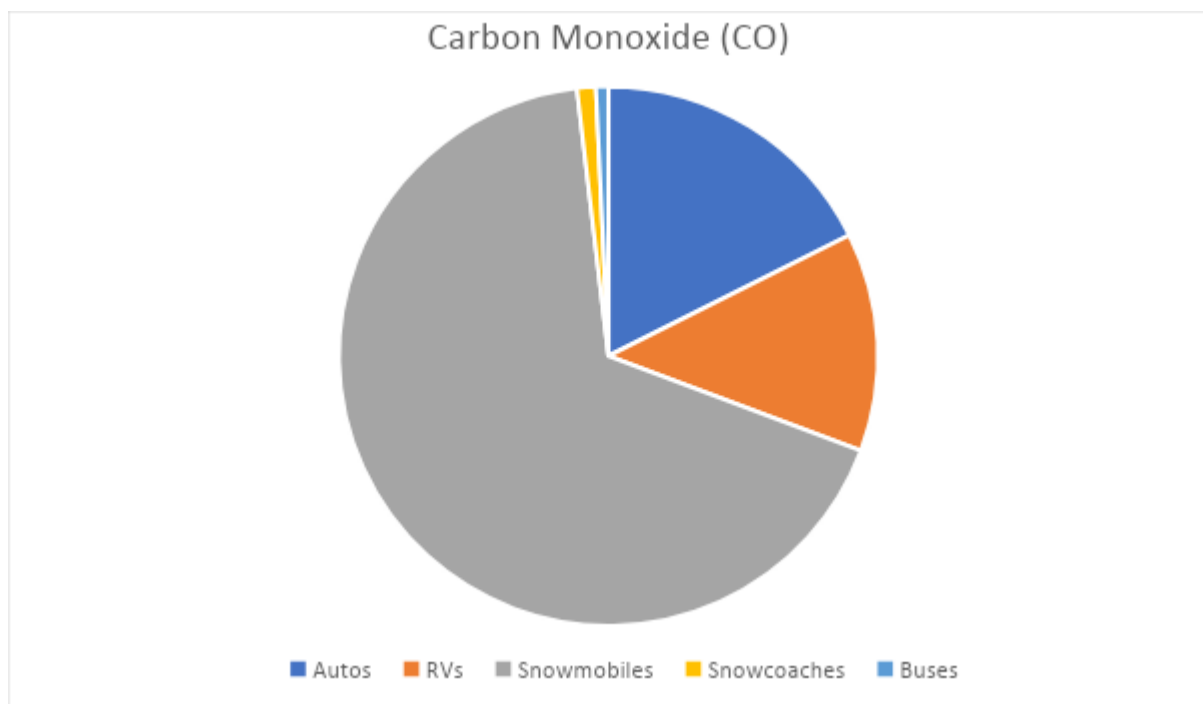
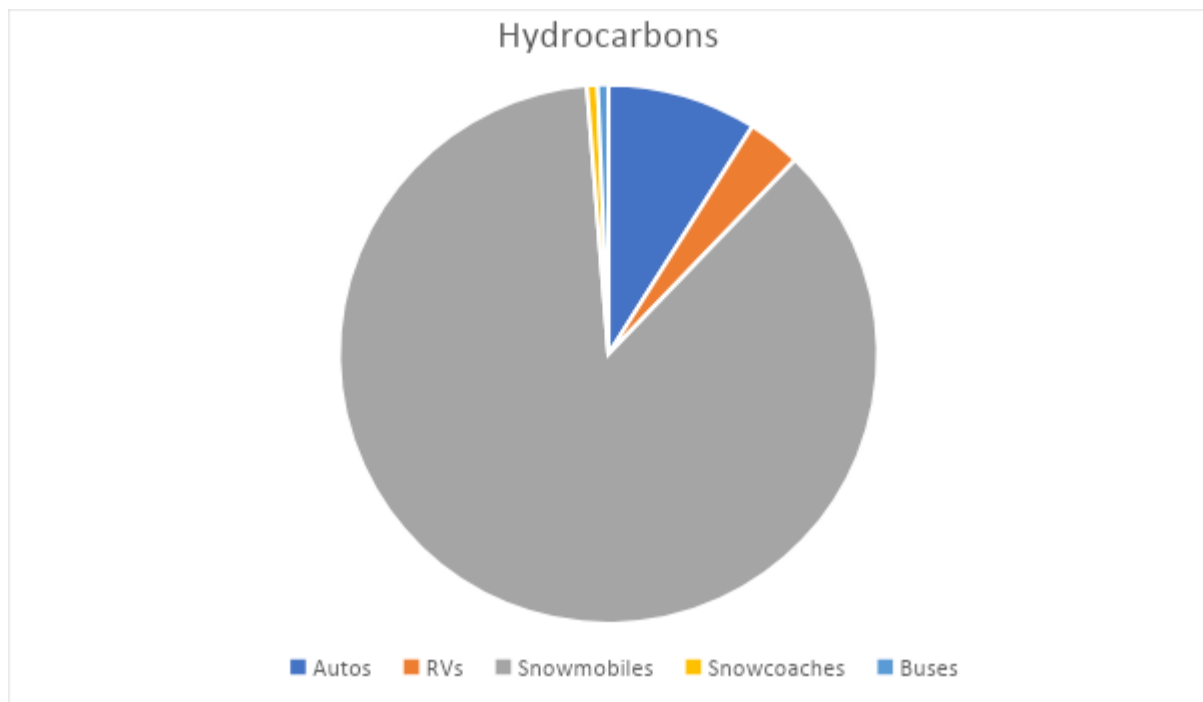
3 Apply Quantitative Tools + - × ÷

Between 2002 and 2011, the National Park Service released eight different reports on the impact of winter use on air quality. You can see a complete list of all of their reports [here](#). The following explanation shows just a small part of the data examined by Park officials as they used data to make decisions on Park policy.

At the beginning of their research in 2000, researchers compared carbon monoxide and hydrocarbon emissions from different type of vehicles entering Yellowstone. The following bar-chart shows the estimated annual emissions from automobiles, RVs, snow machines, snowcoaches, and buses for 1992 through 1999.



They also looked at pie charts that showed the source of all annual hydrocarbons and carbon monoxide emissions:



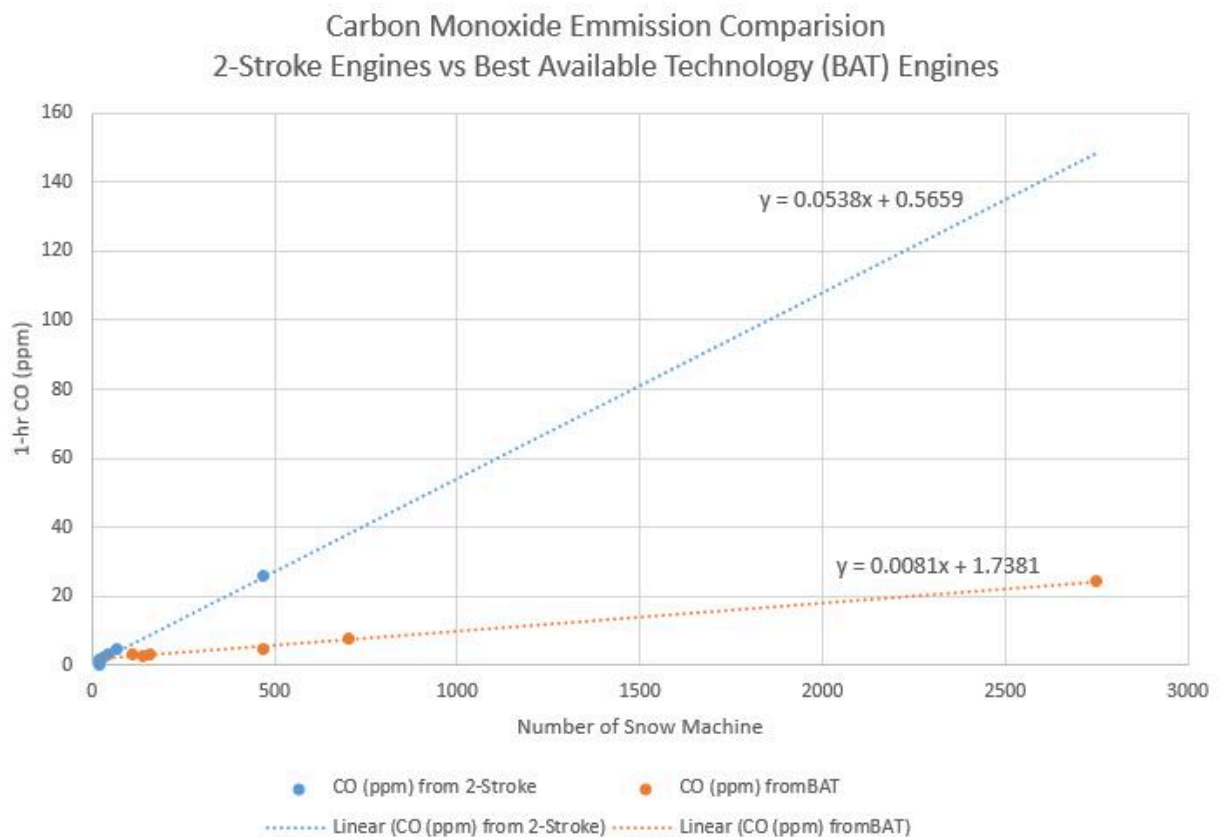
From the bar chart and the pie chart, it is clear that snow machines are the largest source of these harmful emissions. This led Park officials to decide that, although they should not completely ban the use of snow machines in Yellowstone, changes to the winter use policy did need to occur.

They gathered data on how the type of engine a snow machine had impacted the carbon monoxide emissions.

Type of snow machine engine	Number of snow machines entering Park
2-stroke	20

2-stroke	22
2-stroke	28
2-stroke	42
2-stroke	67
2-stroke	470
BAT	110
BAT	140
BAT	160
BAT	470
BAT	701
BAT	2750

In order to test their assumption that snow machines with the best available technology (BAT) have lower emissions than older snow machines with 2-stroke engines, they created a scatter plot of these data and created trendlines for each type of snow machine.



Comparing the slopes of these two lines gave helpful information on how the two different types of snow machines are related to carbon monoxide emissions.

For snow machines with 2-stroke engines (older technology), the slope of the line is 0.0538. This tells us:

$$\text{slope} = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{change in carbon monoxide}}{\text{change in number of snow machines}} = \frac{0.0538}{1 \text{ snow machine}}$$

Each 2-stroke engine adds 0.0538 ppm of carbon monoxide. Similarly, since the slope of the line for BAT engines is 0.0081, we know BAT engines add 0.0081 ppm of carbon monoxide. Because these numbers are small they are hard to compare. If we multiply them each by 1000, we see that 1000 2-stroke engines would add 53.8 ppm of carbon monoxide while 1000 BAT engines would only add 8.1 ppm of carbon monoxide.

4 Make an Informed Decision



Based on these calculations (and others that are not shown here), Park officials developed a new policy for winter use in Yellowstone. In 2003 and 2004 they put new winter use restrictions in place. They were again sued over these plans because they restricted winter use in Yellowstone. They again did more research and in 2007 they completed a new plan that allowed for 540 commercially guided, BAT snow machines and 83 snowcoaches to enter the Park each day.

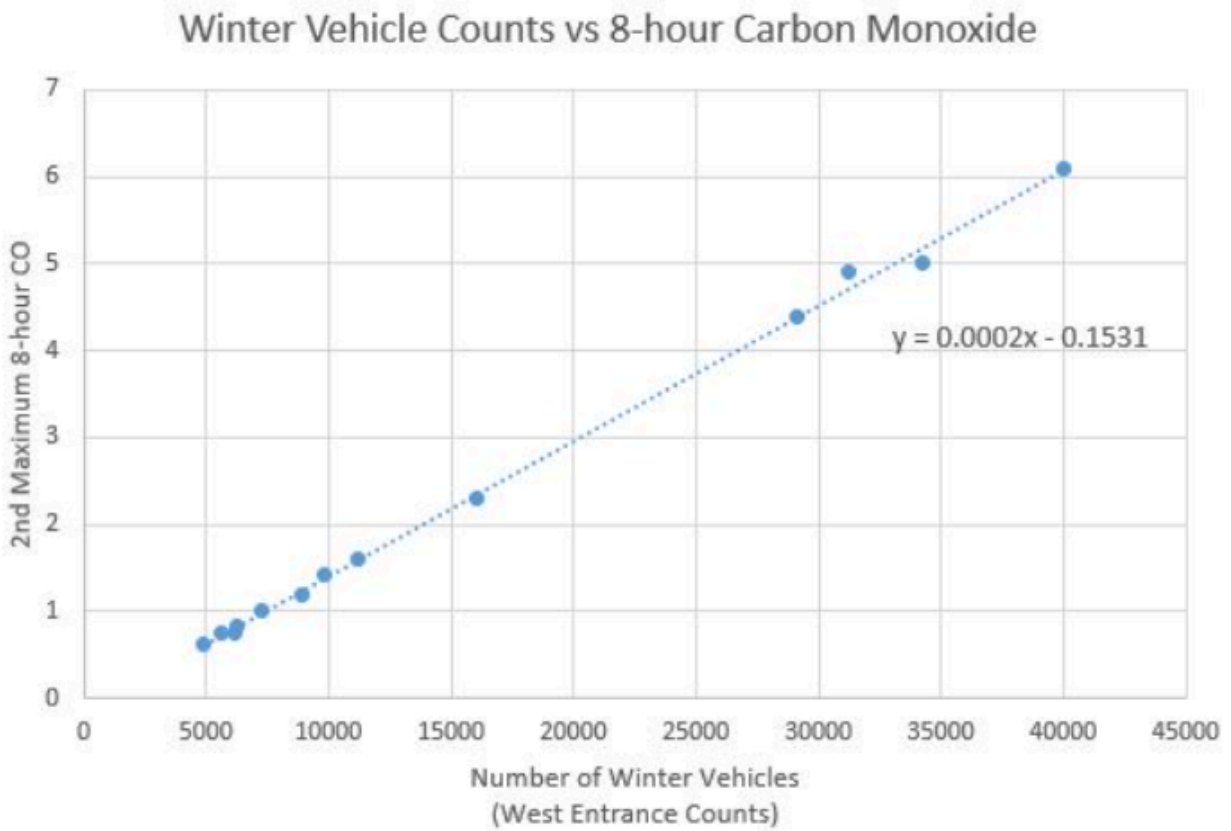
5 Evaluate Your Reasoning



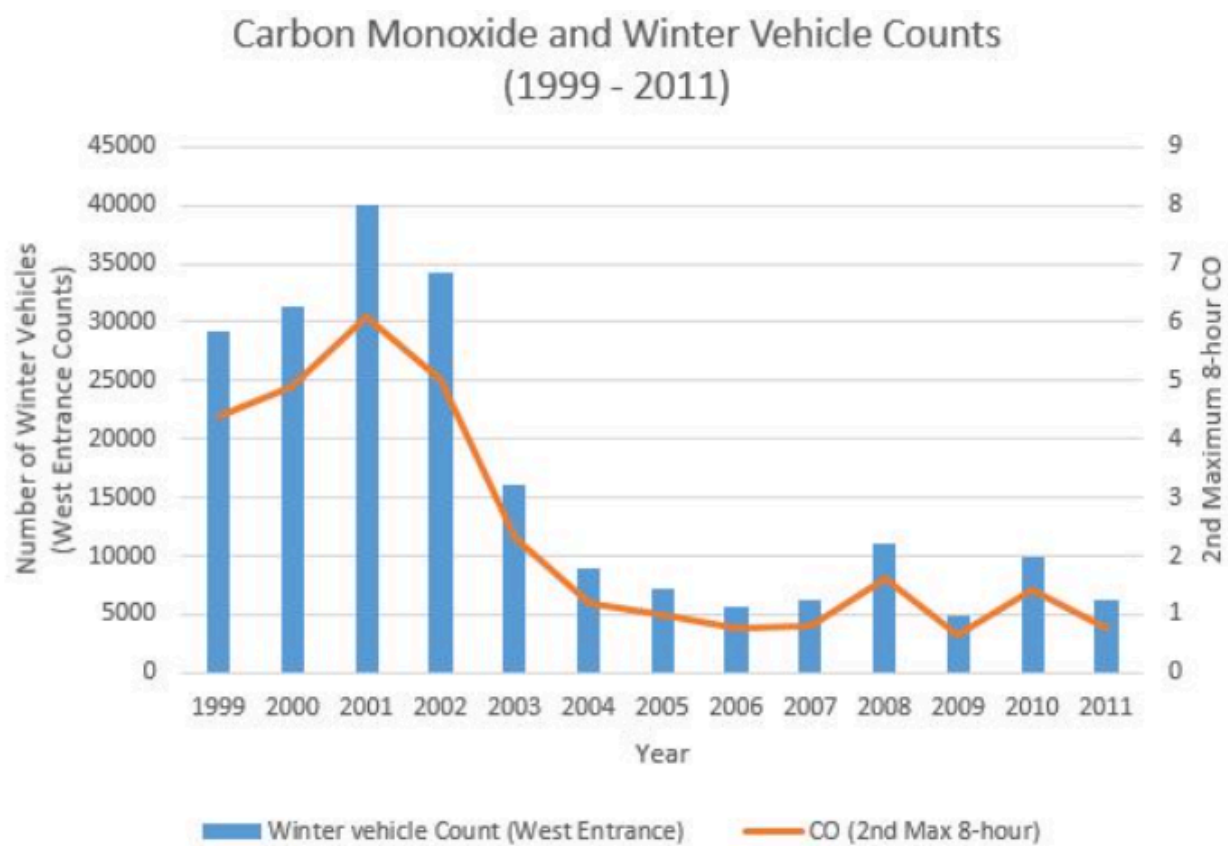
Between 2007 and 2013 the Park reevaluated their winter use plan several more times. Some of the reevaluation occurred as a result of further suits filed in federal court and some of the reevaluation was based on public comment and input.

In 2011, Park officials gathered data that compared data collected on the number of winter vehicles entering the Park through the West Entrance and carbon monoxide measurements taken near the entrance.

This scatter plot shows how the number of winter vehicles correlated to carbon monoxide measurements. There is a strong linear relationship that shows that when more cars enter the park, carbon monoxide measurements also increase.



The following graph shows a bar chart and a time series plot overlayed on top of each other. It shows how the number of winter vehicles and carbon monoxide measurements have changed over the time period the Park has been changing their policy.



These graphs clearly show that the changes made in 2003-2011 are helping reduce carbon monoxide emissions in the Park.

In 2013 they revised their policy again based on further study. One common criticism of the plan prior to 2013 was that the public was not allowed to enter the Park unless they were with a commercially guided tour. Also, the management plan allowed 83 snowcoaches per day, but on some days fewer than 83 snowcoaches entered the Park while snow machine users were turned away.

Park officials revised their guidelines to allow for flexibility in the ratio of snow machines and snow coaches. If fewer snowcoaches enter the park on a given day, they are now able to allow more snow machines to enter. They also added an option for one non-commercially guided group to enter the Park each day.

You can read all of the details of Yellowstone National Park's current winter-use policy on their website. [11](#)

Most of the examples of the Quantitative Reasoning Process have shown how it can be used by individuals to make decisions about their personal and family lives. This example shows how the Quantitative Reasoning Process was used by the National Park Service. The Quantitative Reasoning Process is commonly used by businesses, government agencies, non-profit organizations, communities, and other organizations. Scientists, mathematicians, statisticians, business people, and other professionals use the Quantitative Reasoning Process to help them make decisions in their work.

Lesson Checklist

By the end of this lesson, you should be able to:

- identify the input and output variable from a given scenario.
- create a scatter plot of two quantitative variables in Excel.
- identify for given data whether a linear, exponential, or quadratic trendline would be the most appropriate.
- fit a linear, exponential, or quadratic trendline to a scatter plot of data in Excel.
- use a trendline to make a prediction for the average value of the response variable at a given value of the explanatory variable.
- interpret predictions and draw a contextual conclusion
- assess the appropriateness of a trendline for given data
- explain why correlation does not imply causation

Optional Resources

Paragraph 10-13 of [That the Lost May Be Found](#) given in April 2012 by Elder M. Russell Ballard include an interesting example of cause and effect.

The following video shows more about the research done to establish a causal relationship between smoking and lung cancer.

[Correlation: Joy of Stats Transcript](#)

¹ [Things as They Really Are](#), Elder Bednar, May 3, 2009

² [Jacob 4:13](#)

³ [Americans Serving Their Communities Gain Well-Being Edge](#), Lindsey Sharpe, 2014

⁴ [Ice Cream Sales cause Shark Attacks. A Real Estate Perspective](#), Jeff Tumbarello, 2017

⁵ [Soda a day may lead to heart attacks in men](#), CBSNews, July 25, 2013

⁶ [About Old Faithful, Yellowstone's Famous Geyser](#), Elisabeth Kwak-Hefferanapr 27, 2020

⁷ The Old Faithful Geyser is only a 2.5 hour drive from the BYU-Idaho campus. You can watch a live webcam of Old Faithful [here](#).

⁸ [Old Faithful Geyser](#)

⁹ [Best Available Technology \(BAT\) Snowmobiles](#)

¹⁰ [Snowmobiles in Yellowstone National Park: An American right, or wrong? \(carleton.edu\)](#)

¹¹ [Winter Use Management - Yellowstone National Park \(U.S. National Park Service\) \(nps.gov\)](#)



This content is provided to you freely by BYU-I Books.

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https://books.byui.edu/math_108x/lesson_10_making_predictions_from_data.

Lesson 11: Solving Systems of Equations

Opening Story

Lesson 11 - Opening Story

[L11 Opening Story Transcript](#)

Introduction

Sophia and Emmy need to make a decision where they must compare two options. We often face situations with different options where we can use the Quantitative Reasoning Process to compare our options. There are several different quantitative tools we can use to make this type of comparison. One tool we look at in this lesson is a *system of equations*.

*A **system of equations** is a list of two or more equations that use the same input and output variables.*

We can use graphs or algebra to solve for the values of the variables that satisfy both equations. Let's start by looking at how Sophia and Emmy use a system of equations to decide between renting the monster truck or renting the boat.

1 Understand the Problem

Sophia and Emmy need to find a way back to their hotel from the jungle. They are faced with a decision where they need to compare the cost of two different things, renting the monster truck or renting the motor boat.

2 Identify Variables & Assumptions

We identify the following variables:

- The number of kilometers Sophia and Emmy need to travel
- The total cost of transportation out of the jungle

We can use the following assumptions:

- Sophia and Emmy can drive either the truck or the boat.
- Sophia can use her emergency credit card that was not swallowed by the black caiman.

We can set up two equations that are related to each other because the variables are the same in both cases: the total cost of transportation out of the jungle and the number of kilometers. With either the monster truck or the motor boat we are focused on money and distance.

3 Apply Quantitative Tools $+\div\times$

We can model each scenario with a mathematical equation.

The total cost, y , of the truck is $\$37.50 + \$0.88 \times \text{kilometers}$: $y = 37.50 + 0.88x$

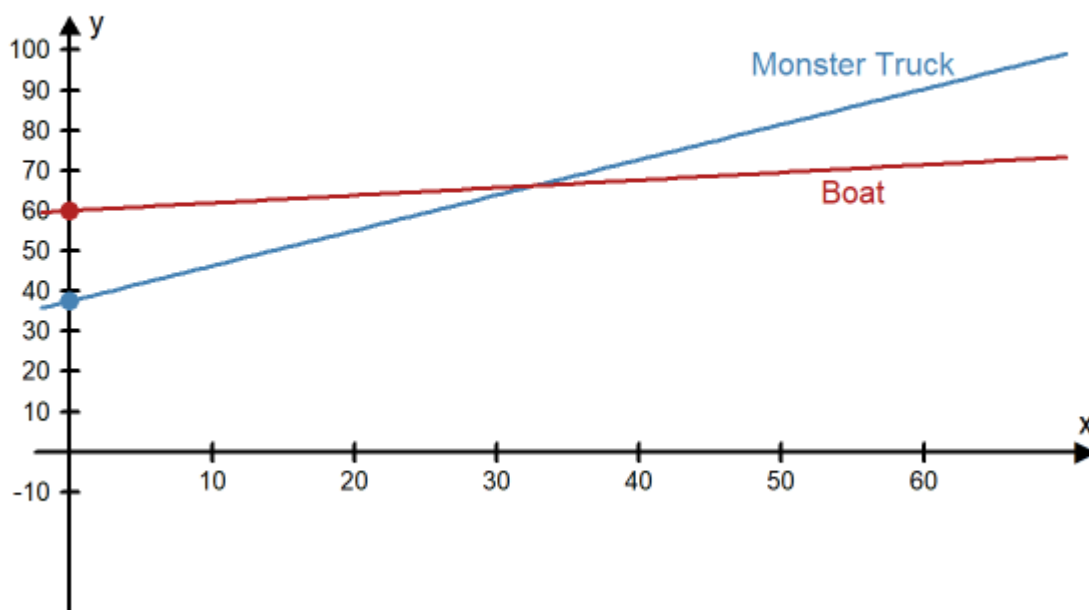
The total cost, y , of the boat is $\$60.00 + \$0.19 \times \text{kilometers}$: $y = 60 + 0.19x$

The input value, x , in each of the equations is the number of kilometers they travel. The output value, y , is the total cost of transportation.

As we learned in Chapter 7 the two equations in this example are linear equations. The rate of change (or slope) in each equation is constant. When we plot this type of equation we get a straight line.

Let's look more carefully at the graph of the two lines representing the Emmy and Sophia's situation. The blue line represents the cost of the monster truck as the kilometers increase. The red line represents the cost of the boat as the kilometers increase.

How to Get Out of the Jungle



We can see from the graph that the two lines cross at around 33 kilometers on the x -axis and just above \$65 on the y -axis.

If Sophia and Emmy travel less than 33 kilometers, then the truck is the least expensive. If they travel more than 33 kilometers, then the boat is the least expensive.

4 Make an Informed Decision



Sophia and Emmy need more information to make an informed decision. They really need to know the distance from their jungle position back to town.

After asking for more information, they find that the dirt road back to the hotel is 40 about kilometers long and the river trip is 38 kilometers. Use the equations we built to compute the cost of each rental vehicle.

Practice: Make an Informed Decision

Problem 1

Sophia and Emmy found the following equations. In these equations, the input variable, x , represents the distance, and the output variable, y , represents the total cost.



$$\text{Boat: } y = 60.00 + 0.19x$$



$$\text{Truck: } y = 37.50 + 0.88x$$

If the dirt road back to the hotel is 40 kilometers and the river trip is 38 kilometers, find the cost of each rental vehicle:

Truck: total cost = \$

Boat: total cost = \$

How Did I Do?

$$\text{Truck: } y = 37.50 + 0.88(40) = \$72.70$$

$$\text{Boat: } y = 60 + 0.19(38) = \$67.22$$

Based on these calculations, which mode of transportation should Sophi and Emmy use? (Which is less expensive?)

☐ Boat

☐ Truck

5 Evaluate Your Reasoning



Sophia and Emmy tie the boat up at the town marina and ask directions to their hotel. After finding out that they have 5 kilometers still to walk they rethink their decision.

Sophia and Emmy have realized that they made an incorrect assumption in their application of the Quantitative Reasoning process. They mistakenly assumed that both modes of transportation would get them back to their hotel. But since they didn't realize that travel by boat would also require a 5 kilometer walk, they made a decision that didn't turn out to be the best.

Because the difference in price between the rentals was relatively small, they now wish they had rented the monster truck and driven straight to the hotel.

The remainder of this lesson will focus on the powerful quantitative tool, systems of equations, that we used to understand Sophia and Emmy's situation.

There are several different techniques that can be used to solve systems of equations. We will look at two of them: solving systems of equations with graphs and solving systems of equations with algebra.

Solving Systems with Graphs

The graph of an equation represents all the points that satisfy the equation. When we are solving a system of equations we want to find a point that satisfies all of the equations in the system. This means that if you graph both equations in a system of two equations on a graph you can find the solution to the system by finding all points where the graphs intersect. An intersection point satisfies both equations so it represents the solution we are looking for.

Example 1

Graph the following two equations to solve the system of equations:

$$3x + 2y = 5$$

$$x - y = 0$$

Solution:

We want to find a point (x, y) that satisfies both equations. This means the point we are looking for will be the point where the two graphs intersect. If we solve each equation for y , we will see that they fit the pattern of a linear function and we will be able to graph the lines.

Equation 1:

$$3x + 2y = 5$$

$$2y = -3x + 5$$

$$y = -\frac{3}{2}x + \frac{5}{2}$$

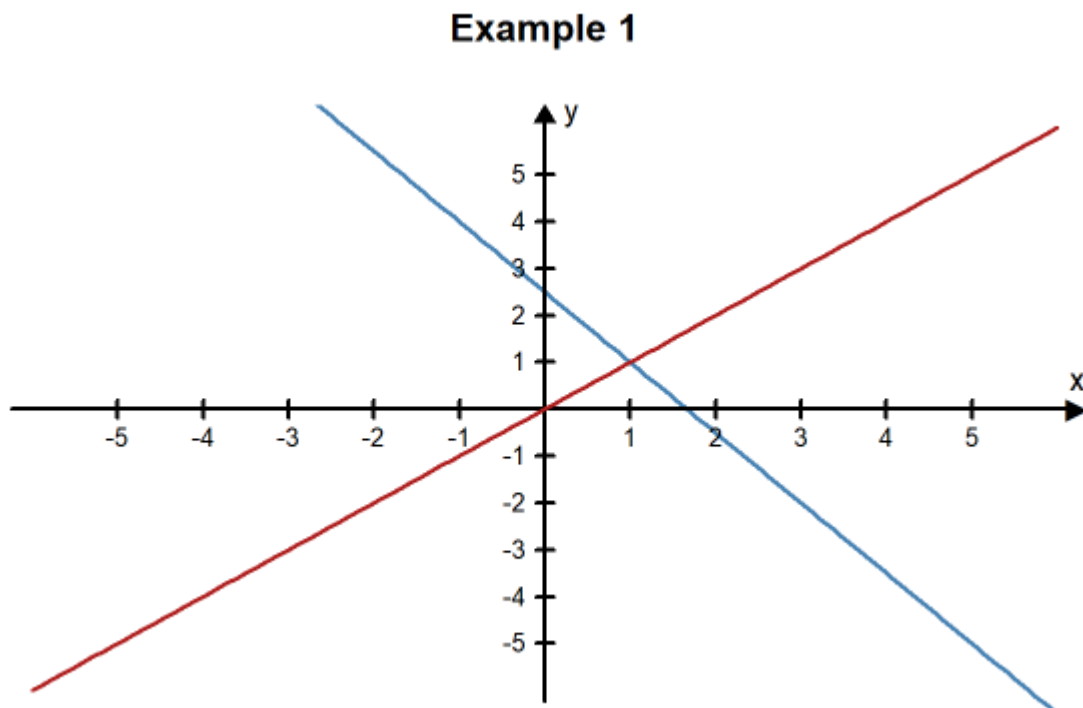
Equation 2:

$$x - y = 0$$

$$-y = -x$$

$$y = x$$

Equation 1 can be represented with a line with a y -intercept of $\frac{5}{2}$ and a slope of $-\frac{3}{2}$. Equation 2 gives the graph of a line with a y -intercept of 0 and a slope of 1. If we plot both lines on the same graph, they look like this:



The solution to the system of equations is represented on the graph by the point where the two lines intersect. By looking at the graph of these two lines, we can see that they cross at the point $(1, 1)$. This means $x = 1, y = 1$ is the solution to the system of equations.

Check Our Answer:

We can verify that $x = 1, y = 1$ is the solution to the system by checking that $(1, 1)$ is exactly the point where they cross. We do this by substituting in these values for x and y in both equations and making sure that both mathematical statements are true.

Equation 1:

$$3x + 2y = 5$$

$$3(1) + 2(1) = 5$$

$$3 + 2 = 5$$

$$5 = 5$$

True Statement

Equation 2:

$$x - y = 0$$

$$1 - 1 = 0$$

$$0 = 0$$

True Statement

The point $(1, 1)$ is the only point where the two lines cross and the values $x = 1, y = 1$ is the only set of x, y values that will yield true statements for both equations. When we get a single unique solution like this, we say that the system of equations is *independent*.

Example 2

Graph the following two equations to solve the system of equations:

$$-x + 2y = 4$$

$$-3x + 6y = -18$$

Solution:

If we solve each equation for y , we can see that each one fits the pattern of a linear function and can graph the lines.

Equation 1:

$$-x + 2y = 4$$

$$2y = x + 4$$

$$y = \frac{x + 4}{2}$$

$$y = \frac{1}{2}x + \frac{4}{2}$$

$$y = \frac{1}{2}x + 2$$

The first equation can be graphed as a line with a slope of $\frac{1}{2}$ and a y -intercept of 2.

Equation 2:

$$-3x + 6y = -18$$

$$6y = 3x - 18$$

$$y = \frac{3x - 18}{6}$$

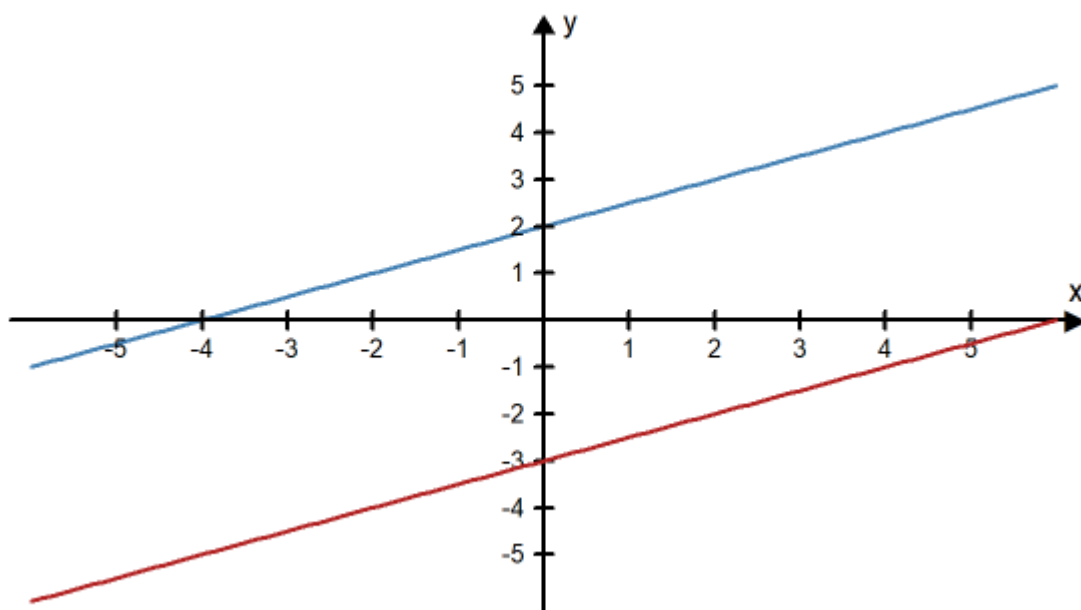
$$y = \frac{3}{6}x - \frac{18}{6}$$

$$y = \frac{1}{2}x - 3$$

The second equation can be graphed as a line with a slope of $\frac{1}{2}$ and a y -intercept of -3 .

It we plot both lines on the same graph, they look like this:

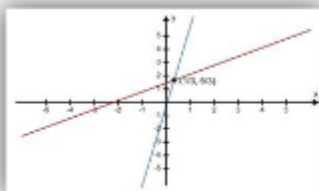
Example 2



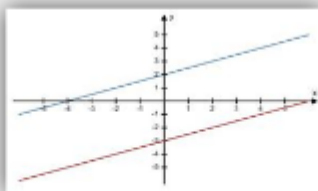
From the graph it looks like the lines are parallel, which means they will never cross. We can see from the equations that the lines are parallel because the two lines have the same slope. This means that there is no (x, y) pair that will satisfy both equations and thus there is no solution to this system of linear equations. When there is no solution to a system of equations we say that the system of equations is *inconsistent*.

Types of Solutions

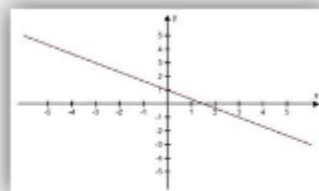
There are three types of solutions for systems of equations:



Independent Systems



Inconsistent Systems



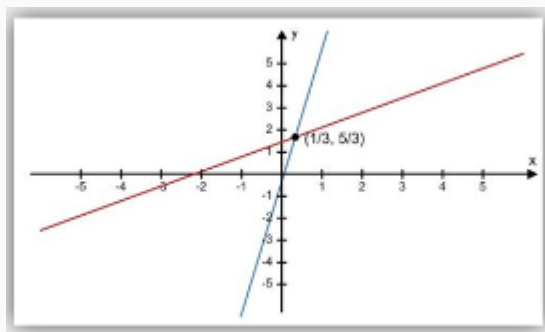
Dependent Systems

Click on each button to learn more.

Independent Systems

An independent system is formed when two lines intersect at one point.

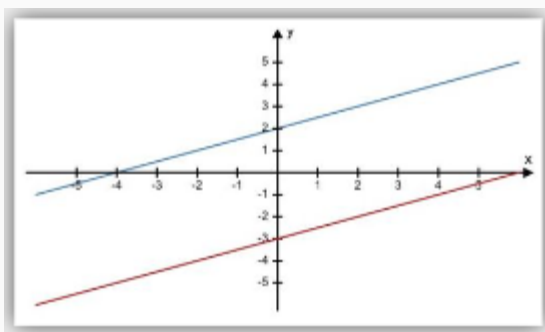
The point where the lines intersect is the solution to the system.



Inconsistent Systems

An inconsistent system is formed when two lines are parallel and never intersect.

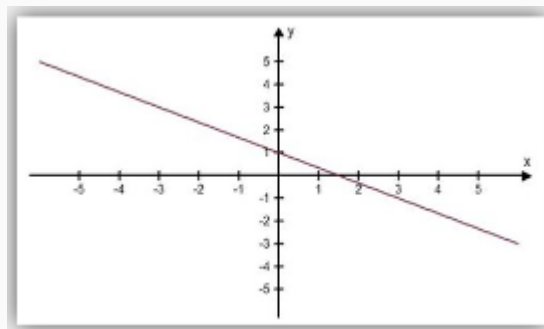
Inconsistent systems have no solution.



Dependent Systems

A dependent system is formed when the two equations in the system actually represent the same line.

Every point on the line is a solution to the system. So the solution has infinitely-many solutions.



Note: We won't really see this type of system in MATH 108X

Solving Systems with Algebra

Graphing systems of equations helps us to visualize the system and the solutions but unless the equations and solutions are fairly simple it can be difficult to obtain accurate results. We can also solve systems of equations using techniques of algebra. The *Substitution Method* is one way to solve a system of equations. To solve a system of equations using the substitution method we eliminate one variable from one of the equations by substituting an expression for that variable from the other equation.

Example 3

Use the substitution method to solve the system of equations:

$$2x - y = -5$$

$$3x + 5y = 12$$

Solution:

To solve this system using the substitution method, we first solve the first equation for y :

$$2x - y = -5$$

$$-y = -2x - 5$$

$$y = 2x + 5$$

Now, we substitute the expression $2x + 5$ in for the y in the other equation:

$$3x + 5y = 12$$

$$3x + 5(2x + 5) = 12$$

Because we have an equation with only x 's in it, we can solve for x :

$$3x + 5(2x + 5) = 12$$

$$3x + 10x + 25 = 12$$

$$13x + 25 = 12$$

$$13x = 12 - 25$$

$$13x = -13$$

$$x = -\frac{13}{13}$$

$$x = -1$$

We know $x = -1$ so we can substitute this value for the x in the y -expression we found earlier, $y = 2x + 5$:

$$y = 2x + 5$$

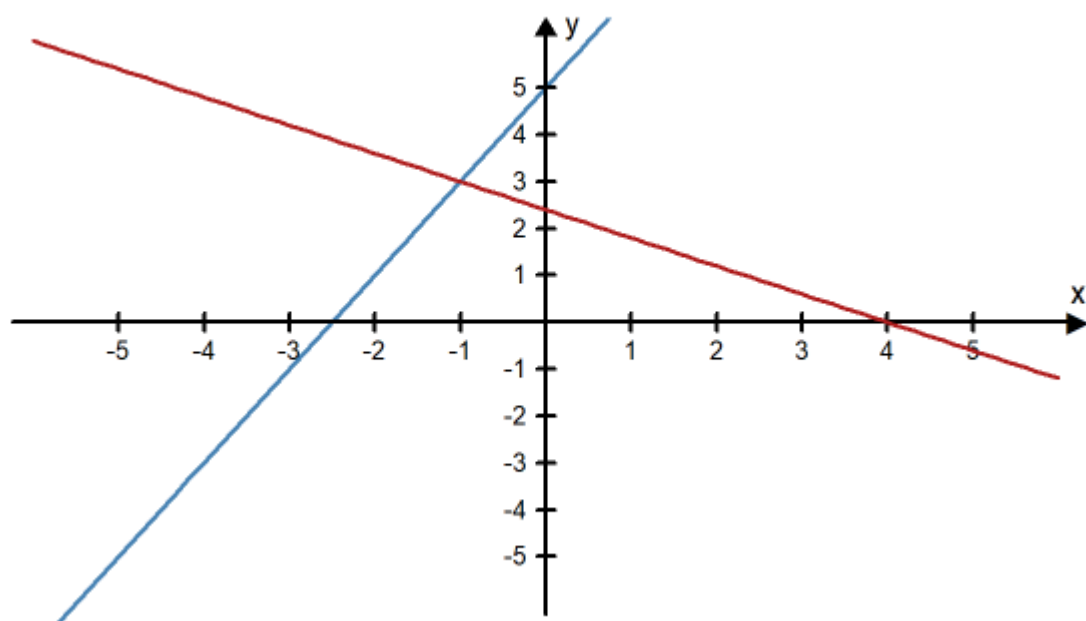
$$y = 2(-1) + 5$$

$$y = -2 + 5$$

$$y = 3$$

We have found the solution to this system of equations to be $x = -1, y = 3$. Because there is a single solution to this system of equations, it is independent. The graph of these two equations verifies this solution because the lines intersect at the point $(-1, 3)$.

Example 3



Example 4

Use the substitution method to solve the system of equations:

$$6x - 9y = -13$$

$$18x - 3y = 1$$

Solution:

We need to solve for one of the variables in one of the equations. We will solve for y in the second equation:

$$18x - 3y = 1$$

$$-3y = -18x + 1$$

$$y = \frac{-18x + 1}{-3}$$

$$y = \frac{-18x}{-3} + \frac{1}{-3}$$

$$y = 6x - \frac{1}{3}$$

Now that we know y in terms of x , we can substitute this into the first equation:

$$6x - 9y = -13$$

$$6x - 9\left(6x - \frac{1}{3}\right) = -13$$

$$6x - 54x + \frac{9}{3} = -13$$

$$-48x + 3 = -13$$

$$-48x = -13 - 3$$

$$-48x = -16$$

$$x = \frac{-16}{-48}$$

$$x = \frac{1}{3}$$

We can now substitute $\frac{1}{3}$ in for x in any of the equations to find y :

$$y = 6x - \frac{1}{3}$$

$$y = 6\left(\frac{1}{3}\right) - \frac{1}{3}$$

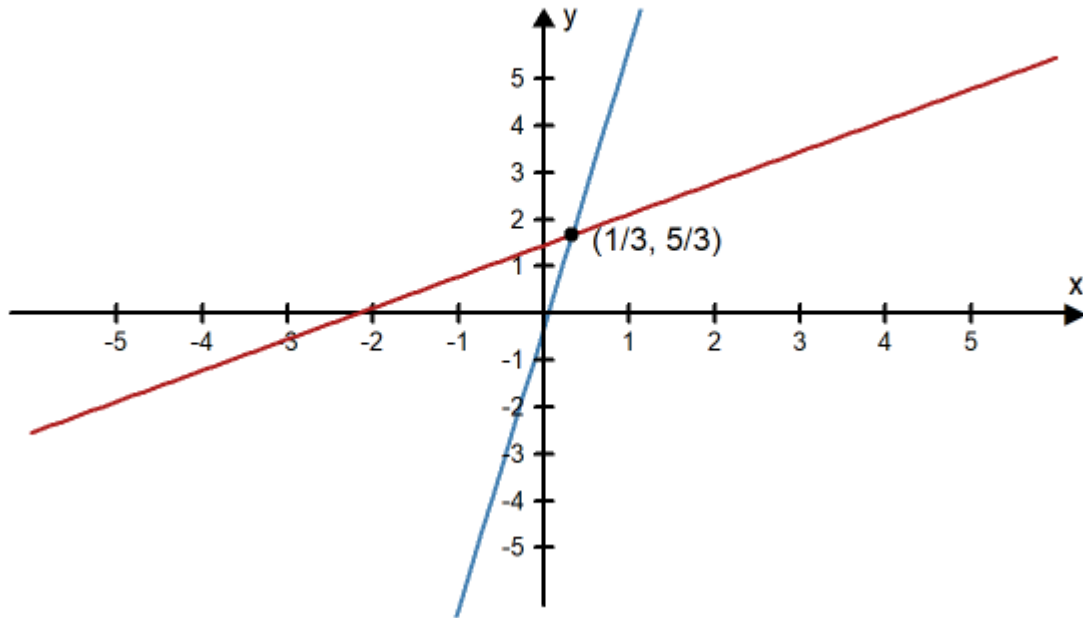
$$y = \frac{6}{3} - \frac{1}{3}$$

$$y = \frac{6 - 1}{3}$$

$$y = \frac{5}{3}$$

This system of equations is independent. The only solution that works for both equations is $x = \frac{1}{3}, y = \frac{5}{3}$. The two lines cross at the point $(\frac{1}{3}, \frac{5}{3})$.

Example 4



Practice: Solving a System

Problem 2

Use the substitution method to solve the system of equations:

$$4x + 2y = 6$$

$$-3x + y = -7$$

$x =$

$y =$

How Did I Do?

$x = 2$

$y = -1$

Hint: Look back at Example 3 to see an example of how to solve this problem. Use the following steps:

- Solve the first equation for y
- Substitute the expression for y into the other equation
- Now you have an equation with only x 's in it, so you can solve for x
- Once you know x , you can substitute the value into the y -expression you found earlier. This will tell you y .

Solving in Context

Piano Sales Example

1 Understand the Problem

Jasper loves music and has recently been hired as a piano salesman. His new boss asked him to choose a commission plan. He can choose Plan A or Plan B. For Plan A Jasper gets a base salary of \$25,000 a year plus a \$500 commission for each piano he sells. In Plan B Jasper gets a base salary of \$30,000 a year plus a \$200 commission for each piano.

2 Identify Variables & Assumptions

Jasper identifies two variables:

- The number of pianos sold per year
- The total amount of money he earns in the year.

3 Apply Quantitative Tools

Because Plan A and Plan B have the same two variables, Jasper can model his situation using a system of equations to better understand the decision he needs to make.

- Plan A: the money he earns in one year = \$25,000 + \$500 for each piano
- Plan B: the money he earns in one year = \$30,000 + \$200 for each piano

This can be turned into the following system of equations where the input variable, x , represents the number of pianos Jasper sells in a year and the output variable, y , represents the total amount of money he earns in one year.

$$y = 500x + 25000$$

$$y = 200x + 30000$$

Because both equations are already solved for y we can substitute $200x + 30000$ in for y in the first equation and solve for x :

$$200x + 30000 = 500x + 25000$$

$$200x - 500x = 25000 - 30000$$

$$-300x = -5000$$

$$x = \frac{-5000}{-300}$$

$$x = 16.67$$

We can now substitute $x = 16.67$ into any of the equations to find y :

$$y = 500x + 25000$$

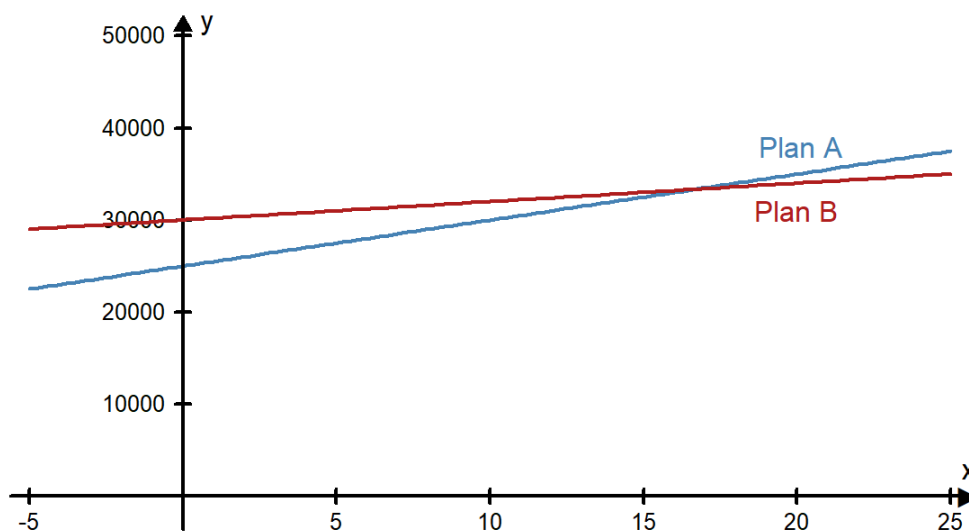
$$y = 500(16.67) + 25000$$

$$y = 33335$$

The solution to the system is $x = 16.67$, $y = \$33,335$

The following graph of this system supports this solution:

Piano Sales



4 Make an Informed Decision



From this solution, Jasper knows if he sells around 16 or 17 pianos the two plans pay about the same and he would make around \$33,335 per year. If he sells more than 16 pianos, Plan A is a better plan because he earns more money in a year. If he sells 16 pianos or fewer, Plan B is a better plan.

He believes he can sell at least 2 pianos per month. Since this a total of 24 pianos he decides to select Plan A.

5 Evaluate Your Reasoning



Before telling his boss his decision, Jasper decides to ask some of the other salesman how many pianos they sold the previous year. He talks to three of them and finds that they sold 18, 31, and 22 pianos each. Because these values are all more than 17, Jasper is satisfied with his decision to chose Plan A and tells his boss of his decision.

Practice: Solving in Context

Problem 3

1 Understand the Problem

Serena is writing an exam for her 5th grade students. The exam is worth 100 points and Serena wants no more than 30 questions. She plans to use two types of questions:

1. Multiple choice questions = 3 points each
2. Essay questions = 8 points each

Serena needs to decide how many questions of each type to put on the exam.

2 Identify Variables & Assumptions

Identify two variables in Serena's situation:

Variable #1: $x =$ _____

Variable #2: $y =$ _____

How Did I Do?

Because Serena wants to know how many of each type of question to put on the exam:

- x = the number of multiple choice questions
- y = the number of essay questions

3 Apply Quantitative Tools

Create a system of equations Serena could use to determine how many questions to put on the exam.

How Did I Do?

- Number of points: $3x + 8y = 100$
- Total number of questions: $x + y = 30$

Now solve the system:

$x =$ _____

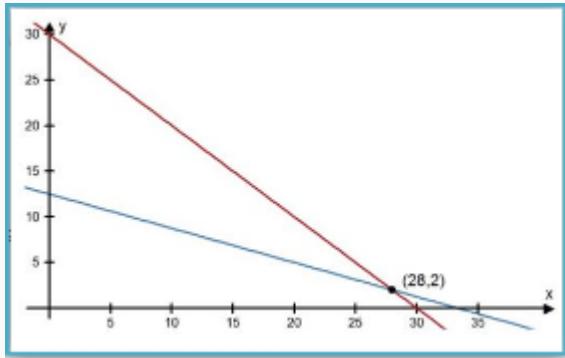
$y =$ _____

How Did I Do?

$$x = 28$$

$$y = 2$$

[Click here to see a graph of the solution](#)



4 Make an Informed Decision



Based on your calculations, how many questions of each type should Serena put on the exam?

Number of multiple choice =

Number of essay questions =

How Did I Do?

28 multiple choice questions

2 essay questions

5 Evaluate Your Reasoning



Serena writes an exam with 28 multiple choice questions and 2 essay questions.

Evaluate your answer:

1. Does this answer make sense in the context of the problem?
2. Does this seem like a reasonable ration of multiple choice and essay questions?

Systems and Trendlines

Often, the equations that make up a system of equations are obtained from data using a trendline. The following example looks at an example from business: the break-even point.

1 Understand the Problem

Jeff and Lucas opened a pizzeria. They have been open for 10 months now and are doing some calculations to determine what they could do to improve their profits. Currently, they charge \$10 for the large pizza, their best-selling item. During the 10 months their restaurant has been open, they collected the following data:

Month	Pizzas Sold	Total Costs	Total Revenue
February	830	\$ 10,240.21	\$ 9,208.84
March	851	\$ 11,013.52	\$ 9,551.70
April	902	\$ 10,985.39	\$ 9,831.73
May	1120	\$ 12,791.87	\$ 12,377.68
June	1291	\$ 13,682.76	\$ 14,239.06
July	917	\$ 11,441.86	\$ 10,556.98
August	1060	\$ 12,004.55	\$ 11,629.34
September	1402	\$ 13,716.38	\$ 15,681.21
October	1289	\$ 12,940.14	\$ 14,461.54
November	1305	\$ 13,850.56	\$ 14,100.33

They want to answer two questions:

- How many pizzas do we need to sell to break even each month?
- Should we change the price of our large pizza?

2 Identify Variables & Assumptions

Jeff and Lucas identify the following variables:

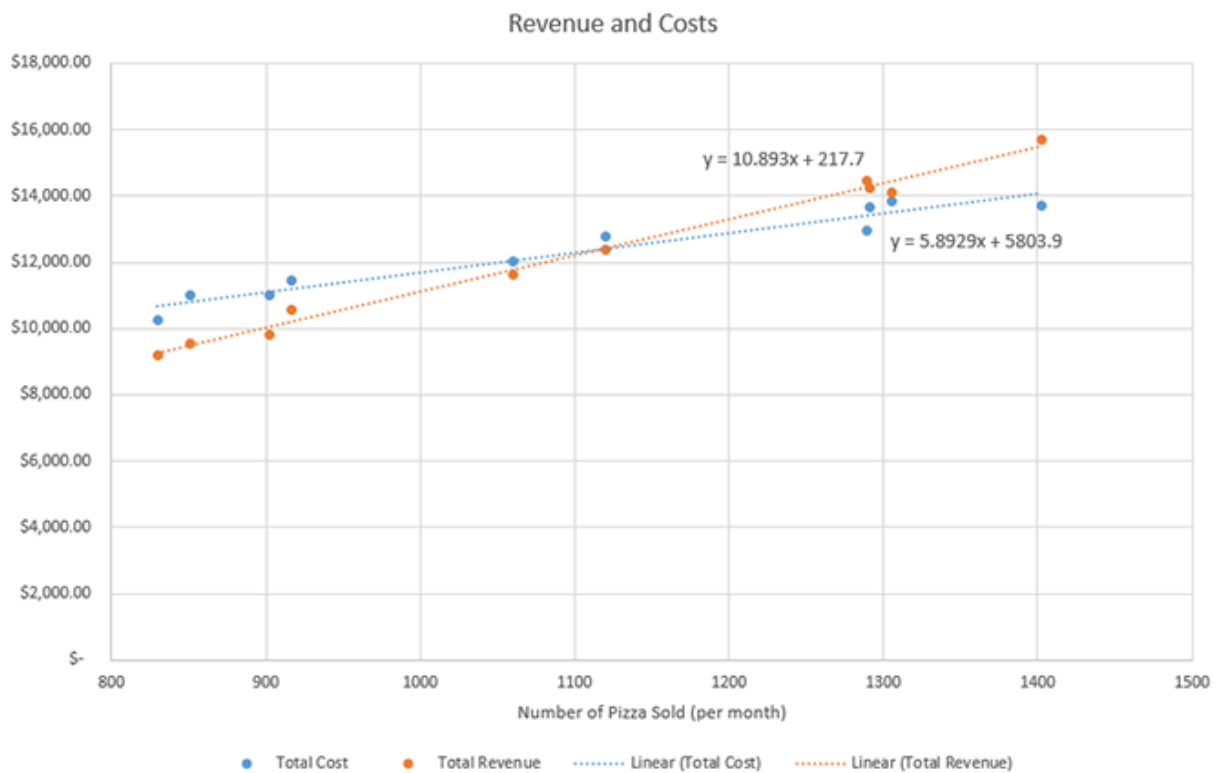
- The number of pizzas sold each month
- Total cost each month (how much money they have to pay to keep the business open)
- Total revenue each month (the total amount of money they bring in during one month)
- Total profit each month (total revenue minus total cost)

They make the following assumptions:

- They will break-even if their monthly revenue is equal to their monthly costs
- If they change the price of the pizza, the number of pizzas sold will stay the same

3 Apply Quantitative Tools + − × ÷

Jeff and Lucas create the following scatter plot and trendlines to compare their revenue and costs. The input variable, x , represents the number of pizzas sold in one month. The output variable, y , represents money. We can see the data and the trendline for both the total revenue (orange) and total cost (blue).



These two trendlines give them a system of equations:

$$y = 10.893x + 217.7$$

$$y = 5.8929x + 5803.9$$

Jeff and Lucas use substitution to solve the system of equations. Because both equations are already solved for y , they can set the equations equal to each other and solve for x :

$$10.893x + 217.7 = 5.8929x + 5803.9$$

$$10.893x - 5.8929x = 5803.9 - 217.7$$

$$5.0001x = 5586.2$$

$$x = \frac{5586.2}{5.0001}$$

$$x = 1117.2177$$

They can then substitute this value of x back into any of the equations to solve for y . They used the first equation:

$$y = 10.893x + 217.7$$

$$y = 10.893(1117.2177) + 217.7$$

$$y = 12387.5524$$

The solution to the system of equations is $x = 1117.2177$, $y = 12387.5524$.

Jeff and Lucas know that x represents the number of pizzas sold and y represents a dollar amount. This solution means that if Jeff and Lucas sell 1,117 pizzas they will break-even and their revenue and costs will both be about \$12,387.55.

4

Make an Informed Decision



These calculations give Jeff and Lucas the answer to their first question: How many pizzas do they need to sell to break even each month? They need to sell around 1,117 pizzas per month. Because they are closed on Sunday, their pizzeria is open for around 26 days per month. They divide the number of pizzas by the number of days and get 42.96. This means they need to sell an average of 43 pizzas every day in order to break even.

Looking back at the data, Jeff and Lucas notice that they sold more than 1,117 pizzas in 5 of the past 10 months: May, June, September, October, and November. They are concerned that they sold fewer than 1,117 pizzas in the other 5 months. But they notice that the most recent three months have had good sales. They hope that this upward trend in pizza sales continues as their pizzeria becomes more popular.

To answer the question of whether they should increase the price of the large pizza, they notice that in the past three months, although they have sold more than 1,117 pizzas their total revenue was not much larger than their total costs. That means their profits were relatively low. They decide to increase the price of the large pizza to \$11. They hope the extra revenue will increase their profits.

5

Evaluate Your Reasoning



As Jeff and Lucas evaluate their reasoning, they re-examine their assumptions. One of their assumptions was: "If they change the price of the pizza, the number of pizzas sold will stay the same." As they think about this assumption, they realize it is probably not very accurate. Reducing prices would probably increase sales, while increasing prices would probably decrease sales. Because their decision was to increase the price of the large pizza to \$11, Jeff and Lucas realize they might see a decrease in sales. They decide to try the higher price for three months and then re-evaluate their decision.

After three more months of sales, Jeff and Lucas reevaluate their decision. In those three months they sold 1410, 1511, and 1620 pizzas. So far, it appears that the increase in their prices has not decreased their sales too much. Their sales remain above the 1,117 pizzas they calculated that they needed to sell in order to break even. Because they raised the

price of the large pizza they find that their revenue has increased, so their profits are also increasing. Jeff and Lucas decide to do more advertising. They know that if they can sell more pizzas their profits will continue to increase.

Non-linear Systems

So far, all of the systems of equations we have encountered in this lesson have involved linear equations. Linear systems are the most common type of system that appear in real life problems. Because a line is simple, both in a graph and in an equation, we often choose to model situations with lines. This can be especially useful when the solution does not have to be precise, like rounding up to the nearest pizza or the nearest dollar.

However, sometimes situations need more precision or sometimes a line is not a close enough model and it is helpful to use non-linear models. There are ways to solve non-linear systems with algebra, but in this class, we will only solve non-linear systems using graphs.

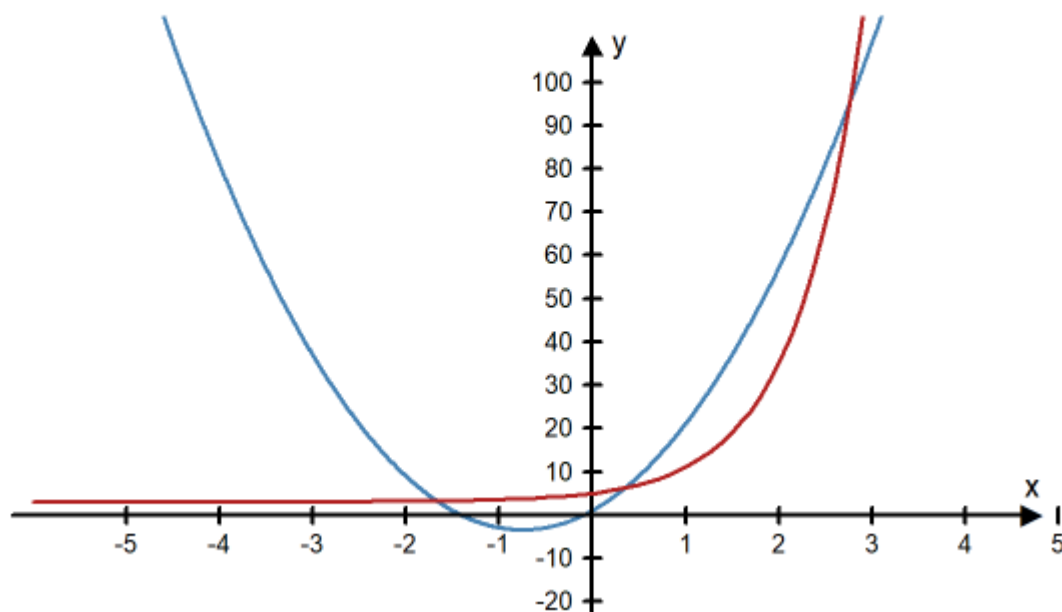
Here is an example of a non-linear system of equations:

$$y = 8x^2 + 12x + 1$$

$$y = 2(4)^x + 3$$

Notice that the first equation is a quadratic function and the second equation is an exponential function. When we graph these functions we get the following:

A Non-linear System



Notice there are three solutions to this system of equations because the graphs intersect at three different points. It is difficult to determine the exact location of the intersection points from the graph, but we can tell they are somewhere around the points (-1.6, 3.5), (0.3, 6), and (2.8, 95).

Practice: Solve a Non-linear System

Problem 4

We will use graphing to solve the following system of equations:

$$y = 2x^2 - 13x + 11$$

$$y = x - 1$$

What type of function is the equation $y = 2x^2 - 13x + 11$

☐ Linear

☐ Quadratic

☐ Exponential

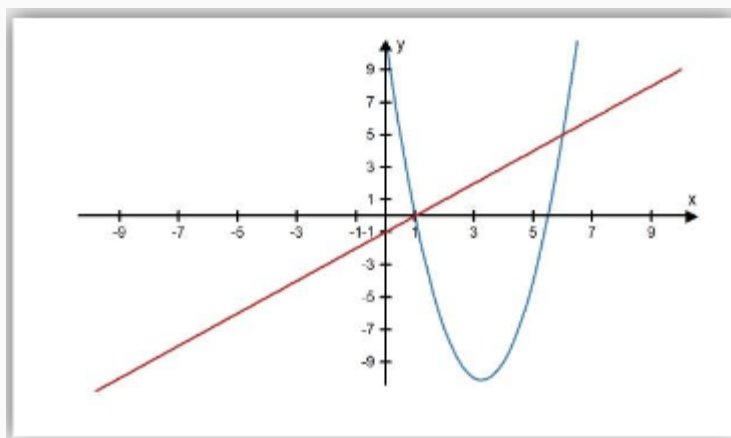
What type of function is the equation $y = x - 1$

☐ Linear

☐ Quadratic

☐ Exponential

Here is the graph of the system.



How many solutions will there be?

How Did I Do?

2

Use the graph to find the solutions.

Hint: In this problem, all the solutions are whole numbers. None of the answers are decimals or fractions.

How Did I Do?

(1,0) and (6,5)

Lesson Checklist

By the end of this lesson, you should be able to:

- create a system of linear equations from information provided in a written format
- solve a system of two linear equations with two unknowns using graphs
- solve a system of two linear equations with two unknowns using substitution
- interpret the solution to a system of equations
- solve a system of equations obtained using trendlines
- solve a non-linear system of two equations using a graph



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Lesson 12: Probability and Confidence Intervals

Opening Story

Lesson 12 - Opening story
[L12 Opening Story Transcript](#)

Introduction

In the introductory video we saw Lucas, Amanda, and Craig interacting on a rather ordinary day. Notice how often they used numbers to predict or to make comparisons.

- Lucas and Amanda used customer ratings to compare different headphones and predict which ones would have the best quality.
- Craig used information about the number of people who interviewed to make a prediction on the likelihood that he would get the job.
- Lucas, Amanda, and Craig used statistics to compare two candidates for governor and predict who might win the election.

All of us commonly use quantitative information to make informal prediction and comparisons, like those we saw Lucas, Craig, and Amanda making. In this lesson we will use probabilities, confidence intervals, and margins of error to help us make better predictions and comparisons. These topics provide us with valuable quantitative tools that can be used as part of the Quantitative Reasoning Process.

Probability

In order to make an accurate prediction or comparison we need to understand how likely it is that a certain outcome will occur.

*The **probability** of an event is a measurement between 0 and 1 of how likely an event is to happen. An event with a probability of 0 will never happen. An event with a probability of 1 is certain to happen.*

Although some events have a probability of 0 (like rolling a 7 on a standard six-sided die) or a probability of 1 (like the sun rising in the morning), most events have a probability somewhere between 0 and 1.

Because probabilities are always a number between 0 and 1, we can report these numbers as percentages. For example, we know 0.4 is the same thing as 40%. So we could say the probability of something happening is 0.4 or that there is a 40% chance it could happen.

Most of us have encountered probabilities often enough that we have a pretty good intuitive sense for what they mean. If we are told there is a 90% chance it will rain today, we know it probably will rain. However, if there is only a 10% chance of rain, we know it probably will not rain.

There are several equivalent ways to think about or explain probabilities:

- If the probability of getting a cold is 0.3, we could also say,
- there is a 30% chance you will get a cold, or
- we expect 3 out of every 10 people to get a cold, or
- out of a group of 100 people, around 30 of them will get a cold.

Informal Use of Probabilities

We commonly use probabilities in a very informal way. For example, if we are walking on a sidewalk on a winter day and see a patch of ice ahead, we subconsciously realize that the probability we will slip is higher if we walk across the ice than the probability we will slip if we walk around the patch of ice. We then adjust our path and walk around the ice rather than over it. In making that decision, we compared probabilities and assessed the risk of slipping, but we did so without even realizing we were doing it.

Informal probabilities are an important part of the assumptions we make in the Quantitative Reasoning Process. When we make an assumption, we automatically accept some risk that the assumption might not be true. The probability that the assumption is not true plays a role in the decision we make.

For example, if we are making a decision about taking out student loans and make the assumption that we will get a job paying \$100,000 per year as soon as we graduate, we need to realize this assumption has a low probability of being true. Although some graduates in very high demand fields may be offered \$100,000 per year upon graduation, for most graduates there is a low probability of being offered a salary that high. In 2023, the average starting salary of new bachelor's degree holders in the United States was \$58,862¹. If we base our student loan decision on the assumption that our starting salary will be \$100,000 per year, we have a high risk of the assumption being false and as a consequence we could make a poor decision.

As we create assumptions, we need to informally assess the probability that they are true. Notice, in the example above we did not ever compute a probability, but by comparing our salary assumption to national data about salaries, we were able to informally conclude the probability was low. This type of informal assessment can be done by looking at historical information, studies, surveys, and other data that helps us determine the likelihood that our assumption is true.

Decisions made using the quantitative reasoning process are only as good as our assumptions. If we make assumptions with a low probability of being true, then the probability we will make a good decision is also low.

Computing Probabilities From Historical Data

One way to assess how likely it is that something will happen in the future is to look at what has happened in the past. Although there are no guarantees that what happened in the past will happen in the future, looking at past behavior gives us some sense of the likelihood of a particular outcome. Using information about the past allows us to compute probabilities.

Example 1 – Basketball

For the 2022-2023 season, the National Basketball Association (NBA) player with the best free throw percentage was Nic Claxton from the Brooklyn Nets. ² Claxton made 414 of the 587 free throws he attempted during the season. Find and interpret the probability that Claxton will make the next free throw he attempts.

Solution:

Based on the historical information provided, we compute the percentage of free throw shots McCollum made:

$$\begin{aligned}\text{Probability of making a free throw} &= \frac{\text{Number of free throws made successfully}}{\text{Total number of free throws attempted}} \\ &= \frac{414}{587} \\ &= 0.705\end{aligned}$$

The probability that McCollum will make his next free throw is 0.705 or 70.5%. Because this number is over 0.50 or 50% it means that McCollum is likely to make his next free throw.

Practice: Probability

Problem 1

During the Winter 2024 semester, 4921 of the 22,251 on-campus BYU-Idaho students were married.

Use this historical data to find the probability that a randomly selected BYU-Idaho on-campus student is married.

Probability =

Please enter your answer as a number between 0 and 1.

How Did I Do?

Probability = 0.22

Problem 2

During the Winter 2024 semester, 10,757 of the 22,251 on-campus BYU-Idaho students were female.

Use this historical data to find the probability that a randomly selected BYU-Idaho on-campus student is female.

Probability =

Please enter your answer as a number between 0 and 1

How Did I Do?

Probability = 0.48

Here is an example where historical information was used to compute probabilities in order to make a decision using the Quantitative Reasoning Process.

1 Understand the Problem

Skylar owns a small convenience store. He knows most of his customers come into the store planning to make one small purchase. In order to increase his sales and his profits, he wants to encourage customers to purchase more items. He heard that where items are positioned in a store can entice customers to purchase items that they were not planning to buy. So he decides to create a plan for the arrangement of the items in his store, with the hope of increasing his sales.

2 Identify Variables & Assumptions

Skylar identifies the following variables:

- Number of customers
- Number of items purchased by each customer
- Total monthly sales
- Total monthly profit

He makes the following assumptions:

- Rearranging the placement of items in his store will increase the number of items each customer purchases.
- If customers purchase more items, his sales will go up.
- If his sales go up, his profits will increase.

As Skylar evaluates his assumptions he determines the second and third assumption have a very high probability of being true. However, he realizes he doesn't have much information about the first assumption. He is basing it entirely on something he has heard, but he thinks it makes sense. So he informally determines that the first assumption had a

medium probability of being true. He decides to look at historical information to get a better sense of whether that assumption is accurate.

3 Apply Quantitative Tools + − × ÷

Skylar looked back over his sales receipts for the previous month. He had 7,215 customers during the month. Many customers purchased one item, but several customers purchased 2 or 3 items. The most items purchased by a single customer at one time was 11. Using the information from his sales receipts, he made the following table showing how many customers purchased a certain number of items.

Items Purchased	1	2	3	4	5	6	7	8
Number of Customers	2670	1587	1010	505	289	216	361	14

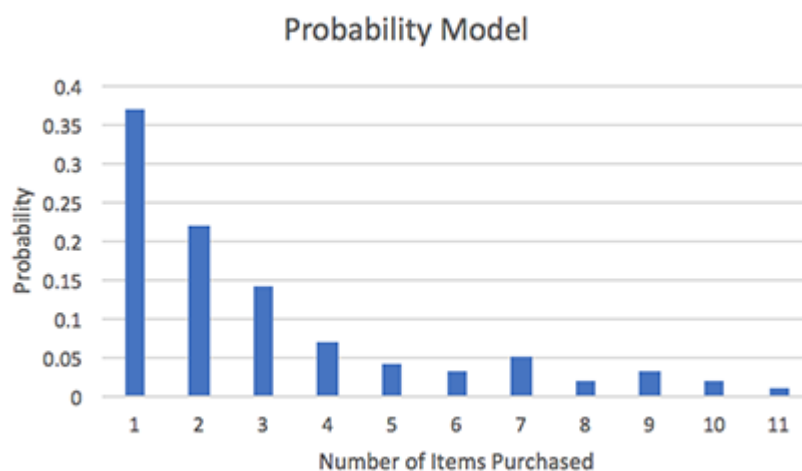
Skylar realized he could use this historical information to compute probabilities.

$$\text{Probability a customer purchases } n \text{ items} = \frac{\text{Number of customers purchasing } n \text{ items}}{\text{Total number of customers}}$$

Using this formula, he found the following probabilities:

Items Purchased	1	2	3	4	5	6	7	8
Probability	0.37	0.22	0.14	0.07	0.04	0.03	0.05	0.02

He graphed this data using a bar chart. This visualization gave him a better sense of what was going on:



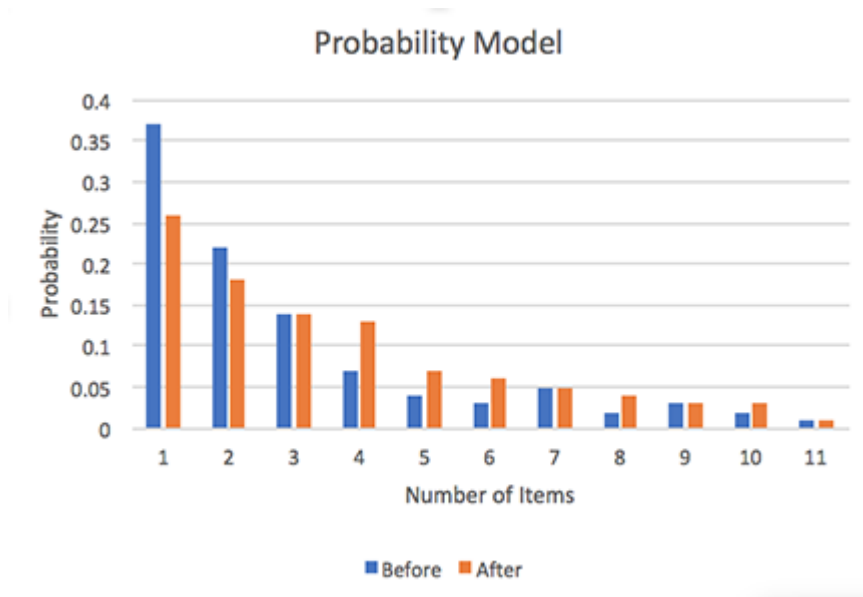
He noticed that the probability that a customer purchased 1, 2, or 3 items was $0.37 + 0.22 + 0.14 = 0.73$. This meant 73% of his customers purchased 3 or fewer items. He also computed the probability that customers purchased 8 or more items: $0.02 + 0.03 + 0.02 + 0.01 = 0.08$. Only 8% of his customers purchased 8 or more items.

Skylar carefully considered the changes he could make to the placement of items in his store. He found that the most commonly purchased item was soda. He couldn't move the soda because it needed to be stored in refrigerators at the back of the store. But he realized the aisle that led most directly to the soda held motor oil, oil filters, and spark plugs. He realized these were not items that customers would buy on impulse. He moved those items to a less convenient aisle. He filled one side of the aisle leading to the soda with candy and gum and the other side of the aisle with donuts and other baked goods. He thought customers were likely to grab these items as they walked to the refrigerators in the back of the store.

One month after making these changes, Skylar again examined his sales receipts. Doing a similar computation to the previous month, he found the following probabilities:

Items Purchased	1	2	3	4	5	6	7	8
Probability	0.26	0.18	0.14	0.13	0.07	0.06	0.05	0.04

In order to compare, Skylar created a new side-by-side bar chart that compared the probability model before he rearranged the store to the probability model after the rearrangement.



This graph showed a lot of information, so it took careful examination in order for Skylar to really see what it was telling him. After studying the graph for a while, he realized that the blue bars were taller than the orange bars for 1-2 items. But then for 4-7 items the orange bars were higher. Because the blue bars represented probabilities before the store was rearranged, he realized this meant that after the rearrangement the probability a customer purchases only 1 or 2 items went down, and the probability a customer purchases 4 to 7 items went up.

He verified what he saw in the graph by comparing probabilities:

Number of Items	Probability Before Rearrangement	Probability After Rearrangement
1-3	73%	51%
4-7	19%	33%

8-11	8%	1
------	----	---

4 Make an Informed Decision

Based on these probability calculations, Skylar concluded that rearranging the store did increase the probability a customer would purchase more items. He also saw a small increase in total sales.

Skylar decided to continue to evaluate what items customers purchased as an impulse buy. He ordered several small items that might entice customers and put them in strategic locations where customers would encounter them near popular items.

5 Evaluate Your Reasoning

After three months, Skylar computed the list of probabilities again. He found that the probability of a customer only purchasing one item had gone back up to 0.31. Skylar realized the new candy and baked good aisle was most effective when it was new to customers. As customers become accustomed to the aisle, they stopped making as many impulse purchases. He decided to rearrange the store regularly hoping to continue to encourage customers to purchase multiple items.

He also noticed that customers could only comfortably carry about 6 or 7 items in their hands. He wondered if the reason so few customers purchased 8 or more items was because it was difficult to carry that many items. Skylar purchased some small baskets to make it convenient for customers to shop.

Sample Spaces and Probability Models

In this example, we saw how Skylar used historical data to compute probabilities and to make a decision about rearranging his store. Skylar's example is a good illustration of some vocabulary that is commonly used when discussing probabilities.

When we compute probabilities, we often use the term event. The dictionary defined the word event as "a thing that happens." We use probabilities to measure the likelihood that "things that happen" (events) will occur. In Skylar's example, the "event" was a customer purchase of a certain number of items. We describe the probability for an event occurring using a sample space and a probability model.

*The **sample space** of an event is a list of all the possible outcomes of an event.*

In Skylar's case, the sample space was the list of possible outcomes when a customer makes a purchase. They could just purchase one item, or two items, or three items, and so on. But based on historical data, no one ever purchased more than 11 items. So Skylar's sample space was:


$$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}.$$

Notice that we describe sample spaces by making a list of the possible outcomes.

*A **probability model** for an event consists of the sample space along with the probability of each possible outcome.*

The tables (and bar charts) that Skylar made represented a probability model:

Items Purchased	1	2	3	4	5	6	7	8
Probability	0.37	0.22	0.14	0.07	0.04	0.03	0.05	0.02



This table is a probability model because it lists the sample space (all the possible outcomes) and the probability of each outcome.

Practice: Sample Spaces and Probability Models

Problem 3

Use the following probability model to answer the given probability question.

Event: For a new car, we observe how many years go by before the car requires its first repair.

Number of Years	1	2	3	4	5	6
Probability	0.05	0.10	0.13	0.24	0.31	0.17

The probability the first repair is needed in 3 years or less =

How Did I Do?

0.28 or 28% chance

Hint: Add the probabilities of the first repair happening in 1, 2, or 3 years.

The probability the first repair is needed in either 4 or 5 years =

How Did I Do?

0.55 or 55% chance.

Hint: Add the probabilities of year 4 and 5.

The probability the first repair is needed after 5 or more years =

How Did I Do?

0.48 or 48% chance.

Hint: Add the probabilities for years 5 and 6.

Empirical and Theoretical Probabilities

There are two types of probabilities that we can compute: empirical and theoretical.

An **empirical probability** is determined by historical data or by trying the event many times and computing how often the desired outcome occurs.

A **theoretical probability** is determined by using your knowledge of the situation and mathematical formulas to predict how often the desired outcome will occur in the future.

Empirical probabilities are computed based on what has happened. Theoretical probabilities are computed based on what could happen. In Skylar's case he used empirical probabilities because he based his calculations on what customers had purchased in the past.

One common example of a theoretical probability is tossing a coin. When we toss a coin there are two possible outcomes: heads and tails. Because the coin is equally likely to land on either side, we know we have a 50% chance of getting heads and a 50% chance of getting tails. This is a theoretical probability because instead of tossing a coin repeatedly and seeing what happens, we used the structure of the coin to determine the likelihood of each outcome.

When computing empirical probabilities it is important to have a very large number of trials. For example, let's toss a coin 10 times and see what happens.



Notice we got 7 tails and 3 heads. This is not an unusual result, even for a equally weighted fair coin. However, if we used this outcome to compute an empirical probability we would say there is a 70% chance of getting tails and a 30% chance of getting heads. This isn't even close to the theoretical probability we would expect.

In order for an empirical probability to be accurate, it must be based on a very large number of trials.

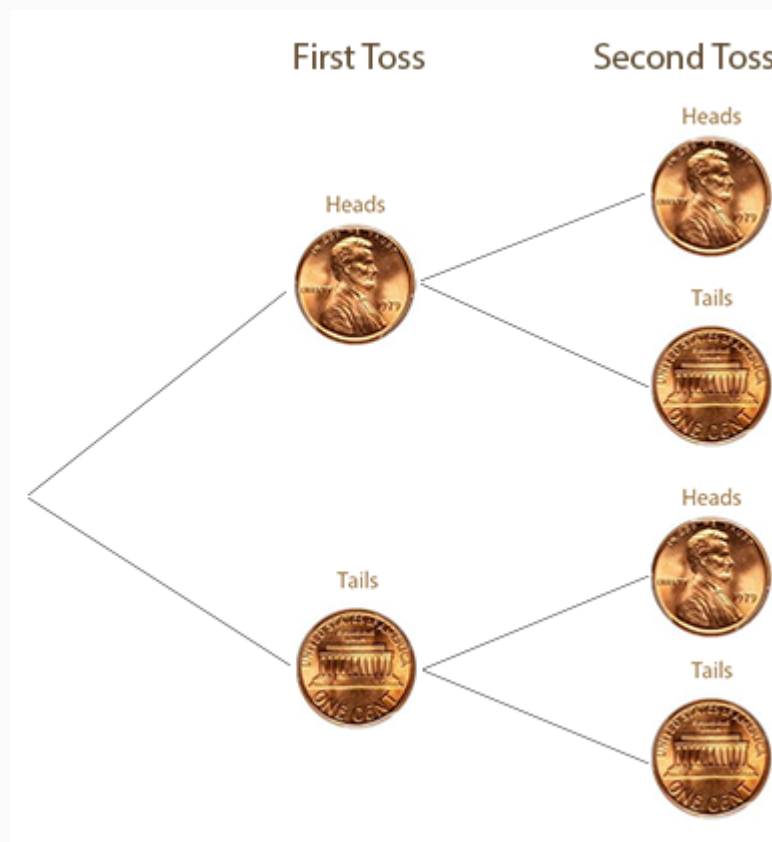
Simple, classic examples, like coins and dice can help us learn about probability.

Example 2 – Tossing a Coin

Consider an event where you toss two coins and look at whether they are heads or tails. Give the sample space and a probability model for this situation. Determine if you used theoretical or empirical probabilities.

Solution:

When we toss the two coins we can represent the possible outcomes with the following tree diagram:



Sample Space:

The sample space of this event is a list of all possible outcomes. Looking at our tree diagram, the four branches of the tree tell us the possible outcomes. If we use H for heads and T for tails, we could write the sample space as:

$$S = \{HH, HT, TH, TT\}$$

Probability Model:

Since there are four possible outcomes that are each equally likely to occur, we assign them each a probability of 1 out of 4, or $\frac{1}{4} = 0.25$. The probability model could be represented by the following table:

Possible Outcome	HH	HT	TH	TT
Probabilities	0.25	0.25	0.25	0.25

What type of probability:












The probabilities in our probability model are theoretical probabilities because we computed them based on our understanding of what could occur when we toss a coin twice.

Example 3 – Rolling Dice

Consider an event where you roll a pair of dice and observe the sum of the numbers on the two dice. Give the sample space and a probability model for this situation. Describe the difference between an empirical and a theoretical probability in this situation.

Solution

When we toss the two dice and add the numbers on the dice, we can represent what could happen with the following chart:

Sums of 2 Dice						
	2	3	4	5	6	7
	3	4	5	6	7	8
	4	5	6	7	8	9
	5	6	7	8	9	10
	6	7	8	9	10	11
	7	8	9	10	11	12

Sample Space:

The sample space of this event is a list of all possible outcomes. Looking at our chart we can see that the sum of the dice can be anything between 2 and 12. We can write the sample space as:

$$S = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}.$$

Probability Model:

There are 36 possible outcomes that are each equally likely to occur. Because they are equally likely, we can count how many times each sum occurs to build our probability model.

Possible Outcome	2	3	4	5	6	7	8	9	10	11	12
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Probabilities	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$
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We can also write the probabilities as decimals instead of fractions. This gives us the same probability model represented in a slightly different way.

Possible Outcome	2	3	4	5	6	6	8	9	10	11	12
Probabilities	0.028	0.056	0.083	0.111	0.139	0.167	0.139	0.111	0.083	0.056	0.028

From this chart, we see the most likely outcome is a sum of 7 which occurs 16.7% of the time. The least likely outcomes are 2 and 12, which each occur only 2.78% of the time.

What type of probability?:

The probabilities in our probability model are theoretical probabilities because we computed them based on our understanding of what *could* occur when we toss two dice

Customer Ratings

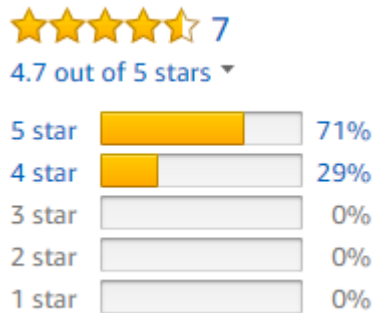
When we looked at empirical probabilities we saw the importance of basing our probabilities on a large number of trials. Let's return to the example of customer ratings given in the introductory video. Lucas and Amanda were shopping for headphones for a birthday gift. They used customer ratings to compare between different pairs of headphones. Many people look at customer ratings when they make a purchase online. These ratings allow us to look at information provided by a large number of previous customers and use that information to make a decision about what to purchase. These ratings are a type of empirical probability based on historical data.



Lucas and Amanda were looking at ratings similar to the one shown above. Notice that 71% of the customers rated Item #1 with 5 stars. But there is variability in the ratings. Not everyone had the same opinion about this item. Some people gave it a 5 star rating, while other people gave it a 1 star rating. There is a big difference between 1 star and 5 star ratings. Because the most common rating was 5 stars, we probably trust the five star rating most. In this example, 71% is an empirical probability based on asking 2798 people their opinion of the item.

Compare the customer ratings for Item #1 shown above with the rating for Item #2 shown below.

Item #2



Notice that the both items have 71% of individuals giving a 5-star rating. However, we would be less confident in the rating given for Item #2. The first item was rated by 2,798 people while the second item was only rated by 7 people. We can trust information more when it is confirmed by large number of people with the same opinion.

This example shows that we can't just look at the percentage of people who gave a 5-star rating. We also need to consider variability in the ratings and the number of people who gave a rating. This is important anytime we look at statistics. We can take variability and the number of trials into account by using a margin of error.

Estimating with Confidence

In the introductory video we heard Craig, Lucas, and Amanda having a conversation about a political poll. A news report stated that 51% of the voters were going to vote for Andrews. But this percentage was based on a sample. The news agency had not contacted all registered voters and asked them how they planned to vote. They only contacted some of them. The news agency knew the answer based on their sample could be different from the true percentage for all the registered voters, the entire population. So in addition to giving the percentage of voters planning to vote for Andrews, the news report also gave a margin of error.

Statisticians distinguish between two types of numbers: "parameters" and "statistics".

*A **parameter** is a number based on a survey of the entire population.*

*A **statistic** is a number based on a survey of just a sample of the population.*

The percentage reported in the news "51% of voters" is an example of a *statistic*. It is based on a sample of the population, so it is a statistic rather than a parameter. Statistics are used because it is difficult, time consuming, and expensive to contact every individual in a large population. Instead, just a sample of the population is contacted. We assume that the value of a statistic is not exactly equal to the true percentage (parameter) from the population. This is where the margin of error comes in.

*Statistics are used to estimate the true percentage (the parameter) for the whole population. The **margin of error** helps us measure how far our statistic might be from the true percentage (the parameter).*

In the introductory video Lucas said the margin of error for the voter study was 3%. By first subtracting the margin of error from the percentage found in the sample, then adding the margin of error to the percentage found in the sample, we obtain an interval that we can be quite confident includes the true percentage (the parameter) for the whole population. We call this interval of numbers a *confidence interval*.

95% Confidence Intervals

News organizations and polling companies often report a statistic along with the margin of error. We can use this information to find the confidence interval.

$$\text{confidence interval} = \text{the statistic} \pm \text{margin of error}$$

Use this formula to find the confidence of intervals for the political poll in the introductory video.

Practice: 95% Confidence Intervals

Problem 5



Lucas: "It says 51% of voters are going for Andrews, and only 49% are for Garcia."

Amanda: "But what's the margin of error?"

Lucas: "The article says it's 3%. Does that make a difference?"

Use the above information from the introductory video to compute confidence intervals for the percentage of voters planning to vote for each candidate.

- 95% Confidence Interval for Andrews: Between and
- 95% Confidence Interval for Garcia: Between and

How Did I Do?

95% Confidence Interval for Andrews: Between 48% and 54%.

95% Confidence Interval for Garcia: Between 46% and 52%.

When reading this type of news report, it is important to realize that the percentage provided is a statistic, not the true percentage for the whole population (not the parameter). The following examples demonstrate how to convert a published statistic and margin of error into a confidence interval.

Example 4 – Grocery Bags

A Gallup poll [3](#) conducted the week of March 1-5, 2017 asked individuals: “Which of these does LESS harm to the environment: using paper shopping bags or using plastic shopping bags?”

Give a 95% confidence interval for the percentage of U.S. adults who believe paper bags are better for the environment than plastic bags. Interpret the meaning of the confidence interval.

Solution:

Gallup reported that they asked this question to 1018 U.S. adults, age 18 and older and 73% said paper shopping bags were less harmful to the environment.

The 73% number is a **statistic** because it was computed by talking to a sample of adults living in the United States (if Gallup had asked all adults in the U.S. this question, then the number would be a parameter). Gallup stated: “The margin of sampling error is 4 percentage points at the 95% confidence level.”

We can use this margin of error to compute the 95% confidence interval:

$$\begin{aligned}\text{confidence interval} &= \text{the statistic} \pm \text{the margin of error} \\ &= 73\% \pm 4\% \\ &= (73\% - 4\%, 73\% + 4\%) \\ &= (69\%, 77\%)\end{aligned}$$

This shows our 95% confidence interval is (69%, 77%) This means we are 95% certain that the true percentage of adults in the United States who think using paper grocery bags is better for the environment than using plastic grocery bags is between 69% and 77%

Example 5 – Bathroom Cleanliness

The Bradley Corporation conducted a study⁴ on customers' views of the connection between bathroom cleanliness in a business and the quality of the business' products and services. Based on a sample of 1042 American adults, they found "that 88% [of American adults] believe that if a restaurant has unclean restrooms, the likelihood is that the kitchen is also unclean". The margin of error in the study was 3%.

Find a 95% confidence interval for the percentage of Americans who believe there is a connection between the cleanliness of a bathroom and a kitchen in a restaurant. Interpret the meaning of the confidence interval.

Solution:

The 88% number reported by the Bradley Corporation is a statistic because it was computed by talking to a sample of American adults (if they had asked all American adults this question, then their number would be a parameter).

We can use the margin of error to compute a 95% confidence interval:

$$\begin{aligned}\text{confidence interval} &= \text{the statistic} \pm \text{the margin of error} \\ &= 88\% \pm 3\% \\ &= (88\% - 3\%, 88\% + 3\%) \\ &= (85\%, 91\%)\end{aligned}$$

This shows our 95% confidence interval is (85%, 91%). This means we are 95% certain that the true percentage of American adults who think there is a connection between the cleanliness of a bathroom and a kitchen in a restaurant is between 85% and 91%. This provides good statistical evidence that unclean restrooms can give a negative message about a restaurant.

The 88% number reported by the Bradley Corporation is a statistic because it was computed by talking to a sample of American adults (if they had asked all American adults this question, then their number would be a parameter).

We can use the margin of error to compute a 95% confidence interval:

$$\begin{aligned}\text{confidence interval} &= \text{the statistic} \pm \text{the margin of error} \\ &= 88\% \pm 3\% \\ &= (88\% - 3\%, 88\% + 3\%) \\ &= (85\%, 91\%)\end{aligned}$$

This shows our 95% confidence interval is

$$(85\%, 91\%)$$

This means we are 95% certain that the true percentage of American adults who think there is a connection between the cleanliness of a bathroom and a kitchen in a restaurant is between 85% and 91%. This provides good statistical evidence that unclean restrooms can give a negative message about a restaurant.

Example 6 – Technology Use

In 2016, the Pew Research Center conducted a study on the use of technology by U.S. adults age 65 and older⁵. They found that 42% of adults 65 and older own a smartphone. This is a significant increase from the results of a similar study conducted in 2011 that found that only 11% of adults 65 and older owned a smartphone. The margin of error in the 2016 study was 4.1%.

Find a 95% confidence interval for the percentage of U.S. adults age 65 and older who own a smartphone. Interpret the meaning of the confidence interval.

Solution:

The 42% number reported by the Pew study is a statistic because it was computed by talking to a sample of U.S. adults age 65 and older (if they had asked all U.S. adults age 65 and older this question, then the number would be a parameter).

We can use the margin of error to compute a 95% confidence interval:

$$\begin{aligned}\text{confidence interval} &= \text{the statistic} \pm \text{the margin of error} \\ &= 42\% \pm 4.1\% \\ &= (42\% - 4.1\%, 42\% + 4.1\%) \\ &= (37.9\%, 46.1\%)\end{aligned}$$

The 95% confidence interval is (37.9%, 46.1%). This means we are 95% certain that the true percentage of U.S. adults age 65 and older who own a smartphone is between 37.9% and 46.1%. Because this interval is higher than the data from the 2011 survey conducted 5 years prior to this study, we know that smartphone use among older Americans is increasing. However, this interval shows that still fewer than half of older Americans owned a smartphone in 2016.

Note: The percentage of U.S. adults age 65 and older who owned a smartphone in 2021 was up to 61%.⁶

Computing 95% Confidence Intervals

In the previous examples we were given the margin of error so we could use it to find the confidence interval. But what if we are not given a margin of error?

The formula we will use in this class is a simplified version of the formula used by statisticians, but it gives a good approximation and will help us gain a better understanding of confidence intervals and margin of error.

To estimate the percentage of a population with a certain characteristic based on a sample, we use the following formula. The variable x represents the number of people in the sample with the characteristic and the variable n represents the total number of people in the sample:

$$\text{Confidence Interval} = \frac{x}{n} \pm \frac{1}{\sqrt{n}}$$

The following examples show how to use this formula to compute a confidence interval.

Example 7 – Election Results

A school district is holding a special election to pass a school bond. The purpose of the bond is to raise construction funds to pay for a new elementary school. The bond would be paid for by collecting additional property taxes over the next 10 years.

To assess support for the bond, the local newspaper asks a sample of 200 registered voters how they plan to vote on the bond. Seventy-two of those in the sample indicate that they plan to vote in favor of the bond. Find and interpret a 95% confidence interval for this situation.

Solution:

The sample was of 200 registered voters so $n = 200$

Seventy-two people said they would vote in favor of the bond so $x = 72$

Using the confidence interval formula we get:

$$\begin{aligned}\text{Confidence Interval} &= \frac{x}{n} \pm \frac{1}{\sqrt{n}} \\ &= \frac{72}{200} \pm \frac{1}{\sqrt{200}} \\ &= 0.36 \pm 0.07 \\ &= (0.36 - 0.07, 0.36 + 0.07) \\ &= (0.29, 0.43)\end{aligned}$$

This confidence interval tells us we are 95% certain that the true percentage of voters who will vote in favor of the bond is between 29% and 43%. From this sample, it appears that the bond will fail to be approved.

Example 8 – Potential Customers

Lisa and Conner are a married couple who want to start a new restaurant in Rexburg. To determine if their restaurant would be successful, they decide to conduct a survey to see what percentage of Rexburg residents would visit their restaurant at least once a month.

They surveyed 110 Rexburg residents and 27 of them indicated they would visit the new restaurant at least once per month. Find and interpret a 95% confidence interval for this situation.

Solution:

The sample was 110 potential customers so $n = 110$.

Twenty-seven people said they would visit at least once per month so $x = 27$.

Using the confidence interval formula we get:

$$\begin{aligned}\text{Confidence Interval} &= \frac{x}{n} \pm \frac{1}{\sqrt{n}} \\ &= \frac{27}{110} \pm \frac{1}{\sqrt{110}} \\ &= 0.245 \pm 0.095 \\ &= (0.245 - 0.095, 0.245 + 0.095) \\ &= (0.15, 0.34)\end{aligned}$$

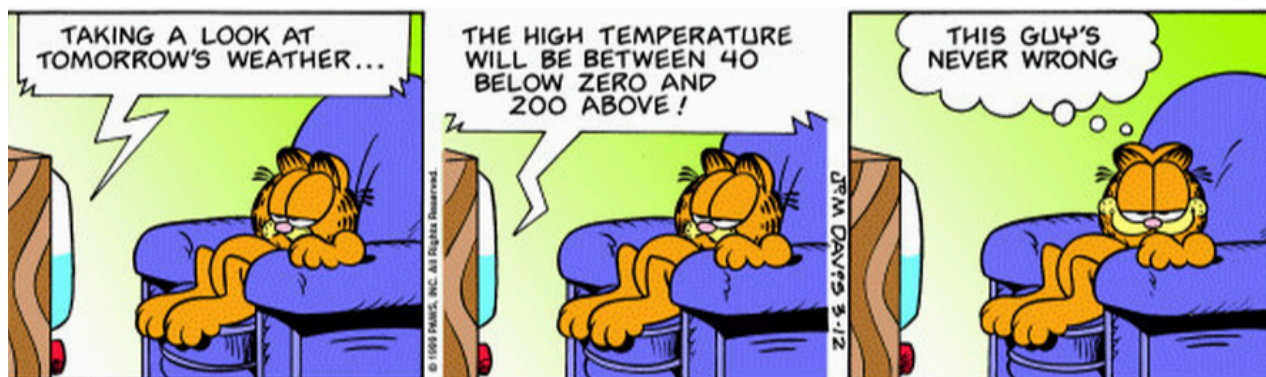
This confidence interval tells us we are 95% certain that the true percentage of Rexburg residents who will visit the new restaurant at least once per month is between 15% and 34%. The confidence interval gives Lisa and Conner an estimate of how many monthly customers to expect.

Sample Size and Margin of Error

The margin of error is used to give us a better understanding of how close we expect a statistic to be to the true percentage (parameter) it is designed to estimate.

If you have a very large margin of error, it is easy to make an accurate prediction. For example, without even doing any mathematical calculations it is easy to predict that the percentage of BYU-Idaho students who are from Idaho is between 0% and 100%.

Here is another example from a Garfield cartoon⁷.

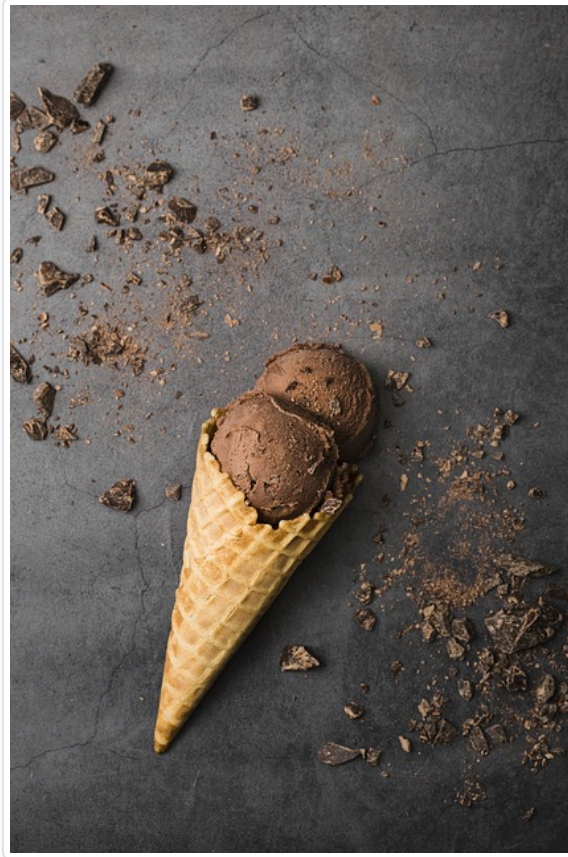


In order to be useful, confidence intervals need to be narrow, which means the margin of error needs to be small. One way to shrink the margin of error is to increase the sample size. Notice that the $\frac{1}{\sqrt{n}}$ in the 95% confidence interval formula tells us the margin of error. Let's compute the margin of error for several different sample sizes:

Sample Size (n)	Margin of Error $\left(\frac{1}{\sqrt{n}}\right)$
10	32 percentage points
100	10 percentage points
500	4.5 percentage points
1000	3.1 percentage points
5000	1.4 percentage points

Notice that the bigger the sample size, the smaller the margin of error. This makes sense because the more individuals you survey, the better sense you have for what is going on in the population. Although collecting data from a larger sample improves the margin of error, it also makes the survey more expensive and time consuming to complete. When designing a survey we have to balance the need to sample enough people to adequately reduce the margin of error while still keeping the size of the sample manageable.

Example 9 – Ice Cream

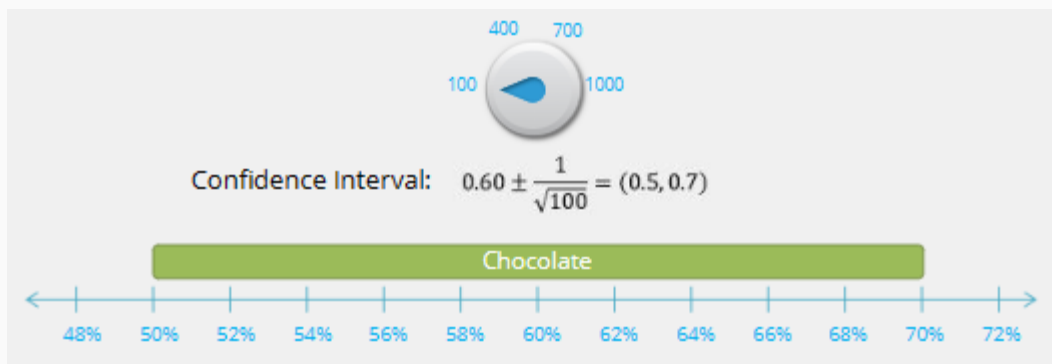


You ask a sample of individuals the question:

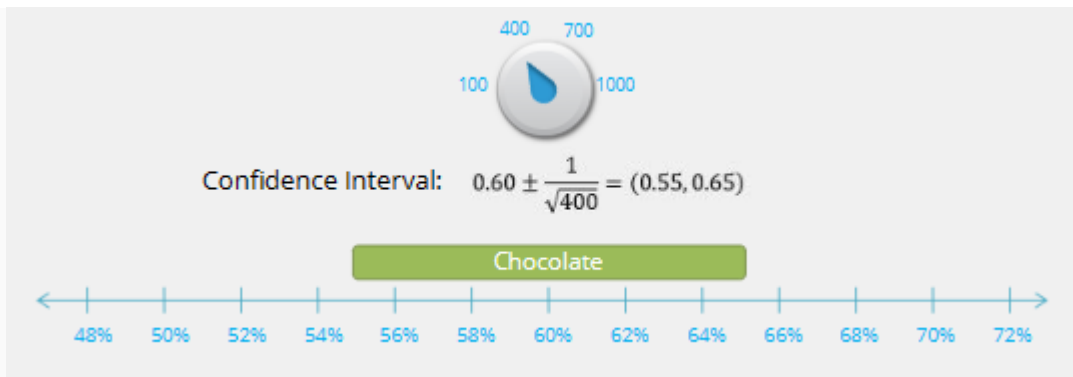
“What is your favorite flavor of ice cream?”

Sixty percent of the individuals in your sample said their favorite flavor of ice cream was chocolate. The following shows how the 95% confidence interval (in green) changes according to the sample size.

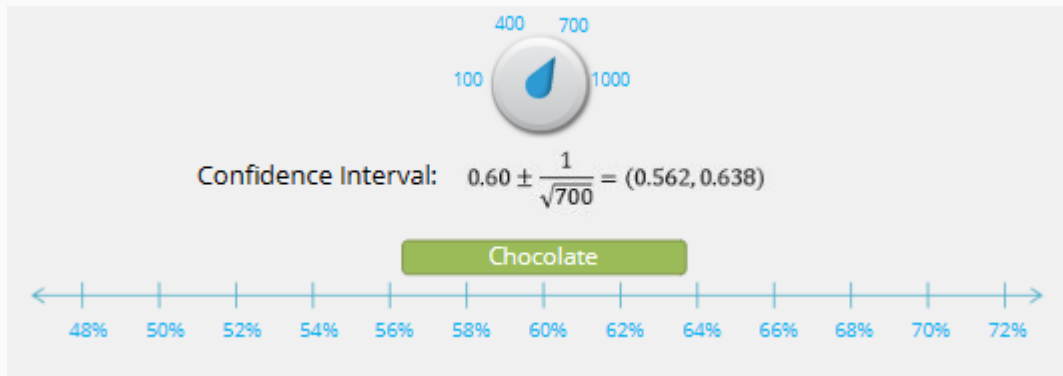
Sample size of 100:



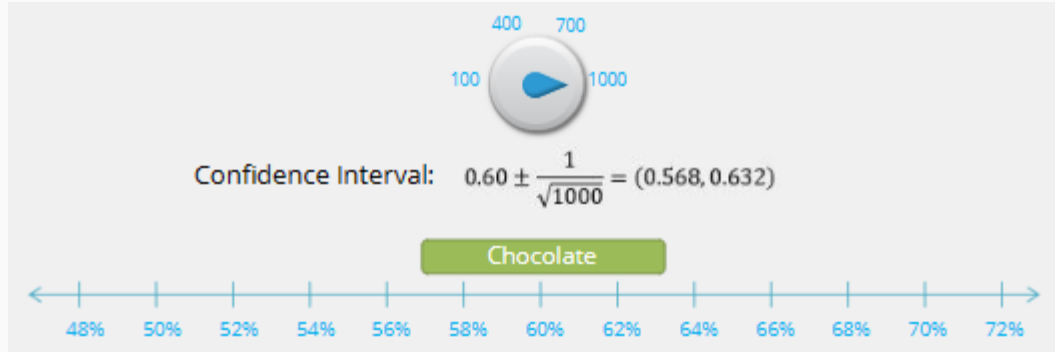
Sample size of 400:



Sample size of 700:



Sample size of 1000:



Comparing Confidence Intervals

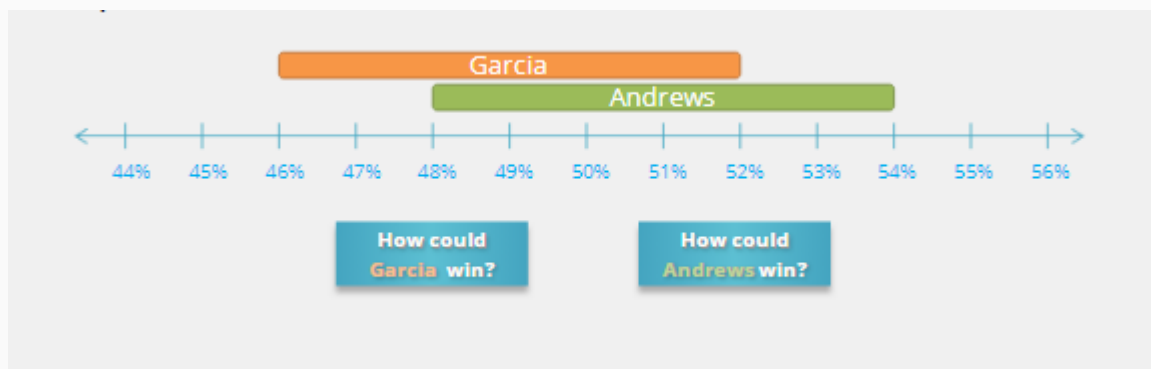
In the introductory video, we heard Craig explain to Lucas and Amanda why the margin of error mattered in a political poll. The follow explanation expands on that example.

Example 10 – Elections

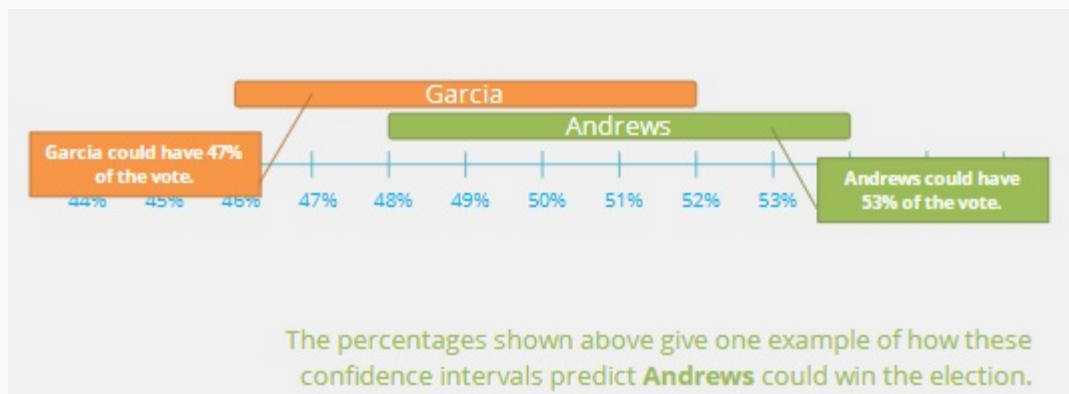


Craig: "Well, with a margin of error of 3%, that just means anywhere between 48% and 54% of voters will choose Andrews and 46 to 52% will go with Garcia. So really it's anyone's game."

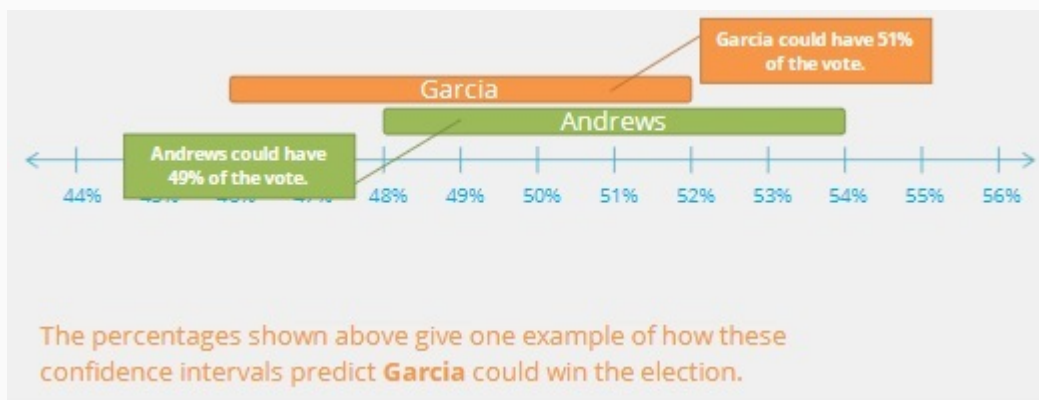
The following graphic shows the two confidence intervals we computed represented on a number line.



Example of how Andrews could win



Example of how Garcia could win



From this example, we see that when two statistics are close to one another, we might not have enough evidence to say the true values for the population (the parameters) are different from one another.

Here is another example:

Example 11 – Well-being and Student Loans

The Gallup Poll organization conducted a poll [8](#) to determine the long-term effects of student loans on the well-being of college graduates. They based their information on a survey of 29,560 college graduates who graduated between 1990 and 2014. Gallup stated the study had a margin of error of 1.4% at the 95% confidence level.

Based on the results of their study, Gallup compared the well-being of graduates in five areas: purpose, social, financial, community, and physical. The following chart gives the percentage of graduates who were “thriving” in each of these five areas based on the amount of student loan debt they had at the time of graduation.

Percentage of U.S. College Graduates Thriving in Five Elements of Well-Being, by Amount of Student Loan Debt (Among adults who graduated between 1990 and 2014)

	No student debt	\$25,000 and below	\$25,001 to \$50,000
Purpose	49	46	40
Social	47	45	42
Financial	40	31	26
Community	43	42	35
Physical	34	30	26

Use the above information to answer the following questions:

a. Is there a statistical difference between the percentage of graduates with no student loan debt who are thriving *financially* and the percentage of graduates with over \$50,000 in student loan debt who are thriving financially? Explain.

b. Is there a statistical difference between the percentage of graduates with no student loan debt who are thriving *socially* and the percentage of graduates with over \$50,000 in student loan debt who are thriving socially? Explain.

Solution (part a, *financially*):

No Student Loan Debt

Among graduates in the survey with no student loan debt, 40% were thriving financially. Because we know the margin of error was 1.4%, we can find the confidence interval:

$$\begin{aligned}\text{confidence} &= 40\% \pm 1.4\% \\ &= (40 - 1.4, 40 + 1.4) \\ &= (38.6, 41.4)\end{aligned}$$

We are 95% confident that the true percentage of graduates with no student loan debt who are thriving financially is between 38.6% and 41.4%.

Over \$50,000 in Student Loan Debt

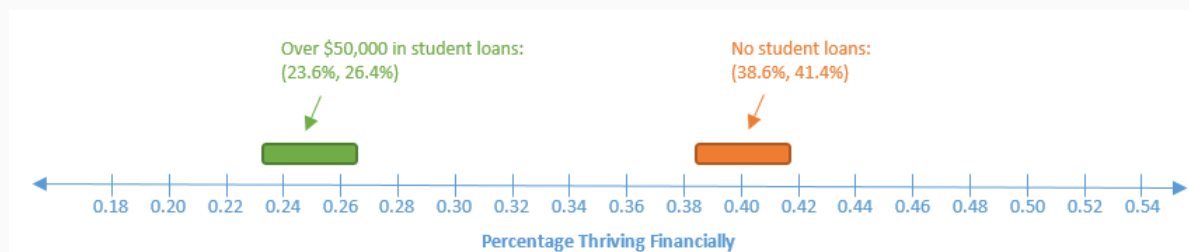
Among graduates in the survey with over \$50,000 in student loan debt, 25% were thriving financially. Because the margin of error was 1.4%, we can find the confidence interval:

$$\begin{aligned}\text{confidence} &= 25\% \pm 1.4\% \\ &= (25 - 1.4, 25 + 1.4) \\ &= (23.6, 26.4)\end{aligned}$$

We are 95% confident that the true percentage of graduates with over \$50,000 in student loan debt who are thriving financially is between 23.6% and 26.4%.

Is there a difference?

Let's graph these intervals on a number line:



Because these two confidence intervals do not overlap, there is enough statistical evidence to conclude that the percentage of students who are thriving financially is greater among students with no student loan debt than it is among students with over \$50,000 in student loan debt.

Solution (part b, *socially*):

No Student Loan Debt

Among graduates in the survey with no student loan debt, 47% were thriving socially. Because we know the margin of error was 1.4%, we can find the confidence interval:

$$\begin{aligned}\text{confidence} &= 47\% \pm 1.4\% \\ &= (47 - 1.4, 47 + 1.4) \\ &= (45.6, 48.4)\end{aligned}$$

We are 95% confident that the true percentage of graduates with no student loan debt who are thriving socially is between 45.6% and 48.4%.

Over \$50,000 in Student Loan Debt

Among graduates in the survey with over \$50,000 in student loan debt, 45% were thriving socially. Because the margin of error was 1.4%, we can find the confidence interval:

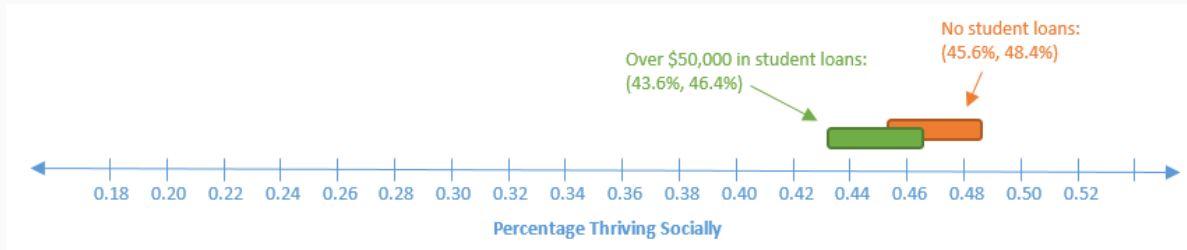
$$\begin{aligned}\text{confidence} &= 45\% \pm 1.4\% \\ &= (45 - 1.4, 45 + 1.4)\end{aligned}$$

$$= (43.6, 46.4)$$

We are 95% confident that the true percentage of graduates with over \$50,000 in student loan debt who are thriving socially is between 43.6% and 46.4%.

Is there a difference?

Let's graph these intervals on a number line:



These two confidence intervals *do* overlap. This means we cannot say there is a statistical difference in the percentage of students who are thriving socially between the two groups.

As these examples show, we have to be careful when comparing statistics obtained from a sample. It is easy to mistakenly conclude that because one statistic is larger than another statistic, the corresponding parameter is also larger than the other parameter. If the confidence intervals overlap, we do not have enough statistical evidence to say the parameters are different from one another.

Practice: Unemployment Rates

Problem 6

The governor of a state wants to compare the employment rates in two large cities in his state. He conducts a survey of the two cities. Here are the results he obtains:

City	Number of Individuals in the Sample	Number of Individuals in the Sample who are Unemployed
City A	642	45
City B	918	53

Find the 95% confidence interval for the percentage of individuals in each city who are unemployed.

- City A: Between and .
- City B: Between and .

Please give each answer as a number between 0 and 1. Round to two decimal places.

How Did I Do?

City A: Between 0.03 and 0.11.

City B: Between 0.02 and 0.09.

Hint : confidence interval $= \frac{x}{n} \pm \frac{1}{\sqrt{n}}$

Is there enough evidence to say the two cities have different unemployment rates?

- ☐ Yes. The true percentages are different
- ☐ No. There is not enough evidence to say the true percentages are different

Explanation

Because the confidence intervals overlap, there is not enough evidence to say the true percentages are different.

Lesson Checklist

By the end of this lesson, you should be able to:

- Define probability.
 - For a given event, give the sample space of the event.
 - For a given event, give a probability model for the event.
 - Explain the difference between empirical and theoretical probabilities.
 - Create a 95% confidence interval for the percentage of individuals in a population who have a particular characteristic.
 - Interpret a confidence interval.
 - Determine whether two confidence intervals overlap and make an appropriate conclusion
-

¹ [Average College Graduate Salaries | Bankrate](#)

² [2022-23 NBA Player Stats: Totals | Basketball-Reference.com](#)

³ [Americans Say Paper Bags More Eco-Friendly Than Plastic \(gallup.com\)](#)

⁴ [Healthy Handwashing Survey - Bradley Corp](#)

⁵ [Tech Adoption Climbs Among Older Americans | Pew Research Center](#)

⁶ [Share of tech users among Americans 65 and older grew in past decade | Pew Research Center](#)

⁷ [Garfield by Jim Davis for March 12, 1999 - GoComics](#)

⁸ [Student Debt Linked to Worse Health and Less Wealth \(gallup.com\)](#)



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Lesson 13: Applying the Quantitative Reasoning Process

Opening Story

Lesson 13 - Opening Story

[L13 Opening Story Transcript](#)

Introduction

As we heard President Hinckley say in the introductory video: “You will be expected to put forth great effort and to use your best talents to make your way to the most wonderful future of which you are capable.”¹ As you are completing this Math 108X course, we hope that you can see how the Quantitative Reasoning Process can complement your talents and magnify your efforts to improve your own situation in life and bless the lives of those around you.

The purpose of the Quantitative Reasoning Process is to assist in making good decisions based on a careful consideration of the problem, an analysis of the variables and assumptions that could affect the outcome, and the use of quantitative tools.



Quantitative Tools

We have learned a variety of tools that can be used to make informed decisions in our personal lives and in our families, church, and community activities. As we conclude the semester, let's review the quantitative tools we have learned.

Quantitative Tools

(Click on each one)

Lesson 2 - Introduction to the Quantitative Reasoning Process

- Variables and Assumptions



Lesson 3 - Logical Reasoning

- Logical Reasoning
- Conditional Statements
- Logical Fallacies



Lesson 4 - Tools for Building Better Budgets and More

- English-to-Math translations
- Units and unit conversions
- Percentages
- Budgets



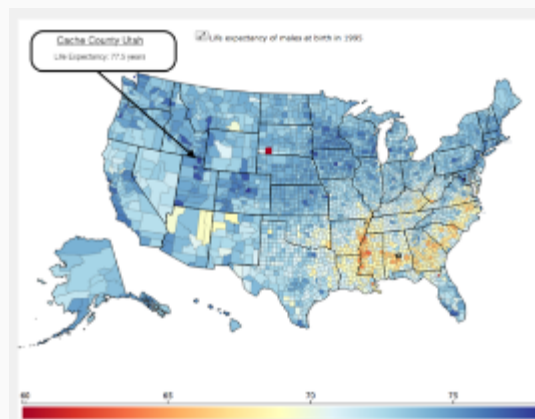
Lesson 5 - Summarizing Data

- Mean, Median, and Mode
- Maximum and Minimum
- Standard Deviation
- Percentiles



Lesson 6 - Visualizing Data

- Bar Charts
- Pie Charts
- Time Series Plots
- Scatter Plots



Lesson 7 - Functions in Excel

- The Payment (PMT) Function
- The Future Value (FV) Function
- Amortization Schedules
- Savings and Loans

Workbook1.xlsx

Home Layout Tables Charts SmartArt Formulas Data

Font Paragraph Alignment Number Styles Cells

File Home Insert References Send To Layout Styles Formulas Data Review View

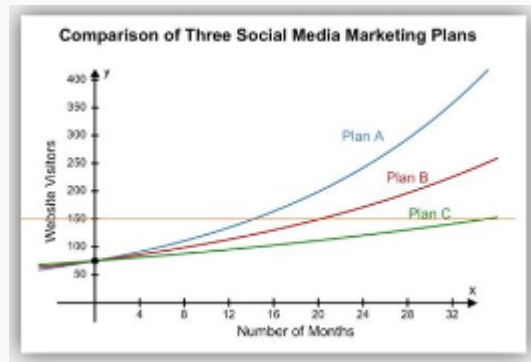
Formulas

Amortization Schedule

Month	Beginning Balance	Payment	Interest	Principal	Ending Balance
1	\$5,000.00	\$150.00	\$166.67	\$333.33	\$4,666.67
2	\$4,666.67	\$150.00	\$160.00	\$340.00	\$4,326.67
3	\$4,326.67	\$150.00	\$153.33	\$346.67	\$3,980.00
4	\$3,980.00	\$150.00	\$146.67	\$353.33	\$3,626.67
5	\$3,626.67	\$150.00	\$140.00	\$360.00	\$3,266.67
6	\$3,266.67	\$150.00	\$133.33	\$366.67	\$2,900.00
7	\$2,900.00	\$150.00	\$126.67	\$373.33	\$2,526.67
8	\$2,526.67	\$150.00	\$120.00	\$380.00	\$2,146.67
9	\$2,146.67	\$150.00	\$113.33	\$386.67	\$1,760.00
10	\$1,760.00	\$150.00	\$106.67	\$393.33	\$1,366.67
11	\$1,366.67	\$150.00	\$100.00	\$400.00	\$966.67
12	\$966.67	\$150.00	\$93.33	\$406.67	\$560.00
13	\$560.00	\$150.00	\$86.67	\$413.33	\$146.67
14	\$146.67	\$150.00	\$80.00	\$420.00	\$0.00

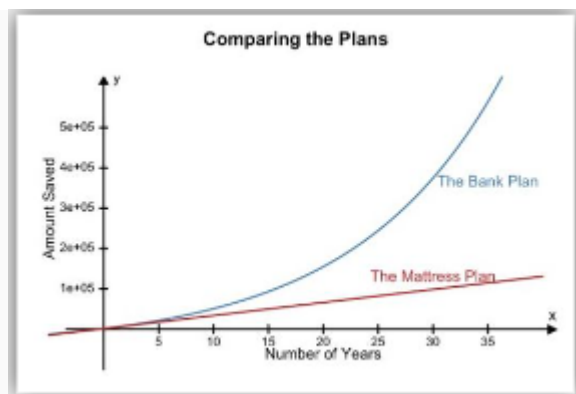
Lesson 8 - Functions and Their Graphs

- Order of Operations
- Rules of Exponents
- Linear Functions
- Quadratic Functions
- Exponential Functions
- Slope and Intercept
- Vertex, Direction, and Asymptote
- Graphs of Functions



Lesson 9 - Growth Over Time

- Constant and Variable Rates of Change
- Compound Interest



Lesson 10 - Making Predictions from Data

- Trendlines
- Predictions



Lesson 11 - Solving Equations

- Solving Systems of Equations with Graphs
- Solving Systems of Equations with Algebra
- Solving Systems of Equations from Trendlines



Lesson 12 - Probability and Confidence Intervals

- Probability Models
- Parameters and Statistics
- 95% Confidence Intervals
- Margin of Error



This list is just the beginning of the quantitative tools that can be used to make decisions. As you continue your studies, you will learn more specific tools used in your chosen career. Adding more quantitative tools will increase the usefulness of the Quantitative Reasoning Process in making informed decisions in all areas of your life.

Foresight and Follow-Through

This week's lesson focuses on the importance of foresight and follow-through. As we use the Quantitative Reasoning Process to make decisions, it is vital to consider the problem carefully and attempt to foresee the possible obstacles you might encounter. After making a decision it is essential to follow-through with the actions that are part of that decision.

A Lack of Foresight

In Lesson 11 we saw an example of a lack of foresight. Remember that Sophia and Emmy made a decision to take a motor boat to get back to their hotel from the Amazon jungle. As they used the Quantitative Reasoning Process they failed to consider the fact that the motor boat would not take them all the way to their hotel. This lack of foresight led to them making a less-than-desirable decision as they had to walk 5 extra kilometers to get back to their hotel. Hopefully, Sophia and Emmy, like the rest of us, will learn from their lack of foresight and be better prepared to foresee similar obstacles in the future.

While it is impossible to consider all future obstacles we might encounter, the Quantitative Reasoning Process helps us spend time considering variables and assumptions. We try to consider which assumptions might not be true, and consider unexpected outcomes that might arise.

The following case study is an example where lack of foresight affected the quality of the decision made through the Quantitative Reasoning Process.

1 Understand the Problem

Johanne loves the thought of traveling around the United States. Because travel can be expensive, a year ago she decided to make it a priority and put a travel category in her monthly budget. After carefully saving and planning, she

now has \$1,300 she can spend for a trip. Johanne is lives in South Carolina and is deciding among three options:

- A trip with three friends to visit the Oregon coast for 7 days.
- A trip alone to visit Washington D.C. for 10 days.
- A trip with her cousin to visit relatives in Miami, Florida for 5 days.

2 Identify Variables & Assumptions

Johanne identifies the following variables:

- Cost of travel
- Cost of lodging
- Cost of activities
- Cost of food
- Her interest in the location

She assumes that she will strictly follow whatever budget she sets for herself. She also makes the following assumptions in comparing the three travel options:

Assumptions	Transportation	Lodging	Activities	
Oregon Coast	She would fly, then split the cost of a rental car with her friends.	Inexpensive, because they will camp on the beach.	Inexpensive, because they are mostly planning to spend time hiking and visiting the beach.	She mea
Washington D.C.	She can drive from her home.	Expensive, because she will have to pay for a hotel room by herself.	Inexpensive, most of the locations she wants to visit are free.	She mea trip).
Miami, Florida	She can ride with her cousin and split the cost of gas.	Inexpensive, because she can split the cost of a hotel room with her cousin and stay with relatives on some nights.	Inexpensive, she will be spending much of her time with family.	Beca mea able \$15

3 Apply Quantitative Tools

Johanne does careful calculations of the costs for each potential trip. Based on her assumptions, she identifies the following costs:

Cost	Travel	Lodging	Activities	Food	Total
------	--------	---------	------------	------	-------

Oregon Coast	\$600	\$250	\$100	\$210	\$1160
Washington D.C.	\$150	\$1600	\$100	\$300	\$2150
Miami, Florida	\$80	\$150	\$100	\$75	\$405

She also considers where she would most like to travel. She decides her first choice is Washington D.C. because she has always wanted to visit the Capital. Her second choice is the Oregon Coast with friends and her last choice is the trip with her cousin. Although Johanne always loves spending time with her family, she has been to Miami many times in the past and wants to see someplace new. She also knows she will have opportunities to travel to Miami with her cousin in the future.

4 Make an Informed Decision

After reviewing her options, Johanne decides the cost of paying for her own hotel room in Washington D.C. makes it prohibitively expensive. Although the Florida trip is the least expensive, it is also her last choice.

She decides to use the money she has saved to travel to Oregon with her friends.

What are some things you think Johanne has failed to consider?

5 Evaluate Your Reasoning

As planning for the trip continues everything goes well. Johanne is excited to see the Oregon coast and is excited to travel with her friends. But as they arrive in Oregon, things begin to go differently from Johanne's plans:

- At the car rental agency, the friends want more space in the car so Johanne and her friends decide to rent a more expensive car.
- The weather is very cold and rainy. Camping on the beach seems miserable, so they they decide to stay in a hotel.
- Because of the rain Johanne's friends want to visit an aquarium and the Columbia River Maritime Museum. When it warms up they want to go on a whale watching excursion and a guided ocean kayaking trip. None of these activities were ones Johanne had planned to pay for.
- Johanne's friends enjoy eating at expensive restaurants and Johanne spends close to \$80 per day on food.

Although Johanne enjoyed the trip, she spent \$600 more than she planned to spend. This caused her added stress and she realized she could not afford to travel again for a long time. As she made her plans, Johanne did not think enough about her friends' spending behaviors and she did not ask them about their plans for activities on the trip. This lack of foresight made it hard to her to follow-through with her spending plan. It also affected her travel plans for the future. Hopefully, after reevaluating her assumptions and variables, she will use what she has learned to make better decisions about future trips.

The gospel gives us an important eternal perspective that is helpful in making informed decisions. In a BYU-Idaho devotional, President Russel M. Nelson noted: "You have been reserved for this particular time to accomplish your mission of mortality...You need to understand how really important you are in the eyes of your Maker. Most of you do not fully appreciate who you really are. And most of you do not fully foresee your future potential for greatness."² As we

begin to fully understand our divine potential and appreciate the Lord's plan for us, we can better include foresight in our life decisions.

A Lack of Follow-Through

After applying the Quantitative Reasoning Process to make an informed decision, your work will go to waste if you don't follow-through with the actions and the decision you made. Follow-through usually requires discipline, self-constraint, and financial modesty. As we live by the decisions we make through the Quantitative Reasoning Process we reap benefits from our informed decision-making process.

The following case study is an example where lack of follow-through affected the quality of the decision made through the Quantitative Reasoning Process.

1 Understand the Problem

Laura and Ian were recently married and just moved into a new apartment with no furniture. After shopping at local thrift stores they have found a few things to furnish their apartment, but they haven't found a bed or a couch. They decide to spend some extra money to purchase a high quality bed and a comfortable couch that will last for a long time. The local furniture store is offering special financing. If Laura and Ian are approved for the store financing, they pay nothing at the time of the purchase and they won't have a payment due for six months.

2 Identify Variables & Assumptions

Laura and Ian identify the following variables:

- The cost of the new bed
- The cost of the new couch
- The total loan amount
- The loan interest rate
- The total time to repay the loan

They make the following assumption:

- They can afford to make the minimum monthly payment.
-

3 Apply Quantitative Tools

The total bill for the couch, bed frame, and mattress they select, including taxes and the delivery fee, is \$2435.31. Laura and Ian know that taking on debt can have big consequences. After asking more questions about the furniture store financing, they learn the interest rate is 22% and the minimum monthly payment is \$50. They also learn that interest begins to accrue at the time of the purchase. This means that interests accrues during the first six months, even though they aren't required to make payments.

They use Excel to create the following Amortization Schedule for the store financing:

Month	Beginning Balance	Interest Accrued	Payment	Ending Balance
0	\$ 2,435.31	\$ 44.65	-	\$ 2,479.96
1	\$ 2,479.96	\$ 45.47	-	\$ 2,525.42
2	\$ 2,525.42	\$ 46.30	-	\$ 2,571.72
3	\$ 2,571.72	\$ 47.15	-	\$ 2,618.87
4	\$ 2,618.87	\$ 48.01	-	\$ 2,666.88
5	\$ 2,666.88	\$ 48.89	-	\$ 2,715.78
6	\$ 2,715.78	\$ 49.79	-	\$ 2,765.57
7	\$ 2,765.57	\$ 50.70	\$ 50.00	\$ 2,766.27
8	\$ 2,766.27	\$ 50.71	\$ 50.00	\$ 2,766.98
9	\$ 2,766.98	\$ 50.73	\$ 50.00	\$ 2,767.71
10	\$ 2,767.71	\$ 50.74	\$ 50.00	\$ 2,768.45
11	\$ 2,768.45	\$ 50.75	\$ 50.00	\$ 2,769.21
12	\$ 2,769.21	\$ 50.77	\$ 50.00	\$ 2,769.98
13	\$ 2,769.98	\$ 50.78	\$ 50.00	\$ 2,770.76
14	\$ 2,770.76	\$ 50.80	\$ 50.00	\$ 2,771.56
15	\$ 2,771.56	\$ 50.81	\$ 50.00	\$ 2,772.37
16	\$ 2,772.37	\$ 50.83	\$ 50.00	\$ 2,773.19
17	\$ 2,773.19	\$ 50.84	\$ 50.00	\$ 2,774.04
18	\$ 2,774.04	\$ 50.86	\$ 50.00	\$ 2,774.89
19	\$ 2,774.89	\$ 50.87	\$ 50.00	\$ 2,775.77
20	\$ 2,775.77	\$ 50.89	\$ 50.00	\$ 2,776.66
21	\$ 2,776.66	\$ 50.91	\$ 50.00	\$ 2,777.56
22	\$ 2,777.56	\$ 50.92	\$ 50.00	\$ 2,778.48
23	\$ 2,778.48	\$ 50.94	\$ 50.00	\$ 2,779.42
24	\$ 2,779.42	\$ 50.96	\$ 50.00	\$ 2,780.38

As they looked at the amortization schedule, they noticed that during the first 6 months their account balance would go up from \$2435.31 to \$2765.57. This is an increase of more than \$330. They also noticed that by month 7, the interest payments were slightly higher than the minimum payment of \$50. If they only paid the minimum payment, they would never pay off the loan. After two years they would have paid \$900, but would still owe \$2780.38. They noticed that even after making payments for two years, they would still owe more than they originally borrowed.

Laura and Ian decide the store's financing offer was not a good choice. They had a personal credit card with a lower interest rate of 12% that they could use. Using the payment (PMT) function in Excel they found that if they charged the new bed and couch to their credit card and made monthly payments of \$114.64 they would have the furniture paid off in two years.

	A	B	C	D	E	F	G
1	Inputs						
2	Loan Amount	\$ 2,435.31					
3	Interest Rate	12%					
4	Years	2					
5							
6	Output						
7	Payment	(\$114.64)					
8							
9							
10							
11							

4 Make an Informed Decision



Based on their calculations, Laura and Ian decided to purchase the new furniture using their credit card. They knew they made a wise decision not to use the store financing and were confident in their ability to pay off the credit card. They felt sure the lower interest rate on their credit card would save them a significant amount of money over the interest they would have paid for the store financing.

They added monthly furniture payments of \$114.64 to their budget.

5 Evaluate Your Reasoning



In the first few months, as Laura and Ian evaluated their reasoning, they were pleased with their decision.

Their use of the Quantitative Reasoning Process and their decision to avoid the store financing were good choices.

However, as time progressed, they did not follow-through with their decision. Here is a summary of what happened in the first year after they made the purchase:

- For the first two months, they faithfully made the \$114.64 payments on their credit card.
- Month #3 was December and they used their credit card to purchase Christmas gifts. In addition to adding to the card balance, they only made the minimum payment on the card instead of the \$114.64 they had planned.
- In January Laura noticed that Ian loved the amazing new TV their neighbors got for Christmas. Laura purchased the same TV for Ian's birthday gift. It was even on sale. She charged the entire cost of the TV to the credit card.
- In February they overspent on eating out and a really expensive Valentine's day dinner. They were only able to make the minimum payment on their credit card.
- In March they realized they were overspending and were careful to not add any new charges to the card. They made the planned payment of \$114.64.
- In April, Ian used the credit card to purchase an expensive birthday gift for Laura. They only made the minimum payment on their credit card.
- For the rest of the year, they continued their pattern of overspending, not following their budget, and making only minimum payments on their credit card.

Over the first year, the balance on their credit card grew to \$4,537. This was over the credit limit on the card and they were no longer able to use the card to make purchases. Because of the high balance, the minimum payment on the card increased. Laura and Ian could no longer even make the minimum payment each month.

Laura and Ian found themselves in an unfortunate situation with long-term consequences. In addition to financial consequences, like lowering their credit scores, their money situation brought added stress in their marriage. Their uncontrolled debt situation caused them to delay saving for retirement and starting their family. By taking on excess debt, Laura and Ian had made a significant financial mistake.

As Ian and Laura reviewed their decisions, they began to see what they could have done differently. They agreed that a good-quality bed was important, but the new couch was far less important. They made a mistake in logical reasoning by allowing an appeal to emotion to convince them to purchase expensive gifts and a Valentine's Day dinner. The more money spent on gifts did not equal more love from the giver. They also saw that they should have pondered council from church leaders who had provided a warning about debt.

Ian and Laura plan to incorporate what they learned through their mistakes into the Quantitative Reasoning Process to make better decisions in the future.

President Gordon B. Hinckely said: "Debt can be a terrible thing. It is so easy to incur and so difficult to repay. Borrowed money is had only at a price, and that price can be burdensome."³

President Thomas S. Monson counseled: "Avoid the philosophy and excuse that yesterday's luxuries have become today's necessities. They aren't necessities unless we ourselves make them as such. Many of our young couples today... enter into long-term debt...It is essential for us to live within our means"⁴

In this example, Laura and Ian started out on a good path. They used the Quantitative Reasoning Process, they avoided expensive store financing, and they planned for the purchase in their budget. However, Laura and Ian made mistakes both in foresight and in follow-through. They didn't foresee the difficulty they would have making the monthly payment of \$114.64 to their credit card. They didn't follow-through with their budget or avoid unnecessary purchases.

A Gospel Perspective

When we use the Quantitative Reasoning Process with a gospel perspective our decision-making becomes better and more insightful.

As we ponder the problem in Step 1 of the Quantitative Reasoning Process, we should seek guidance from the Lord. Step 5 of the Quantitative Reasoning Process, in particular, provides us with an opportunity to incorporate gospel principles as we ponder, review, and evaluate our decisions.

As we evaluate our reasoning in Step 5 of the Quantitative Reasoning Process, we should pray about our decision and confirm that our decision is consistent with the advice and counsel in the scriptures and from prophets and church leaders.

The case study about Laura and Ian's furniture purchase is an example of failing to follow important advice from our church leaders.

Earlier in the semester, we saw a video of Henry J. Eyring, President of BYU-Idaho, who shared a personal experience where he made a decision to follow advice he heard in conference to get out of debt even though his decision did not align with the best financial council of the day (you can re-watch this video [here](#)). His story is a good example of the blessings that come from following the advice of our church leaders.

President Thomas S. Monson said: "Remember that oftentimes the wisdom of God appears as foolishness to men, but the greatest single lesson we can learn in mortality is that when God speaks and a man obeys, that man will always be right." ⁵

It is also essential to seek personal revelation as we make decisions about our life. President Russell M. Nelson said: "We live in a world that is complex and increasingly contentious. The constant availability of social media and a 24-hour news cycle bombard us with relentless messages. If we are to have any hope of sifting through the myriad of voices and the philosophies of men that attack truth, we must learn to receive revelation." ⁶

In step 5 of the Quantitative Reasoning Process we evaluate our reasoning. This step of the process gives us a chance to seek personal revelation and the guidance of the Holy Ghost in order to ensure that the decision we have made is wise and in keeping with the Lord's will for our lives.

Applying the Quantitative Reasoning Process

Our goal in teaching the Quantitative Reasoning Process is to equip you with a sound way to make better, more informed decisions. As you leave this class we encourage you to think about the decisions you will need to make in the next few years and consider how the Quantitative Reasoning Process might help.

What decisions do you have coming up in the next few years where the Quantitative Reasoning Process will be helpful?

As you make decisions in your life, both big and small, we encourage you to use foresight and to include a gospel perspective in the Quantitative Reasoning Process. Pray for confirmation about your decisions and ensure that they are consistent with the advice of our church leaders. Finally, as you act on your decisions, remember that follow-through and regular reevaluation are vital.

Lesson Checklist

By the end of this lesson, you should be able to:

- state the steps of the Quantitative Reasoning Process.
 - use the quantitative tools taught in this course as part of the Quantitative Reasoning Process.
 - explain the importance of foresight in the Quantitative Reasoning Process.
 - explain the importance of follow-through in the Quantitative Reasoning Process
 - explain the importance of a gospel perspective in the Quantitative Reasoning Process.
 - plan to apply the Quantitative Reasoning Process in your life.
-

¹ [Stay on the High Road \(churchofjesuschrist.org\)](http://churchofjesuschrist.org)

² [The Lord Uses the Unlikely to Accomplish the Impossible \(byui.edu\)](http://byui.edu)

³ [Thou Shalt Not Covet \(churchofjesuschrist.org\)](http://churchofjesuschrist.org)

⁴ [Constant Truths for Changing Times \(churchofjesuschrist.org\)](http://churchofjesuschrist.org)

⁵ [Constant Truths for Changing Times \(churchofjesuschrist.org\)](http://churchofjesuschrist.org)

⁶ [Revelation for the Church, Revelation for Our Lives \(churchofjesuschrist.org\)](http://churchofjesuschrist.org)



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https://books.byui.edu/math_108x/lesson_13_applying_the_quantitative_reasoning_process.